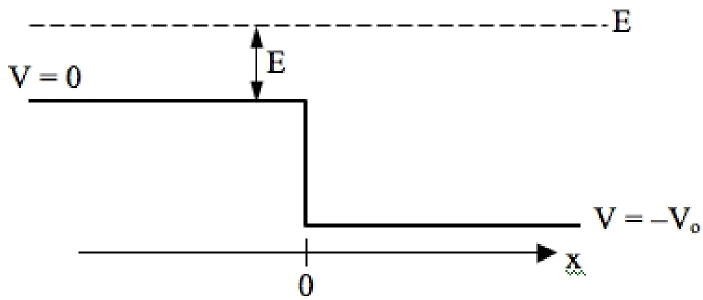
# Physics 3220 – Quantum Mechanics 1 – Spring 2009
# Problem Set #7

### Due Wednesday, March 4 at 9am

**Problem 4.1**: Reflection off a downstep. (20 points)




Consider a "downstep" potential, which drops at $x = 0$ as one goes from left to right:

$$V(x) = \begin{cases} 0 \quad , & x < 0\,, \\ -V_0 \quad , & x > 0\,, \end{cases} \qquad (1),(2)$$

where $V_0 > 0$.

a) In classical physics, consider a particle of mass $m$ coming in from the left with initial velocity $v_i$. What happens to it at the downstep? Find the total energy in terms of $v_i$ and the final velocity $v_f$ in terms of $m$, $v_i$ and $V_0$.

b) In quantum physics, solve the time-independent Schrödinger equation (TISE) for fixed energy $E > 0$ in both regions and impose appropriate boundary conditions at $x = 0$; write down the equations for these boundary conditions. Don't assume anything about where particles are coming from yet.

c) Interpret these solutions as a steady flux of particles, and assume there are no particles coming in from the right. Calculate the reflection coefficient $R$ and transmission coefficient $T$ and write them in terms of $E$ and $V_0$. Does this satisfy $R + T = 1$? Discuss how your answer compares to the classical result.

d) Plot (or sketch) $R$ vs $V_0$ at fixed $E$. (Choose, say, $E = 1$.)

e) A free neutron entering a nucleus experiences a sudden drop in potential energy from $V = 0$ outside the nucleus to about $-12$ MeV inside, due to the strong nuclear force. Suppose a neutron, emitted with KE = 4 MeV by a fission event, strikes a nucleus. What is the probability that the neutron will be absorbed, thereby initiating another fission event? (What would you have guessed classically?)

**Problem 7.2**: Tunneling through a barrier. (20 points)

For the potential with a barrier of height $V_0$,

$$V(x) \;=\; \begin{array}{ll} 0\,, & x < 0\,, \\ V_0 & 0 < x < L\,, \\ 0\,, & L < x\,, \end{array} \qquad \begin{array}{r} (3) \\ (4) \\ (5) \end{array}$$

we found in class the formula for the transmission coefficient for $0 < E < V_0$,

$$T = \frac{4\kappa^2 k^2}{(k^2+\kappa^2)^2 \sinh^2 \kappa L + 4\kappa^2 k^2}\,. \tag{6}$$

a) Demonstrate that this expression can be rewritten as

$$T^{-1} = 1 + \frac{1}{4(E/V_0)(1-E/V_0)} \sinh^2\left(\frac{L}{\hbar}\sqrt{2m(V_0-E)}\right)\,. \tag{7}$$

b) Consider an electron approaching the barrier. Its initial energy is 1/2 of an electron-volt (0.5 eV) and the barrier height is 1 eV, while the width of the barrier is 5 angstroms (5 Å). What is the numerical probability for the particle to make it to the other side of the barrier?

Do this in two ways. One way is by plugging into the formula from the previous part, for which you will need to look up some constants; be careful with units. One helpful relation is

$$1 \text{ eV} \;= 1.602 \times 10^{-19} \text{ J}\,. \tag{8}$$

The other way is to go to phet.colorado.edu and run the "Quantum Tunneling and Wave Packets" sim. Make sure you are using plane waves, not wave packets, and program the sim to calculate the probability for you. Do they agree?

Switch the PhET sim to wave packets and watch the wave packet with the same parameters evolve. Does the sim give you the same result for the probability of transmission? What if you change the initial width of the wave packet? Play with the parameters to get a feel for how things are working.

c) Consider the same system as a model of a baseball being thrown at a wall. A baseball has a mass of about 150 g, and we take it to be thrown at 40 m/s (near 90 mph). Assume that the wall is 0.1 m thick, and let's make the approximation that the ball would have to be 5 times as energetic to punch through the wall classically, so $V_0 = 5E$ with $E$ determined by the quantities above.

What is the order of magnitude for $T$? Lets put this into perspective: if you keep trying, tossing a baseball at the wall once per second, roughly how long do you have to wait until it "pops through" the wall quantum mechanically? Give your answer in seconds, and also in ages of the Universe (current models show the Universe to be about 13.7 billion years old), and comment on your results.

**Problem 7.3**: Scattering states in the finite square well. (20 points)

In class we studied bound states for the finite square well. We can also consider scattering states, which come in from infinity, scatter off the potential and are either reflected or transmitted. An interesting thing happens.

a) Find the transmission coefficient for a finite square well of width $L$ by adapting the result of part a) of the previous problem. To do this, turn the barrier upside down by replacing $V_0$ with $-V_0$ in that formula, and obtain an expression for $T^{-1}$ where everything is real. It may help to know that $\sinh(ix) = i \sin x$.

b) Consider the case where $E = V_0$. Determine $T$ at three different values of the parameters: $V_0 \to 0$, $L^2 m V_0/\hbar^2 = \pi^2/16$, $L^2 m V_0/\hbar^2 = \pi^2/4$. How does this compare to your classical expectation?

c) Assume general values for $E$ again. Keeping $V_0$ fixed, what happens in the limits $E \to 0$ and $E \to \infty$?

d) What is the set of all values of $E + V_0$ where $T = 1$? What is the relationship of these values with an *infinite* square well of length $L$?

e) Sketch $T$ as a function of $E$. (It doesn't have to be precise, but make sure you include all the structure you have observed in the various parts of the problem!)

*The behavior for $T$ found in part d) was observed in the scattering of electrons off noble gas atoms before quantum mechanics could explain it, and is called the* Ramsauer-Townsend effect.

**Problem 7.4**: Odd states in the finite square well. (20 points)

Consider the finite square well that we examined in class:

$$\begin{aligned} V(x) = &\quad -V_0 \qquad && -a < x < a\,, && (9)\\ = &\quad 0 && a < |x|\,, && (10) \end{aligned}$$

with $V_0 > 0$.

a) Consider the bound states with *odd* parity (even parity was done in class). Write down the solution in each of the three regions consistent with odd parity and determine the matching conditions.

b) Use the matching conditions to obtain a transendental equation for the energies of the form

$$f(z) = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}\,, \qquad (11)$$

with $z$ and $z_0$ defined appropriately (write them down) and $f(z)$ determined.

c) At what values of $z$ does $\sqrt{(z_0/z)^2 - 1}$ hit zero for $z_0 = 1$, 3 and 6? How many bound states exist for each of these three values of $z_0$? Plotting or drawing $f(z)$ may help. Is it possible to have zero odd bound states in a finite square well? Is it possible to have zero *total* bound states?