University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#10
due Wed, Oct 28, 2PM at start of class

1. Symmetric potentials (Griffiths, Problem 2.1(c), Total: 10 pts)
   Prove the following statement: If $V(x)$ is an even function (that is, $V(-x) = V(x)$), then the stationary state wave functions $\chi(x)$ can always be taken to be either even or odd.

2. Stationary states in an infinite square well (Total: 20 pts)
   Consider a particle of mass m moving freely between $x = 0$ and $x = a$ inside an infinite square well potential. The normalized stationary states (eigenstates) were found to be

$$\chi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

   with eigenenergies

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

   a) Calculate the expectation values $< x >$, $< p >$, $< x^2 >$ and $< p^2 >$ for the n-th stationary state. Explain the physical meaning of your results for $< x >$ and $< p >$.
   b Calculate the standard deviations $\sigma_x = \sqrt{< x^2 > - < x >^2}$ and $\sigma_p = \sqrt{< p^2 > - < p >^2}$ for the n-th stationary state. One grows more rapidly than the other as n increases. Can you make physical sense of why they behave differently? (Hint: Think about the values that x and p may take.)
   c) Calculate the uncertainty product $\Delta x \Delta p = \sigma_x \sigma_p$ for the n-th stationary state. Does the Heisenberg Uncertainty relation hold?
   d) Use the result of c) to estimate the zero-point energy.

3. Non-stationary states in an infinite square well (Total: 10 pts)
   Using the same conventions for the stationary states of an infinite square well as in the previous problem, the state of a particle of mass m is given by the wave function

$$\Psi(x, t = 0) = A\left(\chi_2(x) + \chi_3(x)\right)$$

   If you measured the energy of this particle at time t, what are the values you might get and what are the probabilities of each of them? Find the expectation value $< E >$ in terms of m and a.

4. An expanding infinite square well (Total: 10 pts)
   A particle of mass m is moving freely inside a infinite square well with walls at $x = 0$ and $x = a$ for $t < 0$. The particle is initially in the ground state of the square well, and we then at $t = 0$ suddenly quadruple the size of the well (i.e. the right hand side of the well is moved instantaneously from $x = a$ to $x = 4a$) leaving the wave function undisturbed. Now the energy of the particle is measured. Calculate the probability that the zero-point energy of the new well is measured.

5. Finite square well (Total: 10 pts)

   a) Determine the energies of the stationary states in a finite square well for the limiting case $V_0 \to \infty$.

   b) Derive an approximate simple expression for the number of bound states in a finite square well, in terms of the width of the well and the depth of the well. Your expression does *not* have to be exact, in fact no long or messy calculations are permitted, just make a quick "intelligent crude guess" (Hint: Think about the energies in an infinite square well!).