Quantum I (PHYS 3220)

concept questions

3-D

Consider a particle in 3D.

Is there a state where the result of position in the y-direction and momentum in the z-direction can *both* be predicted with 100% accuracy?

A) Yes, *every state*

B) Yes, at least one state (but *not* all)

C) No, there is *no* such state

D) Yes, but only for free particles

E) Yes, but only for a spherically symmetric potential (not just free particles)

Is the 3D wave function

$$u(x,y,z) = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right) \sin\left(\frac{n_z \pi z}{a}\right)$$

an eigenfunction of $\hat{H}_x = \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$?

A) Yes B) No

For the particle in a 3D box, is the state $(n_x, n_y, n_z) = (1, 0, 1)$ allowed?

A) Yes

B) No

The ground state energy of the particle in a 3D box is $\left(1^2+1^2+1^2\right)\dfrac{\hbar^2\pi^2}{2ma^2}=3\varepsilon$.

What is the energy of the 2nd excited state?

A) 4ε B) 5ε C) 6ε D) 8ε E) 9ε

Consider three functions f(x) , g(y), and h(z). f(x) is a function of x only, g(y) is a function of y only, and h(z) is a function of z only. They obey the equation $f(x) + g(y) + h(z) = C =$ constant.

What can you say about f, g, and h?

A) f, g, and h must all be constants.

B) One of f, g, and h, must be a constant. The other two can be functions of their respective variables.

C) Two of f, g, and h must be constants. The remaining function can be a function of its variable.

Consider three functions f(x) , g(y), and h (z) which obey the equation f(x) + g(y) + h (z) = C = constant. How many of the functions must be constant?

A) f, g, and h must all be constants.

B) One of f, g, and h, must be a constant.

C) Two of f, g, and h must be constants.

In the 3D infinite square well, what is the degeneracy of the energy corresponding to the state $(n_x, n_y, n_z) = (1, 2, 3)$?

A) 1 B) 3 C) 4 D) 6 E) 9

In Cartesian coordinates, the normalization condition is $\int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \int_{-\infty}^{\infty} \mathrm{d}z |\Psi|^2 = 1$.

In spherical coordinates, the normalization integral has limits of integration:

A) $\int_{0}^{+\infty} \mathrm{dr} \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{\pi} \mathrm{d}\varphi \ldots$

B) $\int_{-\infty}^{+\infty} \mathrm{dr} \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{\pi} \mathrm{d}\varphi \ldots$

C) $\int_{0}^{+\infty} \mathrm{dr} \int_{0}^{2\pi} \mathrm{d}\theta \int_{0}^{2\pi} \mathrm{d}\varphi \ldots$

D) $\int_{-\infty}^{+\infty} \mathrm{dr} \int_{0}^{\pi} \mathrm{d}\theta \int_{0}^{\pi} \mathrm{d}\varphi \ldots$

E) None of these

Separation of variables has gotten us to

$$\frac{1}{\Phi(\phi)}\frac{d^2\Phi(\phi)}{d\phi^2} = -c$$

Is there anything we can say about the sign of the constant "c" in that equation?

A) c must be ≥ 0

B) c must be ≤ 0

C) c can be + or –, but it *cannot* be 0

D) Can't decide without knowing more:
(what's the potential, what are the boundary conditions for our *particular* problem?)

angular momentum

The angular stationary state wave fns for central potentials are:

$$Y_l^m(\theta,\phi) \propto P_l^m(\cos\theta)e^{im\phi}$$

If the quantum number m is *large*, what can you conclude about the wave function and probability density as you vary *just* the azimuthal angle ϕ?

Wave function	Prob density
A) rapidly varies	rapidly varies
B) no variation	rapidly varies
C) rapidly varies	no variation
D) no variation	no variation

E) We need to know more about $P_l^m(\cos\theta)$

If $\exp(+i\,m\,2\pi) = 1$

then it must be true that

A) m = 0, 1, 2, …

B) m = 0, 1/2, 1, 3/2, 2, …

C) m = 0, ±1, ±2, …

D) m = 2pn where n = 0, ±1, ±2, …

E) None of these

Apart from normalization, the Y_l^l spherical harmonics are: $$Y_l^l(\theta,\phi) \propto (\sin\theta)^l e^{il\phi}$$

Normalization says: $$\int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \left|Y_l^m(\theta,\phi)\right|^2 = 1$$

Thus, Y_0^0 (θ,ϕ)= ?

A) 1

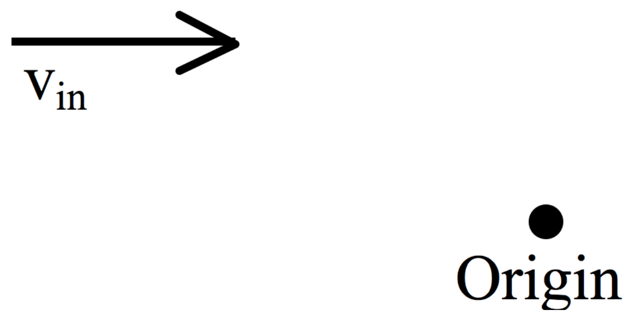
B) 4π

C) $1/4\pi$

D) Sqrt[$1/4\pi$]

E) Something else entirely!

A classical free particle approaches from the left, as shown. How do you characterize the motion of the particle in the radial direction, i.e. r(t)?



A) It is a constant with time.

B) Gets smaller, reaches r=0, gets bigger

C) Gets smaller, reaches $r_{min}>0$, gets bigger

D) Gets smaller steadily

E) Gets larger steadily.

The angular momentum operator is

$$\hat{\mathbf{L}} = \hat{\vec{r}} \times \hat{\vec{p}} = (\hat{L}_x, \hat{L}_y, \hat{L}_z)$$

with e.g. $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y$, ..., $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$

Is $\hat{\mathbf{L}}$ Hermitian? (Hint: Is L_z Hermitian?)

A) Yes

B) No

C) Only L_z is (L_x and L_y are not)

D) L_z is not (but L_x and L_y are)

E) Are you joking here? Can I do this as a clicker question?

Is it possible to find a "nontrivial" state (i.e. nonzero angular momentum) for which

$$i) \quad \Delta L_x = \Delta L_y = \Delta L_z = \Delta L^2 = 0$$

$$ii) \quad \Delta L_z = \Delta L^2 = 0$$

A) i yes, but ii no

B) i no, but ii yes

C) i yes, and ii yes

D) i no, and ii no

Hint: Don't vote A. Why not?

True (A) or False (B) ?

Any arbitrary physical state of an electron bound in a central potential can always be written as

$$\psi_{n l m}(r, \theta, \phi) = R_{n l}(r) Y_l^m(\theta, \phi)$$

with a suitable choice of n , l , and m (where $R_{nl}(r)$ solves the radial TISE)

In classical mechanics, kinetic energy is $p^2/2m$. What is the formula for rotational kinetic energy (where I is moment of inertia)

A) $IL^2/2$

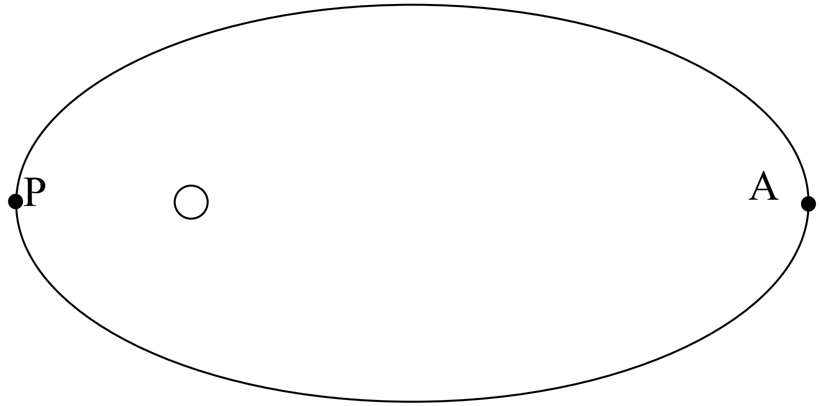
B) $\omega^2/2I$

C) $L^2/2I$

D) $I\omega$

E) Something completely different

A planet is in elliptical orbit about the sun.

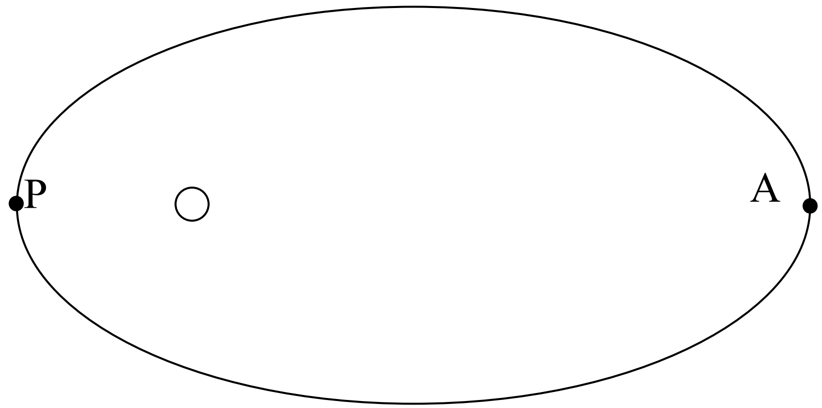


The torque, $\vec{\tau} = \vec{r} \times \vec{F}$ on the planet about the sun is:

A) Zero always

B) Non-zero always

C) Zero at some points, non-zero at others.



The magnitude of the angular momentum of the planet about the sun $\vec{L} = \vec{r} \times \vec{p}$ is:

A) Greatest at the perihelion point, P

B) Greatest at the aphelion point, A

C) Constant everywhere in the orbit

Is the commutator, $\left[\hat{x}, \hat{p}_y\right]$

zero or non-zero?

A) Zero

B) Non-zero

The commutator, $\left[\hat{y}\hat{p}_z, \hat{x}\hat{p}_z\right]$

zero or non-zero?

A) Zero

B) Non-zero

C) Sometimes zero, sometimes non-zero

The commutator, $\left[L_z^2, \hat{L}_z\right]$

zero or non-zero?

A) Zero

B) Non-zero

C) Sometimes zero, sometimes non-zero

In Cartesian coordinates, the volume element is dx dy dz. In spherical coordinates, the volume element is

A) $r^2 \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

B) $\sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

C) $r^2 \cos\theta \sin\varphi \, dr \, d\theta \, d\varphi$

D) $r \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

E) $r^2 \sin\theta \, dr \, d\theta \, d\varphi$

Recall that an operator, $\hat{Q}$, is hermitian if $\left\langle f \middle| \hat{Q} g \right\rangle = \left\langle \hat{Q} f \middle| g \right\rangle$ for all normalizable functions f and g. The operator $\hat{L}_z$ is hermitian, since it corresponds to an observable. Is the operator $i\hat{L}_z$ hermitian?

A) Yes

B) No

In the expression, $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial r}\dfrac{\partial r}{\partial x} + \dfrac{\partial f}{\partial \theta}\dfrac{\partial \theta}{\partial x} + \dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial x}$,

what variable(s) are held constant in the derivative $\dfrac{\partial r}{\partial x}$?

A) x, y

B) θ, φ

C) F

D) x, y, θ, φ

E) None of these

Consider a scalar field, $f = f(x,y,z)$ and a unit vector, $\hat{u}$, in some arbitrary direction. Consider the equation $\nabla \mathrm{f} \cdot \hat{\mathrm{u}} = \dfrac{\partial f}{\partial s}$ where s is the distance along the direction $\hat{u}$.

A) This equation is always true.
B) This equation is never true.
C) This equation is sometimes true, depending on the direction of û.

True (A) or False (B)

$$\frac{\partial r}{\partial x} = \frac{1}{\sin\theta\cos\varphi}$$

Recall that $x = r \sin \theta \cos \varphi$

Apart from normalization, the spherical harmonic

$$Y_l^l(\theta, \phi) = (\sin\theta)^l \exp(i\,l\,\phi)$$

The zero-angular momentum state Y_0^0

A) has no θ, Φ dependence: it is a constant

B) depends on θ only; it has no Φ dependence

C) depends on Φ only; it has no θ dependence

D) depends on both θ and Φ

In spherical coordinates,

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{\partial f}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$$, and in QM the angular momentum operator is

$\hat{L} = \frac{\hbar}{i}\vec{r}\times\nabla = \frac{\hbar}{i}r\hat{r}\times\nabla$. What is the $\hat{r}$ component of $\hat{L}$?

A) zero

B) Non-zero but dependent on θ, φ only (independent of r)

C) Non-zero but dependent on r, θ and φ

In QM, the operator $L^2 = \hat{L} \cdot \hat{L}$

A) depends on θ, φ only (independent of r)

B) depends on r, θ and φ

C) depends on θ only (independent of r, φ)

In classical mechanics, the translational kinetic energy of a particle is $p^2/2m$.

What is the classical formula for rotational kinetic energy (where I is moment-of-inertia)?

A) $\frac{1}{2}IL^2$ B) $\frac{L^2}{2I}$ C) $I\omega$ D) $2IL^2$

In the 1D Simple Harmonic Oscillator, which formula below tells us that the operator a_- lowers the energy of a state by $\hbar\omega$?

A) $[a_-,a_+]=1$

B) $\hbar\omega\, a_- |u_0\rangle = 0$

C) $\hbar\omega\, (a_-)^+ = \hbar\omega\, (a_+)$

D) $H|u_{n-1}\rangle = \hbar\omega(n-1/2)\, |u_{n-1}\rangle$

E) $[H,a_-]=-\hbar\omega\, a_-$

Note: **All** of the above formulas are correct (!!) but only one answers the question.

Does the commutator $[L^2, L_+] = 0$?

A) Yes

B) No

The operator for (angular momentum)^2 is

$$L^2 = L_x^2 + L_y^2 + L_z^2,$$ which means

$$\langle L^2 \rangle = \langle L_x^2 \rangle + \langle L_y^2 \rangle + \langle L_z^2 \rangle.$$

If your state is a Y_l^m, this means

$$\hbar^2 l(l+1) = \langle L_x^2 \rangle + \langle L_y^2 \rangle + \hbar^2 m^2.$$

What can you conclude from this?

A) $l(l+1) \geq m^2$, B) $l(l+1) > m^2$

C) $l(l+1) \leq m^2$, D) $l(l+1) < m^2$ E) $l = m$

The operator for (angular momentum)^2 is

$L^2 = L_x^2 + L_y^2 + L_z^2$.

Is it true that $\left\langle L^2 \right\rangle = \left\langle L_x^2 \right\rangle + \left\langle L_y^2 \right\rangle + \left\langle L_z^2 \right\rangle$?

A) Yes, always

B) No, never

C) Sometimes yes, sometimes no, depending on the state function Ψ used to compute the expectation value.

Given $L_z Y_l^M = \hbar M Y_l^M$, with $\mathrm{M} = \mathrm{m}_{\max}$.

Using the right side (below), what is

$$L^2 Y_l^M = (\mathrm{L}_z^2 + \mathrm{L}\_\mathrm{L}_+ + \hbar \mathrm{L}_z) Y_l^M \ ?$$

A) $\hbar^2\big(M(M+1) + M\big)Y_l^M$

B) $\hbar^2 M^2\big(M+1\big)Y_l^M$

C) $\hbar^2\big(M^2 + M\big)Y_l^M + \hbar^2 M Y_l^{M+1}$

D) $\hbar^2\big(M^2 + M\big)Y_l^M$

E) We don't have enough info to decide.

Given that $L_z f_t = \hbar \ell f_t$, what is

$$L^2 f_t = (L_- L_+ + L_z^2 + \hbar L_z) f_t \ ?$$

A) $\hbar^2\left(\ell^2 + \ell\right)$

B) $\hbar^2 \ell^2 \left(\ell + 1\right)$

C) $\hbar^2\left(\ell^2 + 1\right)$

D) Zero

E) None of these

hydrogen

Please sit in a *new spot*, next to *different*) *people than usual*. (Just for today)

True (A) or False (B)

Any arbitrary angular wave function $f(\theta,\phi)$ can always be written in the form $c_{l,m} Y_l^m(\theta,\phi)$, with a suitable choice of l,m and $c_{l,m}$

What is $L^2\psi$? $L_z\psi$? What values of L_z and L^2 could you measure, with what probabilities? How about $\langle L_z \rangle$?

$$1)\ \psi = Y_0^0(\theta,\phi)$$

$$2)\ \psi = (re^{-r/2a})Y_1^{-1}(\theta,\phi)$$

$$3)\ \psi = \frac{1}{\sqrt{3}}Y_0^0(\theta,\phi) + \frac{\sqrt{2}}{\sqrt{3}}Y_1^0(\theta,\phi)$$

$$\psi = \frac{1}{\sqrt{3}}Y_1^1(\theta,\phi) + \frac{\sqrt{2}}{\sqrt{3}}Y_1^0(\theta,\phi)$$

On the back of your “quiz”:
A and B are positive constants. r is radial distance ($0 \le r < \infty$).

Sketch $-\dfrac{A}{r}$ and $\dfrac{B}{r^2}$

What does the graph $y(r) = \dfrac{B}{r^2} - \dfrac{A}{r}$ look like?

We are solving the equation

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2}+\left(\frac{-ke^2}{r}+\frac{\hbar^2 l(l+1)}{2mr^2}\right)u=Eu$$

What, then, is the full 3-D wave function for hydrogen atom stationary states?

A) $u(r,\theta,\phi)$

B) $u(r)Y_l^m(\theta,\phi)$

C) $ru(r)Y_l^m(\theta,\phi)$

D) $r^2u(r)Y_l^m(\theta,\phi)$

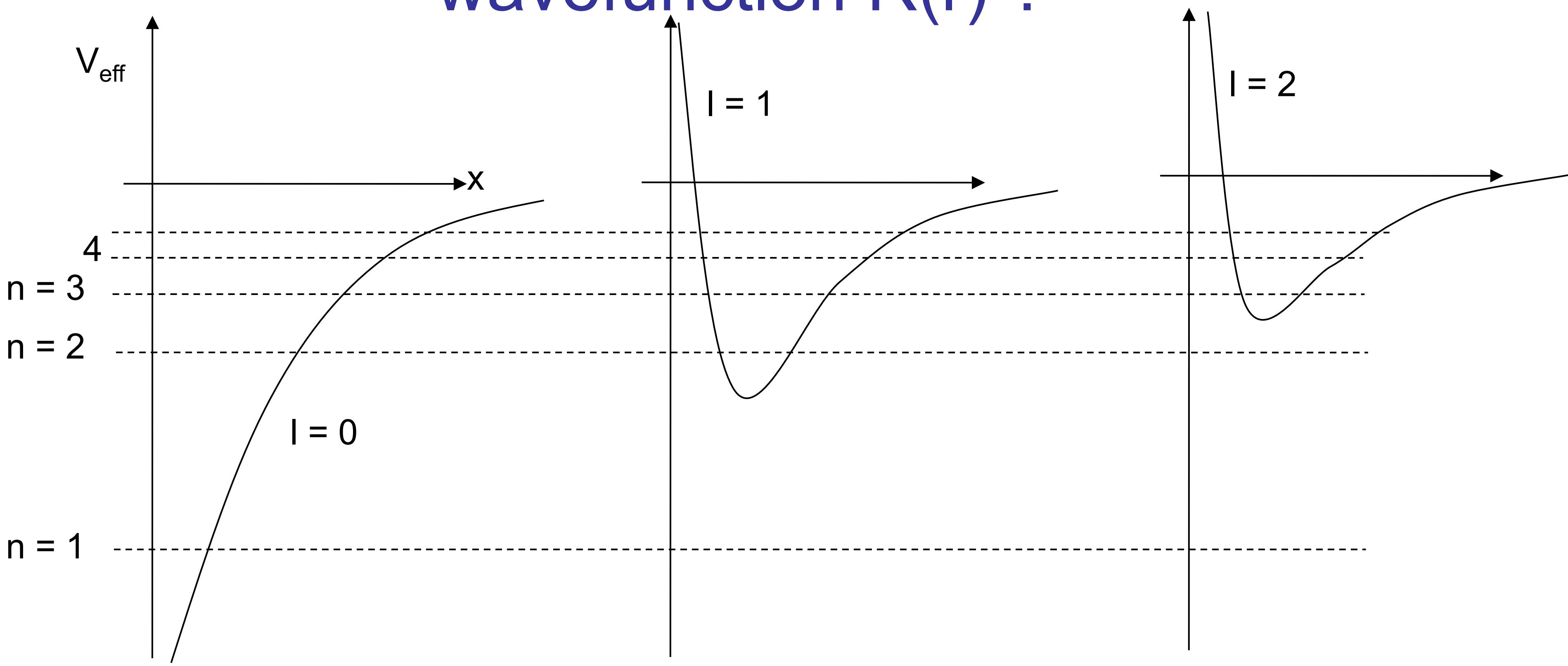
E) None of these

Ignoring spin, what is the angular momentum of the ground state of an electron in a hydrogen atom, in units of h-bar?

A) Zero
B) 1/2
C) 1
D) Something else
E) I don't know

As indicated in the figure, the n = 2, l = 0 state and the n = 2, l = 1 state happen to have the same energy (given by $E_2 = E_1/2^2$).

Do these states have the same radial wavefunction R(r) ?



A) Yes B) No

$$R_{10}(r) \propto e^{-r/a}$$

$$R_{20}(r) \propto (1 - r/2a)e^{-r/2a}$$

$$R_{21}(r) \propto (r/a)e^{-r/2a}$$

$$R_{31}(r) \propto (r/a)(1 - r/6a)e^{-r/3a}$$

What does $\psi_{100}(r,\theta,\phi)$ “look like”?

How about $\psi_{200}(r,\theta,\phi)$? $\psi_{210}(r,\theta,\phi)$? $\psi_{211}(r,\theta,\phi)$?

True (A) or False (B)

Any arbitrary physical state of an electron bound in the H-atom potential can always be written as

$$\psi_{\mathrm{n,l,m}}(\mathrm{r},\theta,\phi) = R_{nl}(\mathrm{r})\mathrm{Y}_{\mathrm{l}}^{\mathrm{m}}(\theta,\phi),$$

with suitable choice of n,l, and m.

True (A) or False (B)

Any arbitrary stationary state of an electron bound in the H-atom potential can always be written as

$$\psi_{n,l,m}(r,\theta,\phi) = R_{nl}(r)Y_l^m(\theta,\phi),$$

with suitable choice of n,l, and m.

Suppose at t=0,

$$\Psi(r,t=0) = \frac{1}{\sqrt{2}}(\psi_{210} + \psi_{200})$$

Is $\Psi(r,t)$ given very simply by $\Psi(r,0)e^{-iEt/\hbar}$?

A) Yes, that's the simple result

B) No, it's more complicated (a superposition of two states with different t dependence => "sloshing")

Recall, for hydrogen:

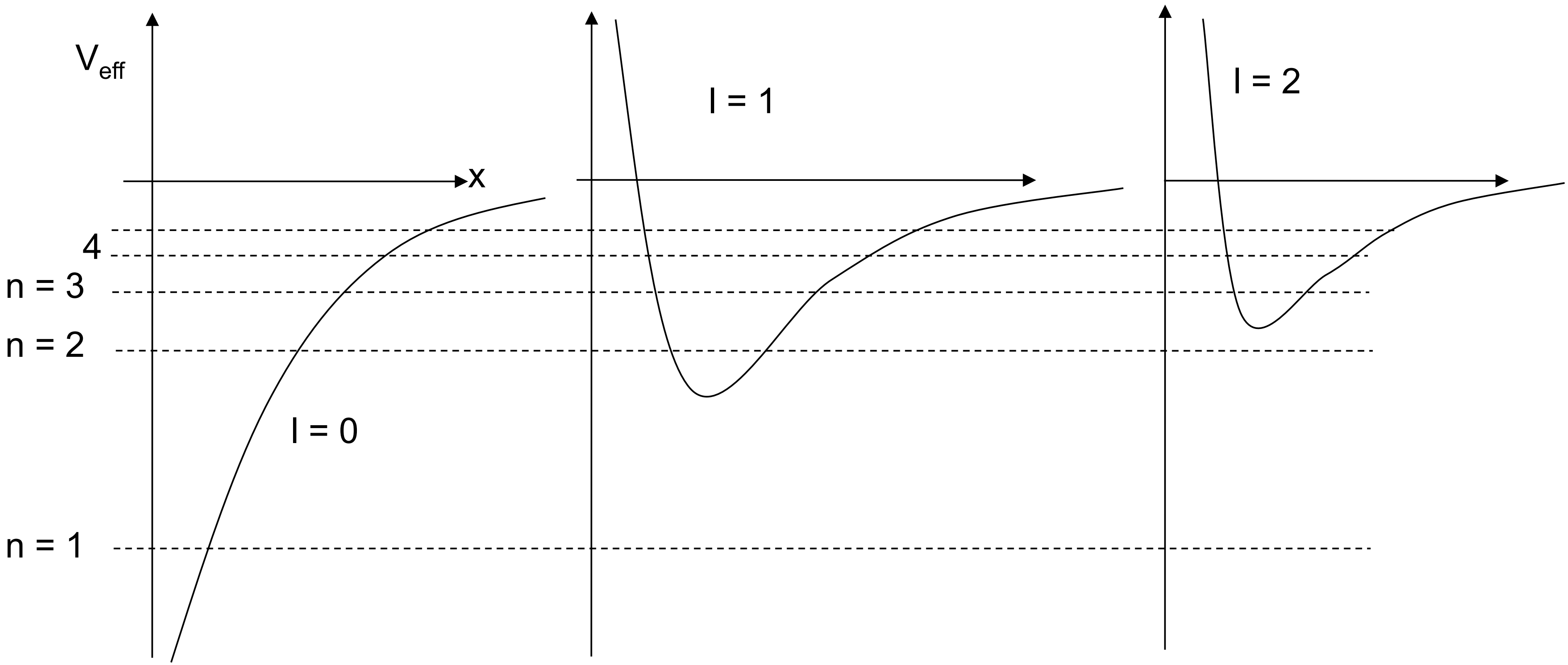
$$E_n = \frac{E_1}{n^2}, \quad \text{with } E_1 = -(ke^2)^2 m_e / 2\hbar^2$$

Consider He+ (1 e- around a nucleus, Q= 2e).
If you look at "Balmer lines" (e- falling from higher n down to n=2) what part of the spectrum do you expect the emitted radiation to fall in?

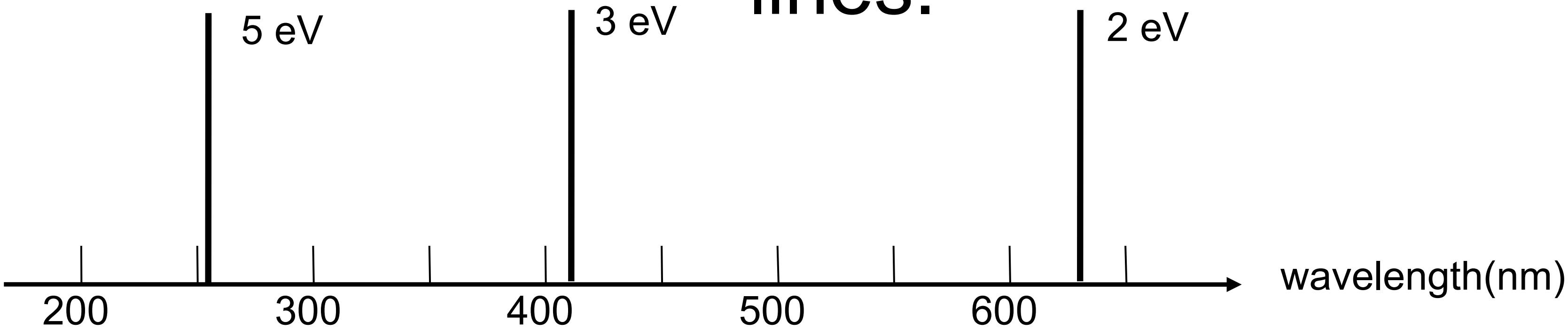
A) Visible
B) IR
C) UV
D) It's complicated, not obvious at all.

How many nodes do you expect to find in $R_{nl}(r)$?

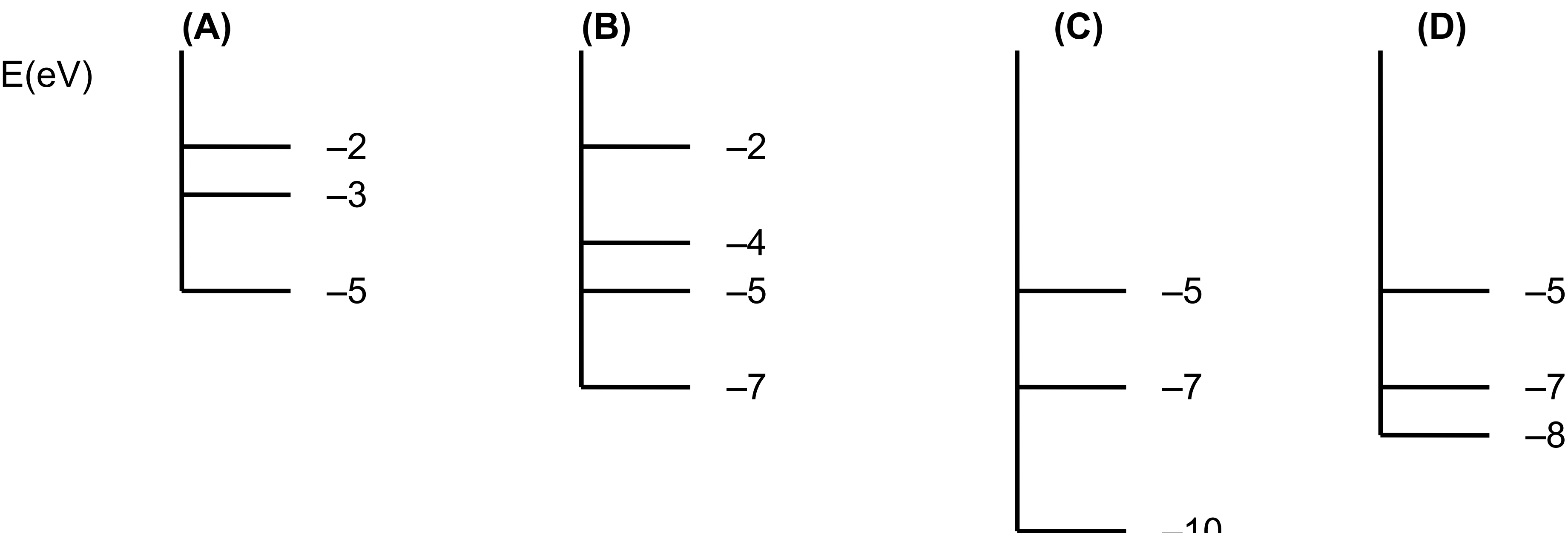
How does this relate to the order of the corresponding "associated Laguerre" polynomial?



The spectrum of "Perkonium" has 3 emission lines.



Which energy level structure is consistent with the spectrum?



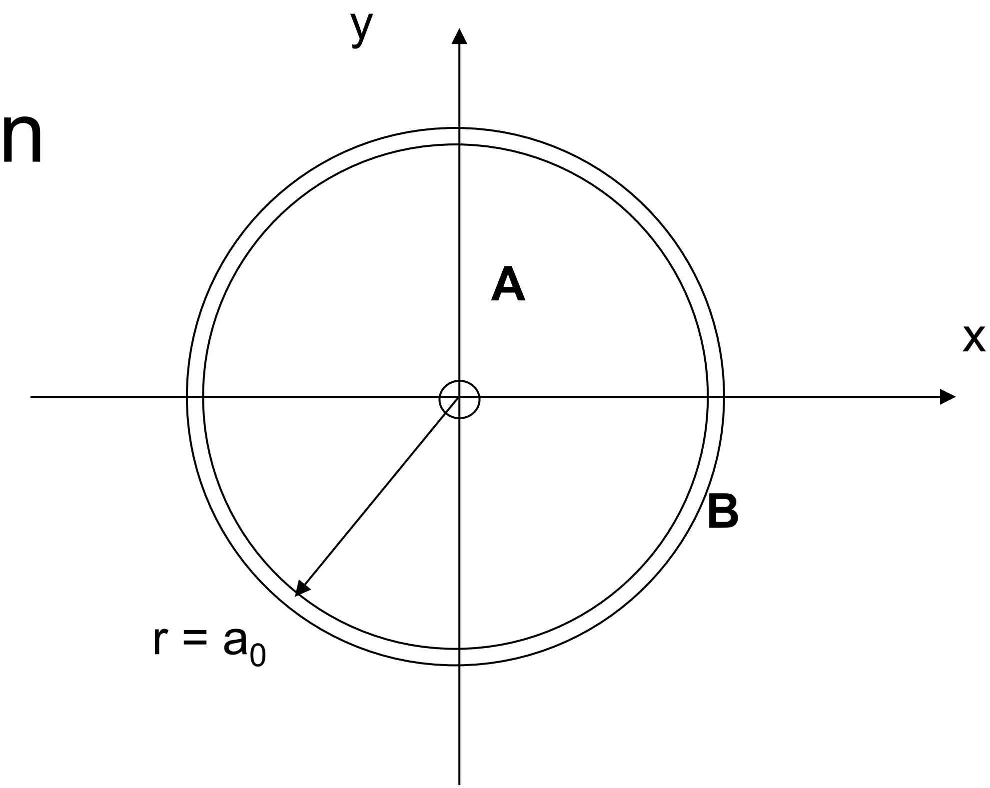
Consider an electron in the ground state of an H-atom. The wavefunction is

$$\psi(r) = A\exp(-r/a_0)$$

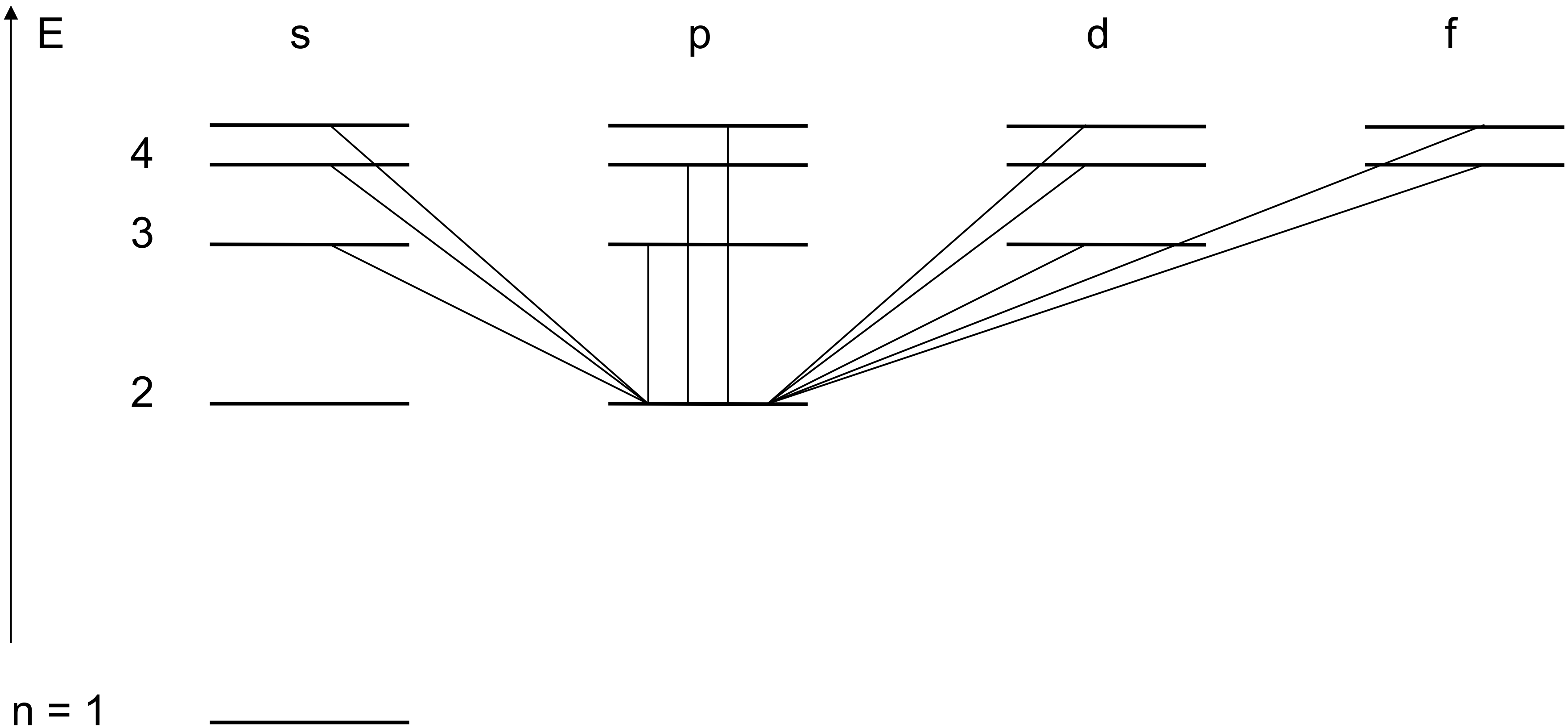
Where is the electron more likely to be found?

A) Within dr of the origin (r = 0)

B) Within dr of a distance $r = a_0$ from the origin?



How many of the following transitions to the 2p in an H-atom will result in emission of a photon ?



A) all of them: 11 B) None of them: 0 C) 8

D) 9 E) 6

Suppose at t=0,

$$\Psi(r,t=0) = \frac{1}{\sqrt{2}}(\psi_{200} + \psi_{300})$$

Is $\Psi(r,t)$ given very simply by $\Psi(r,0)e^{-iEt/\hbar}$?

A) Yes, that's the simple result

B) No, it's more complicated (a superposition of two states with different t dependence => "sloshing") spin

Pick the states $|u_n\rangle$ as our basis.

A general $|\psi\rangle = \Sigma c_n\, |u_n\rangle$ will be written as

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{pmatrix}$$

In this basis, what is $|u_1\rangle$ written as?

Pick the states $|u_n>$ as our basis.

In this basis, what state does $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$ represent?

Consider two kets and their corresponding column vectors:

$$|\Psi\rangle \Leftrightarrow \begin{pmatrix} 1 \\ i \\ \sqrt{2} \end{pmatrix} \qquad |\phi\rangle \Leftrightarrow \begin{pmatrix} 1 \\ i \\ -\sqrt{2} \end{pmatrix}$$

Are these two states orthogonal? Is $\langle \psi | \phi \rangle = 0$?

A) Yes

B) No

Consider a ket and its corresponding column vector:

$$|\Psi\rangle \Leftrightarrow \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix}$$

Is this state normalized?

A) Yes

B) No

Consider two kets and their corresponding column vectors:

$$|\Psi\rangle = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} \qquad |\phi\rangle = \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$$

Are these two states normalized?

A) Yes

B) No

Matrix multiplication: What is the matrix element?

$$\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} 3 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 8 & ? \end{pmatrix}$$

A) 0

B) 3

C) 2

D) 4

E) None of these

Consider a Hilbert space spanned by 3 energy eigenstates: $\hat{H}|n\rangle = E_n|n\rangle, \quad n = 1, 2, 3$

In this space, what is the matrix corresponding to the Hamiltonian operator?

A) $\begin{pmatrix} E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \end{pmatrix}$

B) $\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

D) $\begin{pmatrix} E_1 & E_1 & E_1 \\ E_2 & E_2 & E_2 \\ E_3 & E_3 & E_3 \end{pmatrix}$

E) None of these

What physics does the operator equation $[L_z, L_+] = \hbar L_+$ tell us?

A) That L_+ raises the m–value of an angular momentum eigenstate by *one.*

B) That L_+ raises the l–value of of an angular momentum eigenstate by *one.*

C) That L_z raises the m–value of an angular momentum eigenstate by *one.*

D) That L_z raises the l–value of an angular momentum eigenstate by *one.*

E) None of the above.

Given the classical formula $F_z = \frac{q}{2M_q} L_z \frac{\partial B_z}{\partial z}$

What pattern would you expect to see for a thin beam of neutral atoms passing through a Stern-Gerlach device?

A) 1 beam spot (if the atoms are neutral)
B) A continuous smear at various angles
C) An ODD number of spots
D) Any number of spots
E) None of these!

Which of these is a projection operator?

A) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

C) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

D) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

E) None of these

Consider the matrix equation: $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda\begin{pmatrix} a \\ b \end{pmatrix}$

This is equivalent to

A) $\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

B) $\begin{pmatrix} 0 & 1-\lambda \\ 1-\lambda & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

C) $\begin{pmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

D) $\begin{pmatrix} -\lambda & 1-\lambda \\ 1-\lambda & -\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

E) None of these

Consider the set of H-atom energy eigenstates

$$\{n = 2;\ l = 1;\ m = -1\,K\,+1\} =$$

$$|2, 1, -1\rangle, \ |2, 1, 0\rangle, \ |2, 1, 1\rangle$$

Does this set of 3 states form a vector space?

A) Yes

B) No

Consider the set of H-atom energy eigenstates

$$\{n = 2;\ l = 1;\ m = -1\, K\ +1\} =$$

$$|2, 1, -1\rangle,\ |2, 1, 0\rangle,\ |2, 1, 1\rangle$$

Does this set of 3 states span a vector space?

A) Yes

B) No

Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_2|\psi\rangle$, where $\hat{P}_2 = |2\rangle\langle 2|$

is the projection operator for the state $|2\rangle$?

A) c_2

B) $|2\rangle$

C) $c_2|2\rangle$

D) $c_2{}^*\langle 2|$

E) 0

Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_{12}|\psi\rangle$, where $\hat{P}_{12} = |1\rangle\langle 1| + |2\rangle\langle 2|$?

A) $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$

B) $|1\rangle + |2\rangle$

C) 0

D) $\langle\psi| = c_1^*\langle 1| + c_2^*\langle 2|$

E) None of these

If the state $|\psi\rangle = c_1 |1\rangle + c_2 |2\rangle$ as well as the basis states $|1\rangle$ and $|2\rangle$ are normalized, then the state $\hat{P}_1 |\psi\rangle = |1\rangle\langle 1|\psi\rangle = c_1 |1\rangle$ is

A) normalized.

B) not normalized.

Consider the state $|\psi\rangle = \sum_{n} c_n |n\rangle = \sum_{n} |n\rangle\langle n|\psi\rangle$

and the projection operator onto the state $|n_0\rangle$:

$\hat{P}_{n_0} = |n_0\rangle\langle n_0|$. What is $\langle\psi|\hat{P}_{n_0}|\psi\rangle$?

A) 1

B) 0

C) c_{n0}

D) $|c_{n0}|^2$

E) None of these

How many of these matrices are Hermitian?

I. $\begin{pmatrix} 1 & 3i & 4 \\ -3i & 2 & 0 \\ 4 & 0 & 3 \end{pmatrix}$ II. $\begin{pmatrix} 1 & i & 4 \\ -i & 2 & 0 \\ -4 & 0 & 3 \end{pmatrix}$ III. $\begin{pmatrix} 3 & 3i & 1 \\ -3i & 2i & 0 \\ 1 & 0 & 1 \end{pmatrix}$

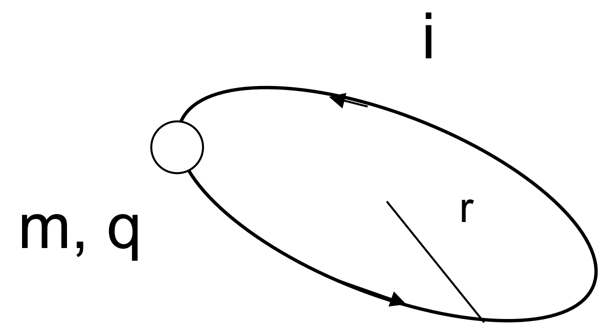
A) All of them

B) None of them

C) 2 of them

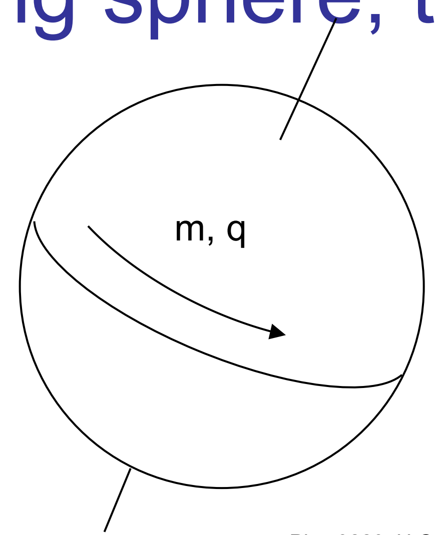
D) 1 of them

The usual classical model of a magnetic moment with orbital angular momentum.



Consider the gyromagnetic ratio $\gamma = \dfrac{\mu_z}{L_z}$

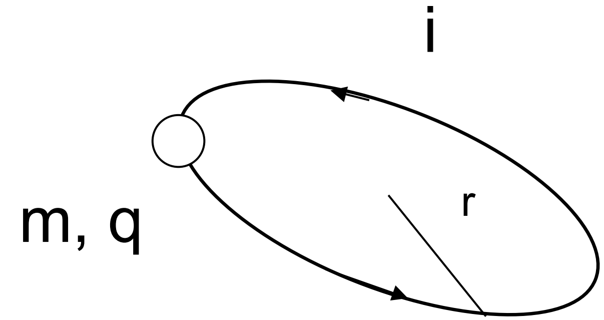
If we take the mass m and charge q and spread both uniformly throughout a rotating sphere, the gyromagnetic ratio



A) increases or decreases

B) remains unchanged

The usual classical model of a magnetic moment with orbital angular momentum.



Consider the gyromagnetic ratio $\gamma = \dfrac{\mu_z}{L_z}$

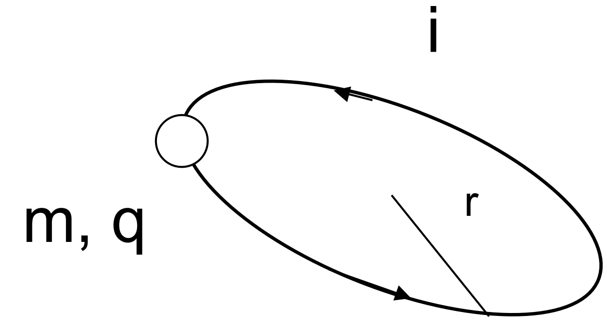
If we double the radius r and double the speed v of the particle, the ratio gamma

A) increases

B) decreases

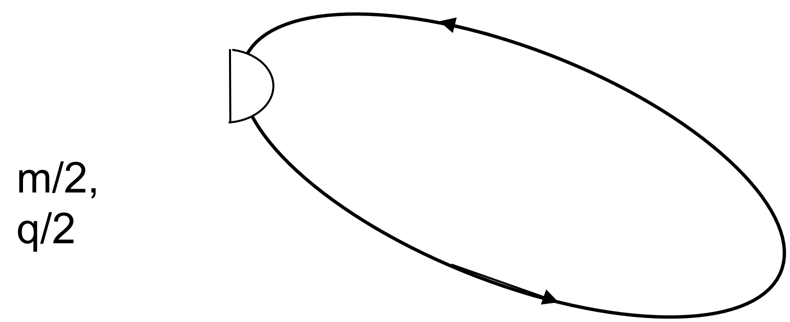
C) remains constant

The usual classical model of a magnetic moment with orbital angular momentum.



Consider the gyromagnetic ratio $\gamma = \dfrac{\mu_z}{L_z}$

If we cut the particle in half so that it has half the mass and half the charge (throw away other half), the ratio

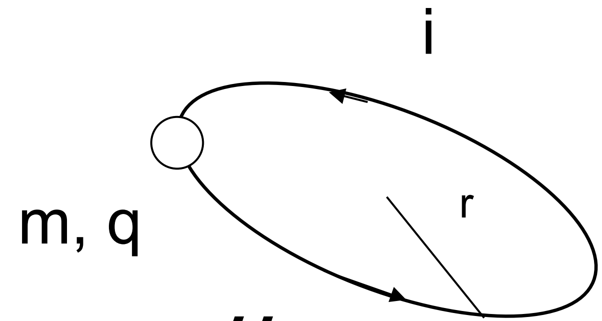


A) increases

B) decreases

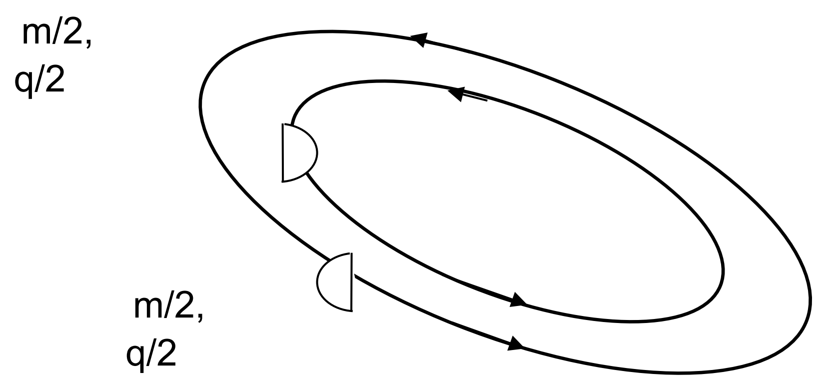
C) remains constant

The usual classical model of a magnetic moment with orbital angular momentum.



Consider the gyromagnetic ratio $\gamma = \dfrac{\mu_z}{L_z}$

If we cut the particle in half, keep both halves, with different radii and speeds but in the same plane, same sense of rotation, the gyromagnetic ratio



A) increases

B) decreases

C) remains constant

Given the classical formula $F_z = \frac{q}{2M_q} L_z \frac{\partial B_z}{\partial z}$

What pattern would you expect to see for a thin beam of neutral atoms passing through a Stern-Gerlach device?

A) 1 beam spot (if the atoms are neutral)
B) A continuous smear
C) An ODD number of spots
D) Any number of spots
E) ??!

Do you plan on coming to Tutorial this afternoon? (On spin, angular momentum, and probabilities)

A) Yes, at 3 PM
B) Yes, at 4 PM
C) Maybe, I'll try...
D) Sorry, can't make it today

In spin space, the basis states (eigenstates of S^2, S_z) are orthogonal: $\langle \uparrow | \downarrow \rangle = 0$.

Are the following matrix elements zero or non-zero?

$$\langle \uparrow | S^2 | \downarrow \rangle \qquad\qquad \langle \uparrow | S_z | \downarrow \rangle$$

A) Both are zero
B) Neither are zero
C) The first is zero; second is non-zero
D) The first is non-zero; second is zero

The raising operator operating on the up and down spin states:

$$S_+\left|\downarrow\right\rangle = \hbar\left|\uparrow\right\rangle \ , \qquad S_+\left|\uparrow\right\rangle = 0$$

What is the matrix form of the operator S_+ ?

A) $\hbar\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ B) $\hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ C) $\hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

D) $\hbar\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ E) None of these

The raising operator is

$$S_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Is the raising operator S_+ Hermitian?

A)Yes, always

B) No, never

C) Sometimes

A spin ½ particle in the spin state

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

A measurement of S_z is made. What is the probability that the value of S_z will be ħ/2?

A) $\left|\langle\uparrow|S_z|\psi\rangle\right|^2$ B) $\left|\langle\uparrow|\psi\rangle\right|^2$ C) $\left|\langle\psi|S_z|\psi\rangle\right|^2$

D) $\left|\langle\uparrow|S_z|\uparrow\rangle\right|^2$ E) None of these

A spin ½ particle is in a spin state (a "spinor")

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

A measurement of S_z is made. What is the probability that the value of S_z will be $-\hbar/2$?

A) $\left|\langle\downarrow|S_z|\psi\rangle\right|^2$

B) $\left|\langle\downarrow|S_z|\downarrow\rangle\right|^2$

C) $\left|\langle\psi|S_z|\psi\rangle\right|^2$

D) $\left|\langle\downarrow|\psi\rangle\right|^2$

E) None of these

A spin ½ particle in the spin state

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

Many measurements of S_z are made. What is the average outcome of those measurements?

A) $\left|\langle\downarrow|S_z|\psi\rangle\right|^2$ B) $\left|\langle\downarrow|S_z|\uparrow\rangle\right|^2$ C) $\left|\langle\psi|S_z|\psi\rangle\right|^2$

D) $\dfrac{\left|\langle\uparrow|\psi\rangle\right|^2 + \left|\langle\downarrow|\psi\rangle\right|^2}{2}$ E) None of these

Consider two possible states for a spin ½ particle:

$$|\psi_I\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \text{ and } |\psi_{II}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Is there any *physical (measurable) difference* between these two states?

A) No, they are indistinguishable
(phases, like -1, don't matter in QM)

B) Yes, they are easily distinguishable

A spin ½ particle is in the +ħ/2 eigenstate of $\hat{S}_x$ (i.e, it has a definite value for the x-component of spin, +ħ/2)

Suppose we immediately measure S_z.
What is the probability that this measurement will yield S_z= +ħ/2?

A) Zero

B) 25%

C) 50%

D) 100%

E) other/Impossible to say

Suppose a spin ½ particle is in the spin state

$$|\chi_+\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ the } +\hbar/2 \text{ eigenstate of } \hat{S}_z .$$

Suppose we measure S_x and then immediately measure S_z. What is the probability that the second measurement (S_z) will leave the particle in the S_z = down state $|\chi_-\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

A) Zero

B) Non-zero

A classical vector is given by:

$$\vec{\mathbf{V}} = \begin{pmatrix} \sin\alpha\cos\phi \\ -\sin\alpha\sin\phi \\ \cos\alpha \end{pmatrix}$$

Visualize/sketch/describe in words this vector.

If $\phi = \phi(t) = \gamma B_0 t$,
how does this affect your visualization?

When adding (combining) two spin ½ objects, we come across a state with z-component of total spin = $+1\hbar$.
(Apparently *each* of the two objects must have had a z-component $+\frac{1}{2}\hbar$)
What can you conclude about the total spin of this combined object?

A) S=1
B) S=0
C) S=1/2
D) S=0 or 1, we can't tell
E) S=0, 1, 2, 3, ... we can't tell.

Do you plan to take Quantum II?

A) Yes, next term

B) Yes, but later

C) Really not sure yet

D) Nope

Suppose the wavefunction for a system is known at t = 0: $\Psi(x,t=0)$

Consider the following statement:

The wavefunction at later times is given by $\psi(x,t) = \psi(x,t=0)e^{-iEt/\hbar}$

This statement is:

A) always true

B) always false

C) true sometimes

Suppose we know the eigenstates and eigenvalues of a Hermitian operator that is NOT the Hamiltonian: $\hat{Q}\, f_n(x) = \lambda_n f_n(x)$

At t = 0, a wave function is known to be

$$\psi(x, t = 0) = \sum_n c_n f_n(x)$$

where the c_n' s are known constants.

True (A) or False (B) $\psi(x, t) = \sum_n c_n f_n(x) e^{-iE_n t/\hbar}$

True (A) or False (B):

$\hat{Q}|\psi\rangle$ is the outcome of measuring an operator $\hat{Q}$ on a state ψ

A spin ½ particle in the spin state

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

A measurement of S_z is made. What is the probability that the value of S_z will be ħ/2?

A) $\left|\langle\uparrow|S_z|\psi\rangle\right|^2$ B) $\left|\langle\uparrow|\psi\rangle\right|^2$ C) $\left|\langle\psi|S_z|\psi\rangle\right|^2$

D) $\left|\langle\uparrow|S_z|\uparrow\rangle\right|^2$ E) None of these