What are they learning in quantum mechanics? A conceptual post test for Quantum I

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Outline

- Why transform upper-division quantum mechanics?
- Quantum mechanics course transformation at CU
- What do we want students to learn in upper-division QM?
- What *are* they really learning?
- Why you should use the QMAT

Acknowledgements
Co-authors and the CU PER group:

Faculty

Paul Beale	Michael Dubson	Noah Finkelstein
Susan Jurow	Ben Kirshner	Valerie Otero
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NSF

SEI The Science Education Initiative at the University of Colorado

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- When the Hamiltonian acts on a state it is a mathematical representation of making a measurement of the energy. (Gire and Manogue, 2008)
- Time development of quantum states (even stationary states) involves diffusion and/or dissipation. (Crouse, 2007)

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- Eighteen faculty members contributed their time and expertise to this process.

Thank You Faculty!

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Chris Greene	Anna Hasenfratz	Ed Kinney
K.T. Mahanthappa	Uriel Nauenberg	John Price
Chuck Rogers	Kevin Stenson	Eric Zimmerman

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What *are* they learning?

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  - Act as a tool to help guide faculty efforts at improving QM instruction.
- The QMAT consists of 14, mostly free-response questions containing 28 separate question items.

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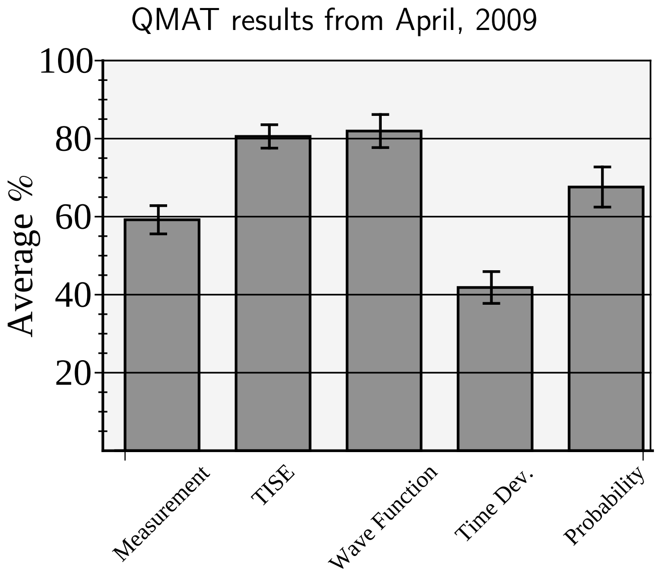
- Revised version given in April, 2009. (N = 36)

Learning Part 3: QMAT Results



Learning Part 4: QMAT Measurement Problem†

A particle in an infinite square well starts in a state given by

$$\Psi(x, t=0) = \left(\sqrt{\frac{4}{5}} u_1(x) + \sqrt{\frac{1}{5}} u_2(x) \right) .$$

a) You make an energy measurement on this system and find the maximum possible value for the energy. **What is the state, ψ(x), of the system after this measurement?**

†Developed from Crouse (2007) and Singh (2008).

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"u_1 because it has the highest probability"

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b) After the energy measurement, you make a position measurement. After this position measurement, you immediately re-measure the energy. **At this point, what value(s) could you get for energy?**

31% correct, (35% of the students who correctly answered part (a)), (Any of the energy eigenvalues is possible).

42% of all students said that the choices were E_1 or E_2.
19% said or implied that the position measurement would not alter the energy.

Summary and Discussion

Why you should use the QMAT

The QMAT as an assessment tool

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- Exposes areas of common student difficulties (e.g., measurement and time development in QM)
- Raises faculty awareness, and guides future reform efforts

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- Students also frequently respond as though sequential measurements on a quantum state retain all original information encoded in the starting state, again consistent with literature. (Crouse, 2007)

Use the QMAT in your Class!

- The full set of learning goals, assessments, and other course materials, are available at http://www.colorado.edu/sei/departments/physics_3220.htm
- If you are interested, contact Steve Goldhaber at Steven.Goldhaber@colorado.edu or sign up on the sheet.

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Thank you – questions?

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"when we use $\hat{H}$, it's to solve the TISE, so $\hat{H}$ doesn't really tell us about time."

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"$\hat{H} = \frac{\hat{p}^2}{2m} + V$ this has no time dependence."

References

D. F. Styer, *Am. J. Phys.* **64**, 31–34 (1996).

C. Singh, *Am. J. Phys.* **69**, 885–896 (2001).

C. Singh *2004 PERC Proc*, AIP, 2005, vol. 790, pp. 23–26.

E. Gire, and C. Manogue *2008 PERC Proc*, AIP Press, Melville, NY, 2008, pp. 115–118.

A. D. Crouse, *Research on Student Understanding of Quantum Mechanics as a Guide for Improving Instruction*, Ph.D. thesis, University of Washington (2007).

P. Shaffer, Private communication (2008).

C. Singh, *Am. J. Phys.* **76**, 277–287 (2008).

E. Cataloglu, and R. W. Robinett, *Am. J. Phys.* **70**, 238–251 (2002).

C. Singh *2006 PERC Proc*, AIP, 2007, vol. 883, pp. 185–188.