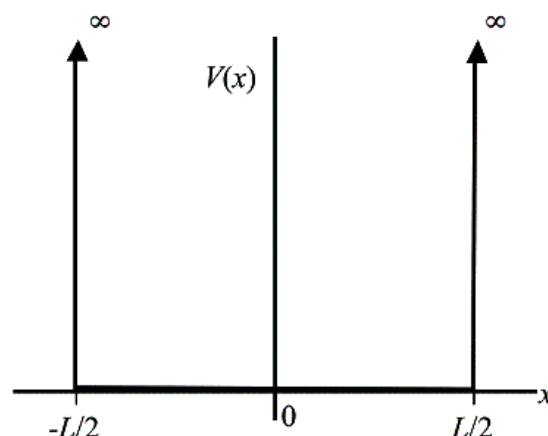


I: Sketching wave functions

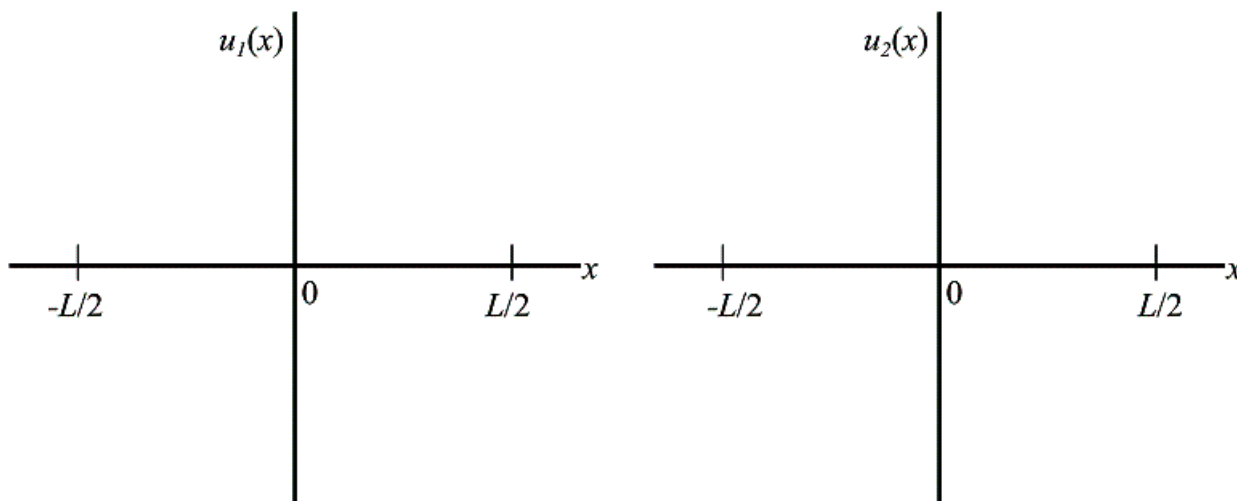
A. Review:

The figure to the right shows an infinite square well potential ($V = 0$ from $-L/2$ to $L/2$ and is infinite everywhere else).



1. Write down the formula for the energies of the first two energy eigenstates, E_1^a and E_2^a .

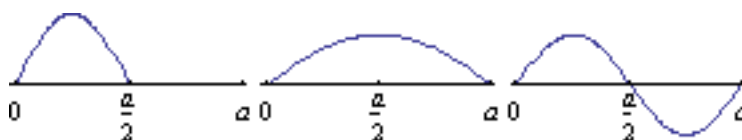
2. On the graphs below, sketch the ground state (u_1 , the lowest energy eigenstate) and first excited state (u_2) for a particle of mass m in this potential.



B. Energetic curves:

The figure below shows normalized energy eigenstate wave functions for three identical particles in (possibly different) infinite square wells. The drawings are to scale with each other.

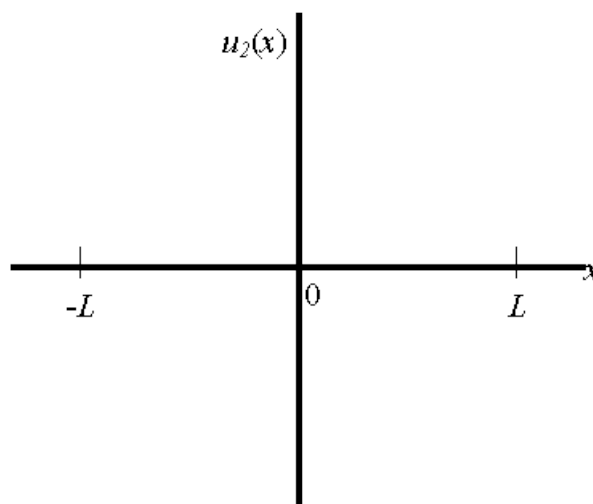
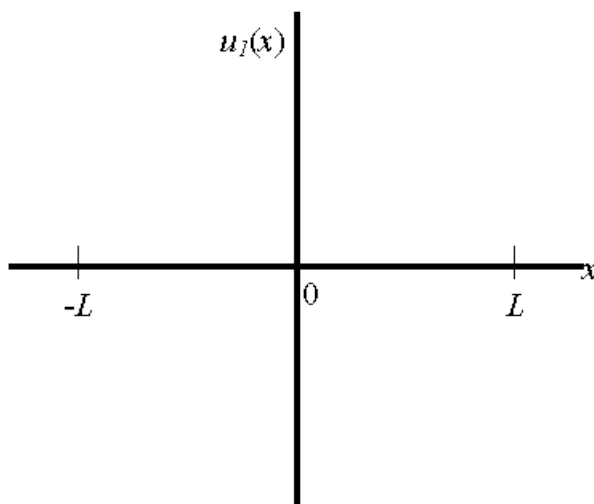
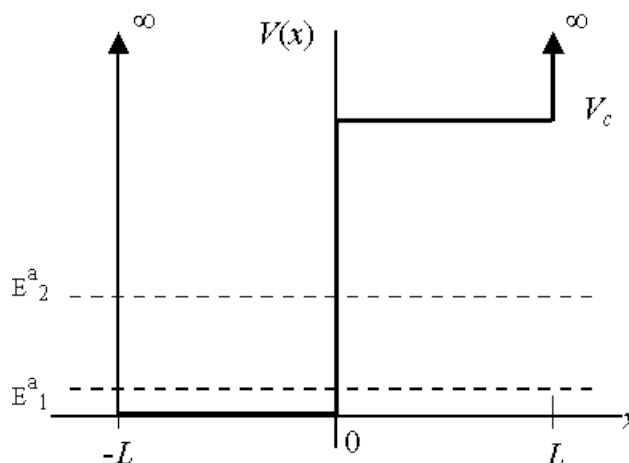
1. Rank the functions from lowest energy (1) to highest energy (2, 3). If you think that two of the functions have the same energy, be sure to indicate the tie.
2. What feature(s) are important in making this determination?



C. Stepped Potential:

Now, consider a slightly more complicated potential, shown on the right. Let's assume that $V_c \gg E_2^a$, i.e., that "step up" in the middle is a *big* step.

1. On the graphs below, sketch the ground state and first excited state for the same particle in this potential. Make sure you use the Schrödinger equation to guide your work. What is the effect of the potential step?



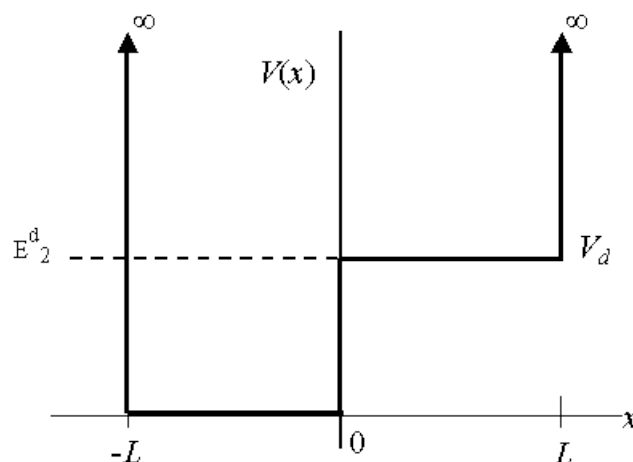
2. Discuss the differences between these sketches and the ones for the infinite square well. How do the new energies compare to E_1^a and E_2^a ?

✓ Check your results with a tutorial instructor.

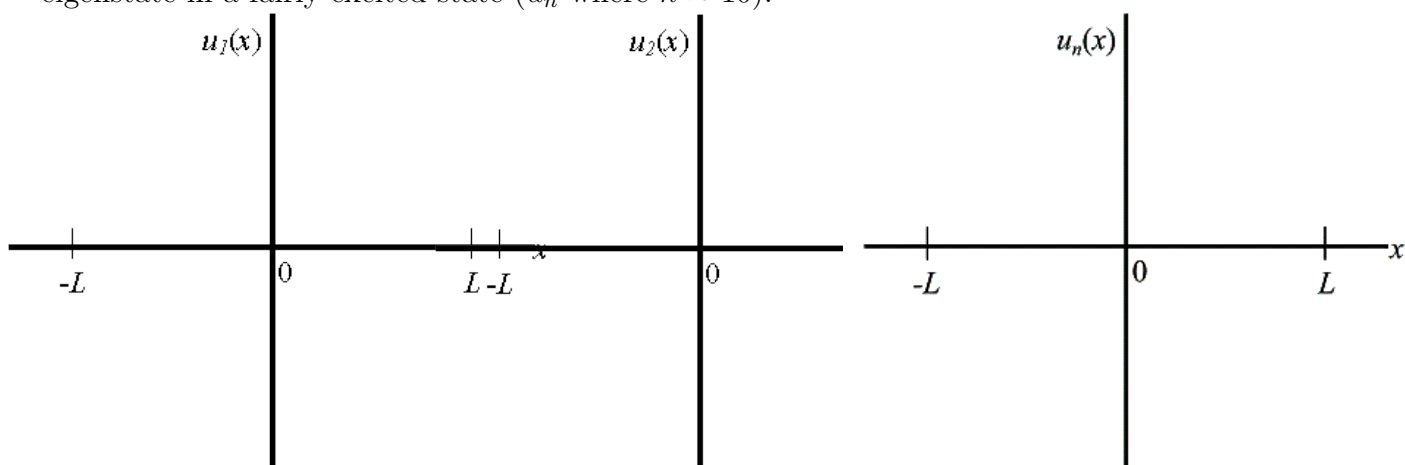
D. Tuned step:

This time the step has changed, it is V_d , which has been carefully tuned so that E_2^d in *this* well turns out to equal V_d .

1. Estimate and draw E_1^d on the drawing.
What is your reasoning for your placement?



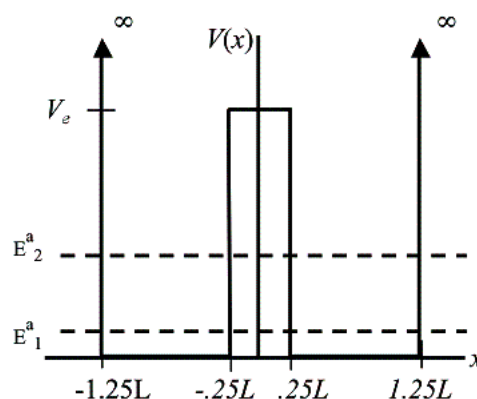
2. On the graphs below, sketch the first two energy eigenstates (u_1 and u_2) as well as an eigenstate in a fairly excited state (u_n where $n \approx 10$).



II: A double well

This time, there are two wells (see figure at right). The potential in the center region is a big step up from the second energy level in the infinite square well ($V_e \gg E_2^a$).

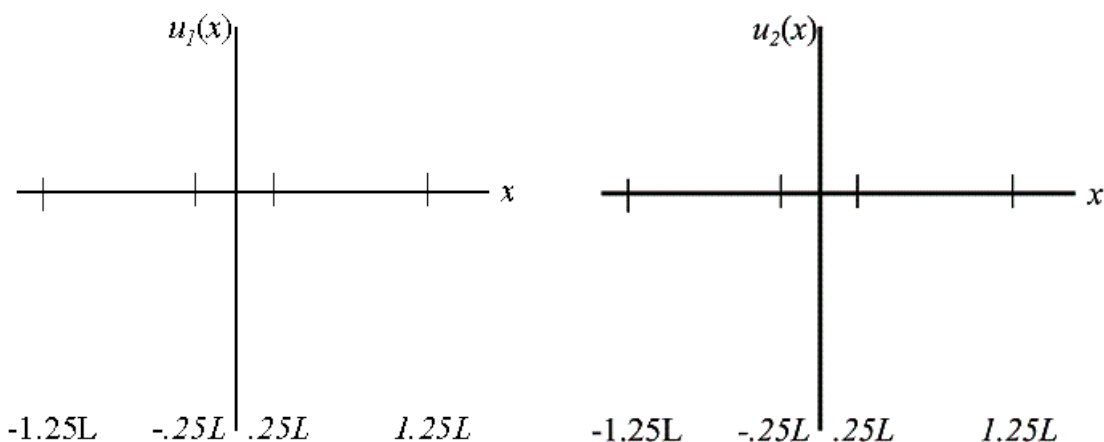
1. Sketch the first two energy eigenstates on a white board.
2. Count the nodes in each sketch. How many nodes would you expect for each of these states? If your sketch does *not* have the number of nodes you expect, can you go back and fix your sketch?



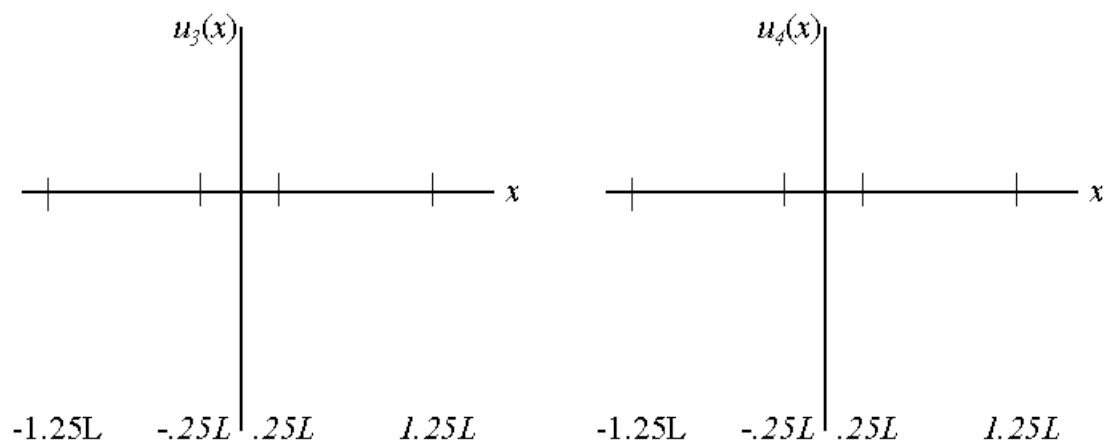
3. Check the curvature of your wave functions in the “bump region”. What is the general general solution of the Schrödinger equation in that region?

4. The potential is symmetric, are your wave functions? *Should* they be symmetric?

5. After answering these questions and coming to consensus with your table, sketch the first two energy eigenstates on the graphs below.



6. Now practice by sketching the third and fourth energy eigenstates for this potential on the graphs below. Again, be careful about how many nodes you end up with.

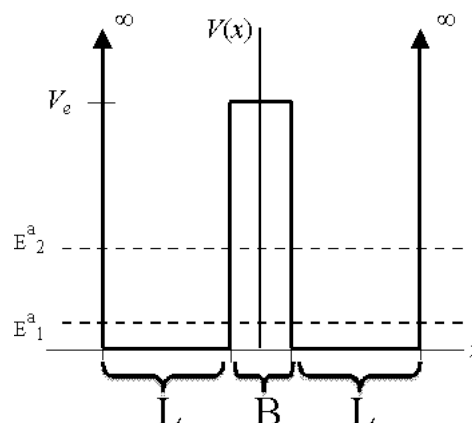


7. Discuss the energy spectrum of this potential. What properties does it share with the spectrum of the single infinite square well potential? How does it differ?

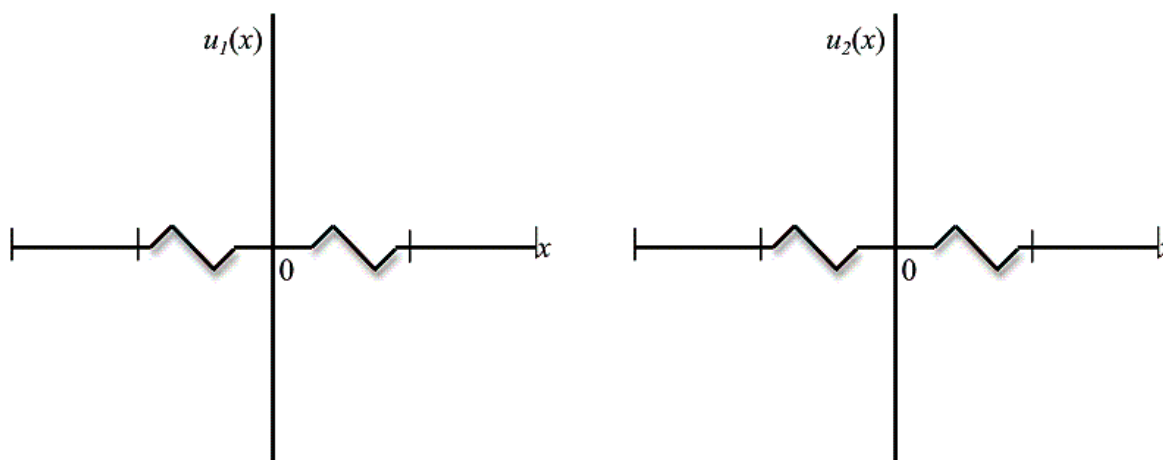
8. What would happen to the energy levels if V_e started dropping? Think about the behavior as it approached zero (what would the potential resemble at that point?). In particular, think about the differences $(E_2 - E_1)$ and $(E_4 - E_3)$.

III: An adjustable double well

Rather than the value of V_e changing, we will now consider what happens to the energy levels if the “bump region” changes in width. In the figure to the right, the two wells will have a fixed width, L , while the central section where the potential is V_e has a width, B , which can vary from almost zero to some very large value, $B \gg L$. As usual, we will assume $V_e \gg E_2^a$.

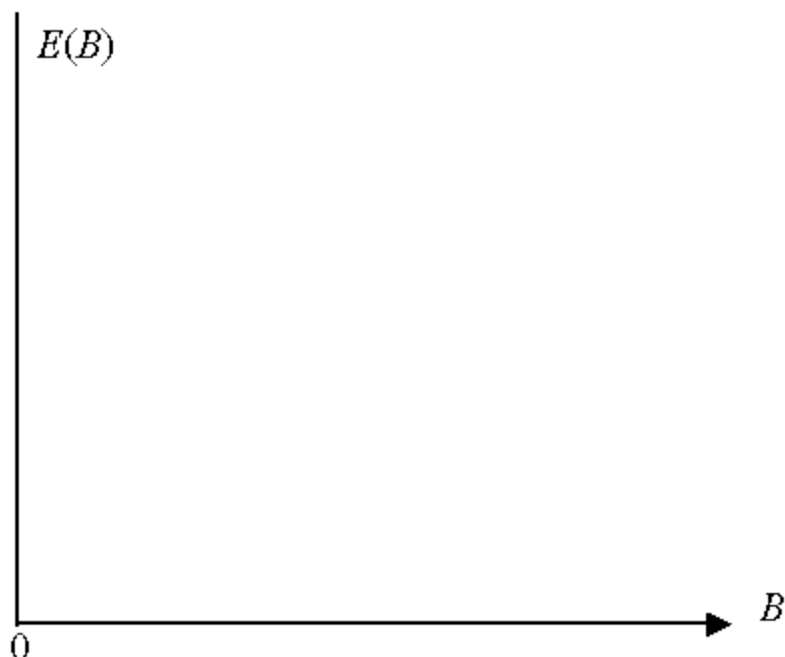


1. What would the energy eigenstate wave functions look like if $B = 0$?
2. Sketch the first two energy eigenstate wave functions for the case of a very wide bump, $B \gg L$. The tick marks indicate the locations where the potential changes value.



✓ Check your results with a tutorial instructor.

3. Now think about the energy levels of these three situations ($B = 0$, $B = L/2$ from the last problem, and $B \gg L$). Make qualitative sketches of $E_1(B)$ and $E_2(B)$ as B varies from zero to infinity. For each sketch, mark the value for $E_1(0)$ and $E_2(0)$ on the graph. Also, indicate the asymptotic values for $E_1(\infty)$ and $E_2(\infty)$.



- ✓ Check your results with a tutorial instructor.
4. Would an experimental physicist be more concerned with the values of E_1 and E_2 or with the value of $(E_2 - E_1)$? Why?
5. **Challenge Problem:** (from Griffiths problem 2.47) The double well is a very primitive one-dimensional model for the potential experienced by an electron in a diatomic molecule (the two wells represent the attractive force of the nuclei). If the nuclei are free to move, they will adopt the configuration of minimum energy. In view of your conclusions in exercise 4 above, does the electron tend to draw the nuclei together, or push them apart? (Of course, there is also the internuclear repulsion to consider, but that's a separate problem.)