$\boxed{\text{Radial Probability Density}}$

$$\int dV\, |\Psi|^2 = 1$$

Prob(find part. in dV about $\vec{r}$) $= |\Psi(\vec{r})|^2 dV$

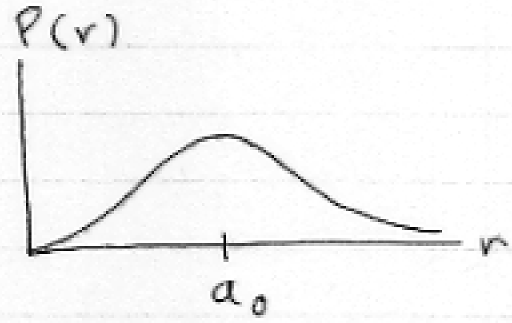
If $l = 0$, $\Psi = \Psi(r)$ then $\int dV |\Psi|^2 = \int dr\, 4\pi r^2 |\Psi(r)|^2$

Prob(find in $r \to r + dr$) $= \boxed{P(r)\,dr = 4\pi r^2 |\Psi(r)|^2 dr}$

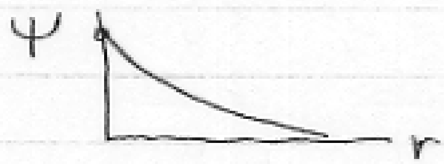
$P(r)$ = radial probability density

Ground state: $\Psi_{100} = A\, e^{-r/a_0}$

$$P(r) = |A|^2 4\pi r^2 e^{-2r/a_0}$$



Notice $P(r)$ very different from $\Psi(r)$:



If $l \neq 0$, $\Psi = \Psi(r, \theta, \varphi) = R(r)\, Y(\theta, \varphi)$, then

"solid angle"

$$\int dV |\Psi|^2 = \underbrace{\int dr\, r^2 |R|^2}_{1} \underbrace{\int d\Omega\, |Y|^2}_{1} = 1$$

Prob(find in $r \to r + dr$) $= r^2 |R|^2 dr$

$\boxed{P(r) = r^2 |R|^2}$ even if $l \neq 0$

Note: if $\Psi = \Psi(r) = R \cdot Y = R \cdot \frac{1}{\sqrt{4\pi}} \Rightarrow |R|^2 = 4\pi |\Psi|^2$

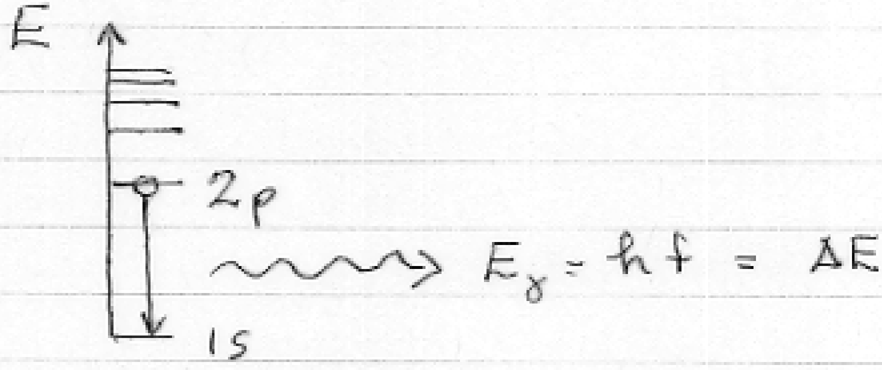
so $P(r) = r^2 |R|^2 = 4\pi r^2 |\Psi|^2$

H-atom and emission/absorption of radiation:

If H-atom is in excited state ($n=2$, $l=1$, $m=0$) then it is in energy eigenstate = stationary state. If atom is isolated, then atom should remain in state ψ_{210} forever, since stationary state has simple time dependence:

$$\Psi(\vec{r},t) = \psi_{210}(r) \cdot e^{-iE_2 t/\hbar}$$

But, experimentally, we find that H-atom emits photon and de-excites: $\psi_{210} \rightarrow \psi_{100}$ in $\approx 10^{-7}$ s
$\rightarrow 10^{-9}$ s



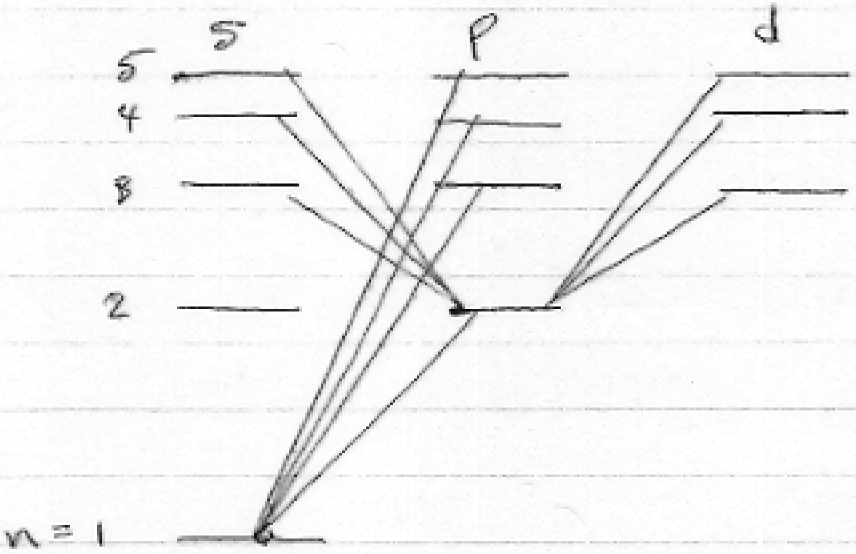
The reason that the atom does not remain in stationary state is that it is not truly isolated. The atom feels a fluctuating EM field due to "vacuum fluctuations". Quantum Electrodynamics is a relativistic theory of the QM interaction of matter and light. It predicts that the "vacuum" is not "empty" or "nothing" as previously supposed, but is instead a seething foam of virtual photons and other particles. These vacuum fluctuations interact w/ the electron in the H-atom and slightly alter the potential $V(r)$. So eigenstates of the coulomb potential are not eigenstate of the actual potential:
$V_{coulomb} + V_{vacuum}$

Photons possess an intrinsic angular momentum (spin) of $1\hbar$, meaning $l=1 \Rightarrow |\vec{L}| = \hbar\sqrt{l(l+1)} = \sqrt{2}\hbar$ and $L_{z_{Max}} = \hbar$

So when an atom absorbs or emits a single photon, its angular momentum must change by $1\hbar$, by Conservation of Angular Momentum, so the orbital angular momentum quantum nbr l must change by 1.

"Selection Rule": $\Delta l = \pm 1$ in any process involving emission or absorption of 1 photon

$\Rightarrow$ allowed transitions are:



If an H-atom is in state 2s ($n=2$, $l=0$) then it cannot de-excite to ground state by emission of a photon. (since this would violate the selection rule) It can only lose its energy (de-excite) by collision with another atom or via a rare 2-photon process