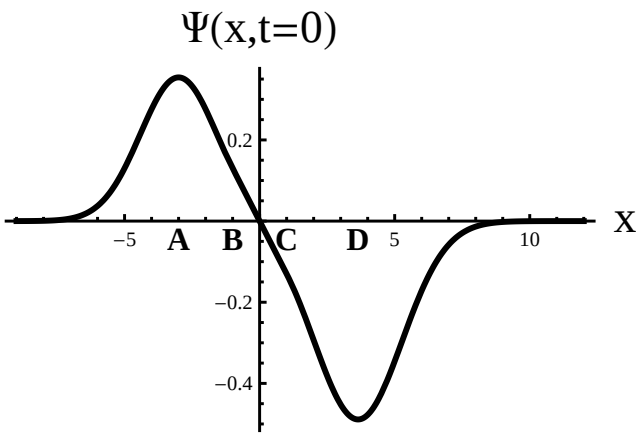
I: Localized wave functions

A. At time, $t = 0$, a free particle in one dimension has the wave function shown at right:



1. How do the probabilities of finding the particle very close (within a very small distance dx) to x = A, B, C, and D compare?

2. What can you say about the integral of this function from $-\infty$ to ∞?

3. What can you say about the integral of $|\Psi(x,t)|^2$ from $-\infty$ to ∞?

4. The wave function shown above is entirely real. Is this a possible wave function for a real, physical particle? Why or why not?

5. Does this wave function represent a localized particle? Describe how you know.

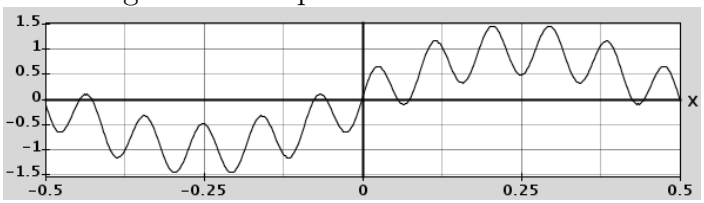
✓ Check your results with a tutorial instructor.

B. Superposition:

Open the 'Fourier: Making Waves' PhET simulation (http://www.colorado.edu/physics/phet/simulations/fourier/fourier.jnlp). You will be working with this simulation to explore how sine and cosine waves add up to make wave packets and other functions.

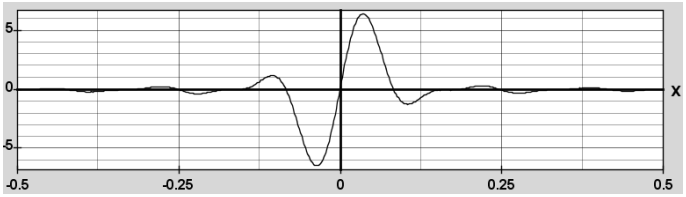
1. How does changing the "Amplitudes" graph change the "Harmonics" and "Sum" graphs? Explain how these three graphs are related.

2. Predict how you could use the sim to produce the following "Sum" graph, then test your prediction. Does the sign of the amplitudes matter?



3. Does this represent a localized particle? Why or why not?

4. Can you reproduce the following "Sum" graph? Write down the amplitudes you chose for your closest fit.



A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11

5. Do you think this represents a localized particle? Write down what you think and why. Then test out your prediction using the horizontal axis controls on the simulation.

✓ Check your results with a tutorial instructor.

C. Next, select the 'Discrete to Continuous' tab in the PhET sim:

1. Does the "Sum" graph represent a wave? A localized particle? Discuss why or why not to each of these questions.

2. How would you interpret the length λ_1 in this graph? What is the source of this length?

3. Predict what will happen if you change the value of the "Spacing between Fourier components" slider control to be 1/2 of its current value.

4. Test your prediction and resolve any discrepancies.

5. Predict what will happen if you change the value of the "Wave packet center" slider control to 10π.

6. Test your prediction and resolve any discrepancies.

✓ Check your results with a tutorial instructor.

D. Continuous distributions:

Reset the "Discrete to Continuous" simulation by hitting the "Reset all" button.

1. Predict what will happen if you change the value of the "Spacing between Fourier components" slider control to be zero.

2. Test your prediction and resolve any discrepancies.

3. Explain the relationship between the mathematical formula at the top of the "Sum" graph and the amplitudes graph. Justify the change in the formula observed when you tested your prediction above.

4. Does the "Sum" graph represent a wave? A localized particle? Discuss why or why not to each of these questions. Also, interpret the meaning of the value of λ_1.

5. Use the "Amplitudes" graph as the representation of the momentum distribution of a quantum wave packet, $\phi(k)$, and write down a plausible formula for this function (hint: it is a Gaussian). Checking the "width indicators" box may help here.

6. What is the expectation value of the momentum for this system?

7. Vary the "Wave packet width" slider controls and notice the changes in the "Amplitudes" and "Sum" graphs. Why can't you manipulate them independently?

8. Vary the value of the "Wave packet center" slider control. Come up with a physical explanation for the resulting changes in the "Sum" graph.

✓ Check your results with a tutorial instructor.