Complex Number Review

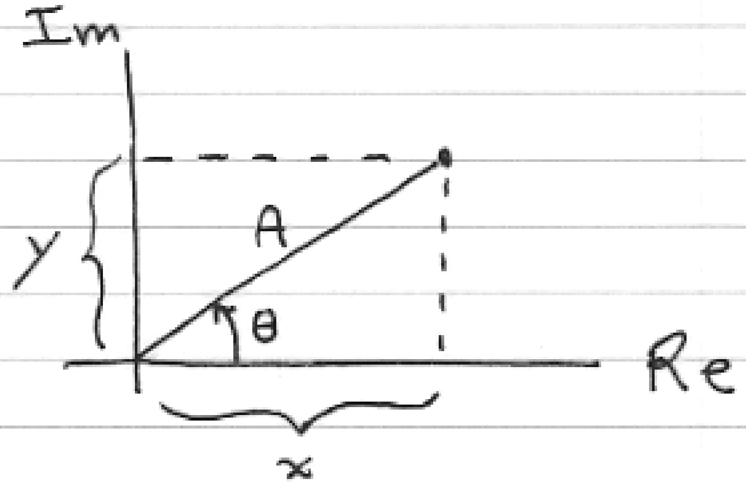
$i = \sqrt{-1}$, $i \cdot i = -1 \Rightarrow i = \frac{-1}{i} \Rightarrow \frac{1}{i} = -i$

Any complex number z can be written in

cartesian form: $z = x + iy$ <u>or</u>

polar form: $z = A e^{i\theta}$

Complex plane:



Euler's relation:

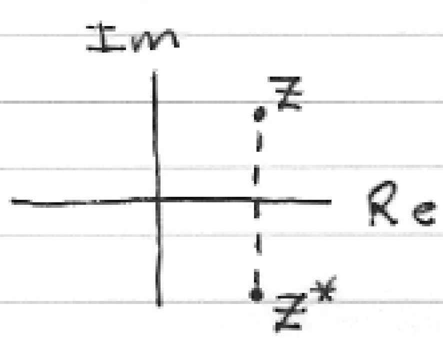
$$\boxed{e^{i\theta} = \cos\theta + i \sin\theta}$$

⇐ can be proven w/ Taylor Series expansion

$\Rightarrow \mathrm{Re}[z] = x = A\cos\theta$

$\mathrm{Im}[z] = y = A\sin\theta$

Complex conjugate of $z = z^* = x - iy$



$z^* = A e^{-i\theta}$

$$z \cdot z^* = (x+iy)(x-iy) = x^2 + ixy - ixy + y^2$$

$$= x^2 + y^2 = \text{pure real}$$

$|z|$ = "modulus" of z or "amplitude" of z

$$= \sqrt{x^2+y^2} = A$$

$$z \cdot z^* = |z|^2$$

Notice $z^2 = z \cdot z \neq |z|^2 = z \cdot z^*$

Useful fact: $\boxed{e^{z_1+z_2} = e^{z_1} \cdot e^{z_2}}$

(z_1, z_2 any 2 complex nbrs)

$\Rightarrow \quad e^{i(\alpha+\beta)} = e^{i\alpha} \cdot e^{i\beta}$ ⟵ can be used to derive trig identities

$\Rightarrow$ if $z_1 = A_1 e^{i\theta_1}$, $z_2 = A_2 e^{i\theta_2}$

then product $z_1 \cdot z_2 = A_1 A_2 e^{i(\theta_1+\theta_2)}$

Useful fact: Any complex number z, written as a complicated expression, no matter how messy, can be turned into complex conjugate z^* by replacing every i with $-i$

$$z = \frac{(5+6i)(-7i)}{2i + 3e^{-i\theta}} \Rightarrow z^* = \frac{(5-6i)(+7i)}{-2i+3e^{+i\theta}}$$