University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#9
due Wed, Oct 22, 2PM at start of class

1. Potential barrier (Total: 20 pts)
For the potential with a barrier of height V_0

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0 & 0 < x < a \\ 0, & a < x \end{cases}$$

the transmission coefficient for $0 < E < V_0$ is given by

$$T = \frac{4\kappa^2 k^2}{(k^2 + \kappa^2)^2 \sinh^2(\kappa a) + 4\kappa^2 k^2}.$$

(Note: It is not part of the problem to derive this formula using the boundary conditions discussed in class. But, you may want to check it.)

a) Demonstrate that this expression can be rewritten as:

$$T^{-1} = 1 + \frac{1}{4(E/V_0)(1 - E/V_0)} \sinh^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 - E)}\right)$$

b) Consider an electron approaching the barrier. Its initial energy is 0.5 eV and the barrier height is 1 eV, while the width of the barrier is 5×10^{-10} m. What is the numerical probability for the particle to make it to the other side of the barrier?
(Hint: You can (but do not have to) study this tunneling problem by going to phet.colorado.edu and running the "Quantum Tunneling and Wave Packets" sim. Make sure you are using plane waves, not wave packets, and program the sim to calculate for you. Does the result agree with your calculation? You may also switch the PhET sim to wave packets and watch the wave packet with the same parameters evolve. Does the sim give you the same probability? What if you change the initial width of the wave packet?)

c) Consider the same system as a model of a baseball being thrown at a wall. A baseball has a mass of about 150 g, and we take it to be thrown at 40 m/s (near 90 mph). Assume that the wall is 0.1 m thick, and let's make the approximation that the ball would have to be 5 times as energetic to punch through the wall classically, so $V_0 = 5E$ with E determined by the quantities above.
What is the order of magnitude for T? Let's put this into perspective: if you keep trying, tossing a baseball at the wall once per second, roughly how long do you have to wait until it "pops through" the wall quantum mechanically? Give your answer in seconds, and also in ages of the Universe (current models show the Universe to be about 13.7 billion years old), and comment on your results.

2. Scattering of a finite well (Total: 20pts)
   Next, consider the case of a particle beam which comes in from infinity with $E > 0$ and scatter of the potential

$$V(x) = \begin{cases} 0, & x < 0 \\ -V_0 & 0 < x < a \\ 0, & a < x \end{cases}$$

   with $V_0 > 0$ (note the minus sign in front of V_0 in the formula).

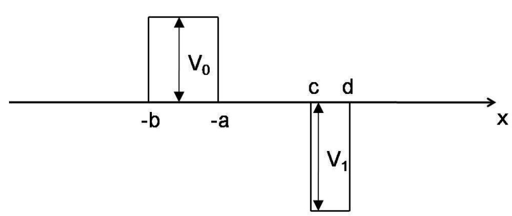
   a) Find the transmission coefficient by adapting the result of part a) of the previous problem. To do this, turn the barrier upside down by replacing V_0 with $-V_0$ in that formula, and obtain an expression for T^{-1} where everything is real. It may help to know that $\sinh(ix) = i \sin x$.

   b) Consider the case where $E = V_0$. Determine T at three different values of the parameters: $V_0 \to 0$, $a^2 m V_0/\hbar^2 = \pi^2/16$, $a^2 m V_0/\hbar^2 = \pi^2/4$. How does this compare to your classical expectation?

   c) Assume general values for E again. Keeping V_0 fixed, what happens in the limits $E \to 0$ and $E \to \infty$?

3. Combinations of two potentials (Total: 20pts)
   Write down the form of the wave function in each region that represents the following two physical systems, give the wave numbers in your general solutions as functions of E, V_0 and V_1, and justify any terms you set to zero. Write down the appropriate boundary conditions using the wave functions you have determined. You do *not* have to solve for the parameters.

   a) First, consider a combination of a potential barrier and a potential well. A particle beam with $0 < E < V_0$ is incident from the *left*:



   b) Next, consider a combination of a delta-function barrier and a potential barrier. A particle beam with $0 < E < V_0$ is incident from the *right*: