Formula Sheet for Exam 1 (These formulas will be given.)

(I will not give the de Broglie relations, or the definitions of the wave number k and the angular frequency ω . I expect you to memorize those.)

The classical wave equation: $\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f(x,t)}{\partial t^2}$

The time-independent Schrödinger Equation:

$$\boxed{i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi}$$

The standard deviation $\sigma = \sqrt{\left\langle (\Delta x)^2 \right\rangle} = \sqrt{\left\langle \left(x - \langle x \rangle\right)^2 \right\rangle} = \sqrt{\left\langle x^2 \right\rangle - \langle x \rangle^2}$

The momentum operator is $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$

Fourier Transform formulae (Plancherel's Theorem) :

$$f(x) = \tfrac{1}{\sqrt{2\pi}}\int dk\ F(k)\exp(i k x)$$

$$F(k) = \tfrac{1}{\sqrt{2\pi}}\int dx\ f(x)\exp(-i k x)$$

A useful form of the delta function: $\delta(x) = \tfrac{1}{2\pi}\int \exp(i k x)\,dk$

The probability current:

$$J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi\frac{\partial \Psi^*}{\partial x} - \Psi^*\frac{\partial \Psi}{\partial x}\right) = \frac{\hbar}{m}\,\mathrm{Im}\left(\Psi^*\frac{\partial \Psi}{\partial x}\right) = \frac{1}{m}\,\mathrm{Re}\left(\Psi^*\hat{p}\Psi\right)$$

Exam 1 Review Topics: **Ch 1 and Ch 2 in Griffiths, Homeworks 1 thru 5, and Lecture Notes up thru/including "Dirac Delta function"**

- probability density and the wavefunction
- normalization of the wavefunction
- computation of expectation values and standard deviation, given a wavefunction

Classical wave equation, the form of a traveling wave : $f(x,t) = f(x - v\,t)$, and superposition of solutions due to linearity of the equation.

- Complex variables: polar form vs. cartesian form; complex conjugation; modulus (amplitude) of a complex variable
- Separation of variables.
- Deriving the TISE from the TDSE, starting with separation of variables
- Relation between solutions to the TISE [$\hat{H}\psi_n(x) = E_n\psi_n(x)$] and solutions to the TDSE [$\Psi(x,t) = \sum_n c_n \exp[-iE_n t/\hbar]\psi_n(x)$.] where expansion coefficients c_n gotten from "Fourier's Trick"
- Solutions of the infinite square well.
- Qualitative solutions to the TISE: Sketching the solutions $\psi_n(x)$, given $V(x)$
- Energy eigenvalue equation and properties of the energy eigenfunctions (they form a complete, orthonormal set)
- Free particles: plane-waves states $\Psi(x,t) = A\, e^{i(kx-\omega t)}$ and wave packets; phase velocity $v_{phase} = \omega / k$, vs. group velocity $v_{group} = d\omega / dk$; relation between free particle wavefunction $\Psi(x,t)$ and the Fourier transform $\phi(x)$.
- Procedure for solving the TISE for specific potentials (such a finite square well or scattering from a step): 1) Write general solutions with unknown constants. 2) Apply boundary conditions to determine the constants.
- Tunneling depth
- Reflection and transmission coefficients and relation to the probability current.