I. <u>The quantum mouse (Part II)</u>

Remember the SETUP: Consider a quantum object (a "quantum lab mouse") and some new properties we can measure:

- "quantum weight", **W**, is a Hermitian operator.
  The eigenvalues are either 1 (skinny mice) or 10 (heavy ones), but nothing else (!)

  W|> = |> , W|> = 10|>

  (We used as icons for skinny and heavy mice|> and |> respectively)
  Being skinny or heavy is orthonormal (and complete).

- "quantum happiness", **H**, is also a Hermitian operator.
  The eigenvalues are +1, or -1

  $\hat{H}$|☺> = |☺> , but

  $\hat{H}$|☹> = -|☹>

  Being happy or sad is orthonormal, and complete)

Let's first get used to this notation again.

Use orthonormality and completeness to expand the H=+1 eigenstate in the "weight basis":

Now, let **P** be the "feed a quantum pizza" operator. This curious operator does the following:

$$\hat{\mathbf{P}}\,|\text{—}\rangle = |\bigcirc\rangle \quad \text{, but}$$

$$\hat{\mathbf{P}}\,|\bigcirc\rangle = 0$$

This means pizza make a skinny mouse heavy, but *kills* the heavy mice. (it gives back 0, nothing)

Are skinny mice eigenstates of **P**? Are heavy mice? Explain.

Show that **P** is NOT a Hermitian operator

Can you observe mice eating pizza in this quantum world? Explain.

What would $\mathbf{P}^{\dagger}$ do to a heavy mouse? What would it do to a skinny mouse?
This is tricky, work it out!

Based on the above, in the spirit of this Tutorial, make up a plausible name for the $\mathbf{P}^{\dagger}$ operator.

Simultaneous measurements:

Do **H** and **W** commute?
*(Again, tricky. If they DO commute, then **HW** gives the same result as **WH** for any state. A single counterexample will disprove it! Consider operating on, say, a skinny state…)*

Based on the above, does measuring the quantum weight of a mouse affect the outcome of a future measurement of its quantum happiness? (Vice versa?)

Given a mouse in the +1 happiness state, if you then measure weight, what is the probability that you will measure 1? (Colloquially, we might phrase this "what is the probability that a happy mouse is skinny?")

Time evolution

Now suppose that the mouse Hamiltonian commutes with W (One way to think of this would be if the *energy* of a mouse is proportional to its measured weight)

Suppose, I give you a mouse which I have carefully prepared. I know what state it is in, but I don't tell you. It might be in a superposition of skinny and heavy states!

You can measure weight at any time you like. Does the probability that your measurement will yield "10" (i.e. heavy) depend on the amount of time that you wait before measuring? Explain.

Suppose instead that you chose NOT to measure W but instead to measure H. Does the probability that your measurement will yield "+1" (i.e, that it's "happy) depend on the amount of time that you wait before measuring? Explain.

(*You might consider the simple case that I handed you a particle prepared in a happy state at t=0.)*