University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#14
due Wed, Dec 9, 2PM at start of class

1. Survey (Total: 20 pts)
   Please take the following survey under

   http://www.colorado.edu/sei/surveys/Fall09/Clicker_Phys3220_fa09-post.html

   Note: Cutting and pasting the link may not work, so type it or go to the link at the web page of the course.

2. Superposition of orbital angular momentum eigenstates (Total: 20 pts)
   Consider a system which is initially in the state

   $$\Psi(\theta,\phi) = \frac{1}{\sqrt{5}} Y_{1,-1}(\theta,\phi) + \sqrt{\frac{3}{5}} Y_{1,0}(\theta,\phi) + A Y_{1,1}(\theta,\phi)$$

   where A is a *real* number.

   a) Find A such that the state is normalized. Is your answer unique? Explain.

   b) Find $< \Psi|\hat{L}_+|\Psi >$. (Hint: We have shown in class, that $\hat{L}_\pm Y_{l,m} \propto Y_{l,m\pm1}$. For the problem you need the proportionality factor. This is $\hat{L}_\pm Y_{l,m} = C^\pm_{l,m} Y_{l,m\pm1}$ with $C^\pm_{l,m} = \hbar\sqrt{l(l+1) - m(m\pm1)}$.)

   c) If $\hat{L}_z$ is measured what values will one obtain and with what probabilities? What is the expectation value of $\hat{L}_z$?

   d) Assume that we measure $\hat{L}_z$ and the result of the measurement is $-\hbar$. What is the state of the system *after* the measurement? Determine the product $\Delta L_x \Delta L_y$ *after* the measurement.

3. And the final problem (Total: 20 pts)

   a) Work out the following commutators:

   $$[\hat{L}_z, \hat{p}_x] = i\hbar\hat{p}_y, \qquad [\hat{L}_z, \hat{p}_y] = -i\hbar\hat{p}_x, \qquad [\hat{L}_z, \hat{p}_z] = 0,$$

   b) Use the results obtained in part a) to evaluate the commutator $[\hat{L}_z, \hat{p}^2]$.

   c) Use that $[\hat{L}_x, \hat{p}^2] = [\hat{L}_y, \hat{p}^2] = [\hat{L}_z, \hat{p}^2]$ (you do *not* have to show these relations) and the result obtained in part b) to show that the Hamiltonian $H = (p^2/2m) + V$ commutes with $\hat{L}^2$ and $\hat{L}_z$, provided that $V = V(r)$ depends only on r.