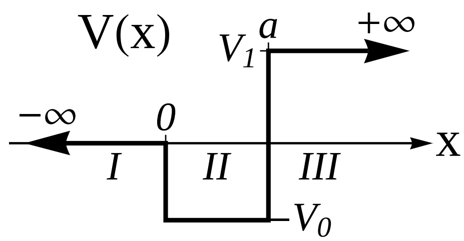
I: An asymmetric potential well

The figure at the right shows a graph of the potential energy for a one-dimensional system. $V(x) = 0$ for $x < 0$ (region I), $V(x) = -V_0$ for $0 \leq x < a$ (region II), and $V(x) = +V_1$ for $x \geq a$ (region III).



A. What are the units of V_0 and V_1? Why?

B. Write down the solution to the Schrödinger equation for regions I, II, and III.

C. Describe in words the boundary conditions for this system for a situation which describes particles with energy $0 < E < V_1$ and which approach from the left.

How would the boundary conditions change for $E > V_1$?

D. Write down an equation for each boundary condition.

E. Consider the transmission coefficient, (T) for this system (i.e., the probability that a particle entering from $-\infty$ will travel into region III). Using physical arguments, but without carrying out calculations, what can you say qualitatively about T?

F. How would you compute the transmission coefficient mathematically? Can you use mathematical arguments to make a specific statement about the transmission coefficient?

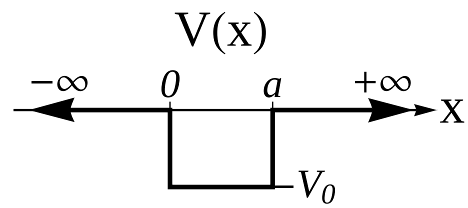
G. What do you think the transmission coefficient would be if the total energy of the particle were large ($E \gg V_1$)?

H. What do you think the transmission coefficient would be if the total energy of the particle was just barely larger than V_1?

✓ **Discuss your answers with a tutorial instructor before continuing.**

II: A symmetric potential well

The figure at the right shows a graph of the potential energy for a one-dimensional system. $V(x) = 0$ for $x < 0$, $V(x) = -V_0$ for $0 \leq x \leq a$, and $V(x) = 0$ for $x > a$.



A. Write down the solution to the Schrödinger equation for all space for a particle with total energy, E, greater than zero.

B. Describe in words the boundary conditions for this system for a situation which describes particles which approach from the right.

C. Consider the transmission coefficient, (T) for this system (i.e., the probability that a particle entering from $+\infty$ will travel into region $x < 0$). Using physical arguments, but without carrying out calculations, what can you say qualitatively about T? For instance, this potential can be thought of as the combination of a downstep followed by an upstep.

D. Obtain an equation sheet from your tutorial instructor. Sketch the transmission coefficient (be sure to label the axes, try for accuracy, and label any “interesting” points). Think about ways to rewrite the equation to make it easier to sketch.

E. Does this sketch match your estimate? Can you try to come up with any physical ideas which might explain the difference?

Instructors:
For I.F, make sure they are starting from the correct definition for transmission using a ratio of probability current densities:

$$\vec{j}(\vec{r},t) = \frac{\hbar}{2m\imath}\left[\Psi^*(\vec{r},t)\vec{\nabla}\Psi(\vec{r},t) - \Psi(\vec{r},t)\vec{\nabla}\Psi^*(\vec{r},t)\right]$$

$$T \equiv \vec{j}_{out}/\vec{j}_{in}$$

Here is the equation for the transmission coefficient for problem II

$$T = \frac{4E(E+V_0)}{4E(E+V_0) + V_0^2\sin^2(k'a)}$$

$$k' = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

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