University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#8
due Wed, Oct 15, 2PM at start of class

1. Minimum energy (Griffiths, Problem 2.2, Total: 10 pts)
   Show that E must exceed the minimum value of $V(x)$, for every normalizable solution to the time-independent Schrödinger equation.

2. An infinitely high potential step (Total: 10pts)
   Examine the limiting case of an infinitely high potential step, i.e. $V_0 \to \infty$ (the energy E kept constant).

   a) Find the wave function in the limiting case.

   b) Are the wave function and its first derivative continuous at the boundary $x = 0$? Explain your result.

3. Some quantitative problems (Total: 10pts)

   a) An electron with a kinetic energy of 10 eV at large negative values of x is moving from left to right along the x-axis. The potential energy is given by:

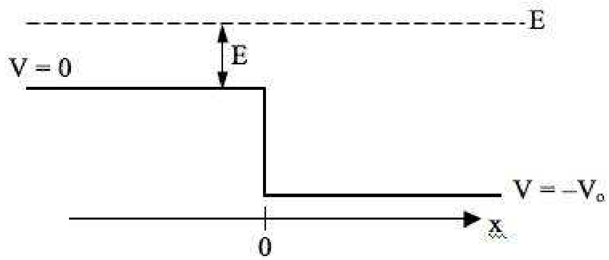
   $$V(x) = \begin{cases} 0 & x < 0 \\ 20\text{eV}, & x \geq 0 \end{cases}$$

   Estimate the order of magnitude of the distance the electron can penetrate the barrier.

   b) Repeat part a) for a 70 kg person initially moving at 4 ms^{-1} and running into a wall which can be represented by a potential step of height equal to four times this person's kinetic energy before reaching the step.

4. Reflection off a downstep potential (Total: 30pts)
Consider a downstep potential with

$$V(x) = \begin{cases} -V_0 & x > 0 \\ 0, & x < 0 \end{cases}$$



a) In classical physics, consider a particle of mass m coming in from the left with initial velocity v_i. What happens to it at the downstep? Find the total energy in terms of v_i and the final velocity v_f in terms of m, v_i and V_0.

b) In quantum physics, solve the time-independent Schrödinger equation for fixed energy $E > 0$ in both regions and impose appropriate boundary conditions at $x = 0$; write down the equations for these boundary conditions. Don't assume anything about where particles are coming from yet.

c) Assume that there are no particles coming in from the right. Sketch $|\chi(x)|^2$ as a function of x.

d) Interpret the solutions as a steady flux of particles. Calculate the reflection coefficient R and transmission coefficient T and write them in terms of E and V_0. Does this satisfy $R + T = 1$? Discuss how your answer compares to the classical result.

e) Sketch R vs. V_0 at fixed E. (Choose, say, $E = 1$.)