A Brief History of Modern Physics
and the development of the Schrödinger Equation

"Modern" physics means physics discovered after 1900; i.e. twentieth-century physics.

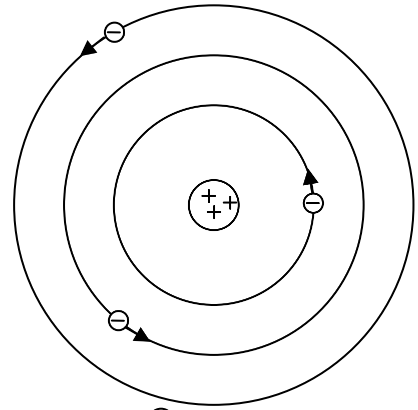
1900: Max Planck (German) tried to explain blackbody radiation using Maxwell's equations and statistical mechanics and found that he could not. He could only reproduce the experimentally-known BB spectrum by assuming that the energy in an electromagnetic wave of frequency f is *quantized* according to

$E_{\text{EM wave}} = n\,h\,f$, where n = 1, 2, 3, … and h = Planck's constant = 6.6×10^{-34} (SI units)

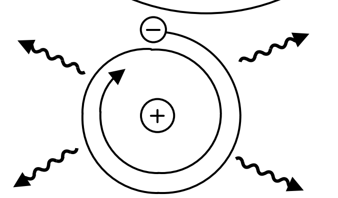
Planck regarding this as a math trick; he was baffled by its physical significance.

1905: Albert Einstein, motivated in part by Planck's work, invents the concept of a *photon* to explain the photoelectric effect. A photon is a quantum (packet) of electromagnetic radiation, with energy

$$\boxed{E_\gamma = h\,f = \hbar\,\omega\,.}$$



1911: Ernest Rutherford (New Zealand/Britain) shows that an atom consists of a small, heavy, positively-charged nucleus, surrounded by small light electrons. But there is a problem with the classical theory of this nuclear atom: An electron in orbit about a nucleus is accelerating and, according to Maxwell's equations, an accelerating charge must radiate (give off EM radiation). As the electron radiates, giving energy, it should spiral into the nucleus.



Classically, an electron in an atom should radiate and spiral inward as it loses energy.

1913: Niels Bohr (Danish), a theorist working in Rutherford's lab, invents the Bohr model. This is essentially a classical model, treating the electron as a particle with a definite position and momentum, but the model has two non-classical, *ad hoc* assumptions:

1) The angular momentum of the electron is quantized: $L = n\hbar$.

2) The electron orbits, determined by (1), are stable ("stationary"), do not radiate, unless there is a transition between two orbits, and then the atom emits or absorbs a single photon of energy $h\,f = |E_f - E_i|$

The predictions of Bohr model match the experimental spectrum of hydrogen perfectly.

It is important to remember that the Bohr model is simple, useful, and *wrong*. For instance, it predicts that the ground state of the H-atom has angular momentum $L = \hbar$, when in fact, the ground state of the H atom (s-state) has L = 0. The Bohr model is a *semi-classical model*, meaning it combines aspects of classical and quantum mechanics. Semi-classical models are frequently used by physicists because they are heuristically useful (easy to understand and often give correct results). But they must always be used with extreme care, because the microscopic world is really purely quantum. We insert classical mechanics into the microscopic world not because it is correct, but because it is convenient.

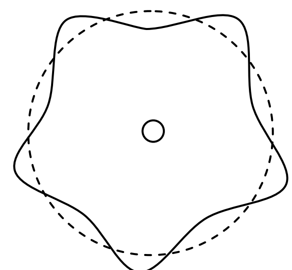
1922: Louis de Broglie (French) proposes wave-particle duality. Theory and experiment indicate that waves sometimes act like particles (photons). Perhaps, argues de Broglie, particles can sometimes act like waves. For photons: $E_\gamma = hf = h\dfrac{c}{\lambda}$. According to Special Relativity (and Maxwell's equations) light of energy E carries momentum $p = \dfrac{E}{c}$. Hence, $p_\gamma = \dfrac{E_\gamma}{c} = \dfrac{h}{\lambda} = \hbar k$. De Broglie argues that the same equations apply to particles and introduces the idea of *matter waves*.

$$\text{de Broglie relations: } E = hf = \hbar\omega \ , \quad p = \frac{h}{\lambda} = \hbar k$$

De Broglie's hypothesis provides a nice explanation for Bohr's quantization condition $L = n\hbar$: assuming that an integer number of wavelengths fit in one orbital circumference (the condition for a standing wave), we have

$$n\lambda = 2\pi r \quad \Rightarrow \quad r = \frac{n\lambda}{2\pi}$$

$$L = r\cdot p \underset{\text{(dB)}}{=} r\cdot\frac{h}{\lambda} = \frac{n\lambda}{2\pi}\cdot\frac{h}{\lambda} = n\hbar$$



n = 5

Soon, there was indisputable experimental verification of the photon concept and of the de Broglie relations. In 1923, the American Arthur Holly Compton observes the Compton Effect, the change in wavelength of gamma-rays upon collision with electrons. This effect can only be explained by assuming that gamma-rays are photons with energy $E_\gamma = hf$ and momentum $p_\gamma = \dfrac{h}{\lambda}$. Then, in 1927, Americans Davisson and Germer diffract a beam of electrons from a nickel crystal, experimentally verifying that $p = \hbar k$ for electrons.

Late in 1925, Erwin Schrödinger, then Professor of Physics at Zurich University, gives a colloquium describing de Broglie's matter wave theory. In the audience is physicist Peter Debye, who called this theory "childish" because "to deal properly with waves, one has to have a wave equation". Over Christmas break, Schrödinger begins developing his equation for matter waves.

1927: Erwin Schrödinger (Austrian) constructs a wave equation for de Broglie's matter waves. He assumes that a *free* particle (potential energy = V = 0) is some kind of wave described by

$$\Psi(x,t) = A\exp[i(kx-\omega t)]$$

{ Recall Euler's relation: $e^{i\theta} = \cos\theta + i\sin\theta$, so $\exp[i(kx-\omega t)] = \cos(kx-\omega t) + i\sin(kx-\omega t)$.} Initially, Schrödinger works with a complex wavefunction purely for mathematical convenience. He expects that, in the end, he will take the real part of Ψ to get the physically "real" matter wave.)

The energy of this free particle is all kinetic so $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$. According the de Broglie relations, this can be rewritten:

$$\hbar\omega = \frac{(\hbar k)^2}{2m}$$

Schrödinger searches for a wave equation that will reproduce this energy relation. He notes that

$$\frac{\partial\Psi}{\partial t} = \frac{\partial}{\partial t}\exp[i(kx-\omega t)] = -i\omega\Psi \quad \text{and}$$

$$\frac{\partial\Psi}{\partial x} = \frac{\partial}{\partial x}\exp[i(kx-\omega t)] = ik\Psi\ , \qquad \frac{\partial^2\Psi}{\partial x^2} = (ik)^2\Psi = -k^2\Psi \qquad \text{so ...}$$

the (trial) equation $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$ leads to $i\hbar(-i\omega\Psi) = -\frac{\hbar^2}{2m}(-k^2\Psi) \Rightarrow \hbar\omega = \frac{\hbar^2k^2}{2m}$

This looks promising. To describe a particle with both KE and potential energy V = V(x), Schrödinger added in the term $V\cdot\Psi$, producing finally

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi}$$

The Schrödinger Equation is really an energy equation in disguise. When you look at the S.E., you should try to see E = KE + PE. For a particle with frequency f (energy E = h f) and wavelength λ (momentum $p = h/\lambda$) in a potential V, this equation appears to correctly predict that

$$E_{tot} = KE + PE = \frac{p^2}{2m} + V \quad \text{which (according to de Broglie) is} \quad \hbar\omega = \frac{\hbar^2k^2}{2m} + V$$

This "derivation" is merely a plausibility argument. Schrödinger immediately used the equation to solve the hydrogen atom and found that he got the right answer for the energy levels. This gave him confidence that the equation was correct.

The physicist Paul Dirac famously asserted that the Schrödinger Equation accounts for "much of physics and all of chemistry". It is probably the most important equation of the 20th Century. Its effect on technological progress has been much, much greater than the more famous equation $E = mc^2$.

Schrödinger was quite puzzled by the nature of the wave function. What is the physical meaning of $\Psi(x,t)$? He wanted to think of it as some kind of physical matter wave, like an electromagnetic wave $E(x,t)$. But this interpretation could not explain a host of experimental results, such as that fact that a particle with a large extended wave function is always found at one small spot when a position measurement is made.

It was German theorist Max Born, who late in 1927 proposed that the wave function is a kind of *information wave*. It provides information about the probability of the results of measurement, but does not provide any physical picture of "what is really going on." Bohr, Heisenberg, and others argued that questions like "what is really going on" are meaningless. Humans live at the macroscopic level, excellently described by classical mechanics, and our brains evolved to correctly describe macroscopic (classical) phenomena. When we ask "what is going on", we are really asking for an explanation in terms that our brains can process, namely, a classical explanation. The microscopic world is fundamentally different from the classical world of large objects that we inhabit, and our brains' internal models simply don't apply at the level of atoms. There is no hope of understanding "what is really going on" in atoms because our brains are not built for that job. All we can know are the results of measurements made with macroscopic instruments.

This view, that the wave function provides probabilistic information, but not a physical picture of reality, is part of the "Copenhagen interpretation" of Quantum Mechanics — so-called because it was largely developed at Bohr's research institute in Copenhagen. Einstein, de Broglie, Schrödinger himself, and others were very, very dissatisfied with this view, and they never accepted the Copenhagen interpretation.

Nobel Prizes for QM

Many of the pioneers of QM eventually received Nobel Prizes in Physics

1918 Max Planck, concept of energy quanta
1921 Albert Einstein, photon concept and explanation of photoelectric effect
1922 Niels Bohr, Bohr Model
1926 James Franck and Gustav Hertz, Franck-Hertz experiment showing quantization of atomic levels
1927 Arthur Compton, Charles Wilson, Compton effect, Wilson cloud chamber
1929 Louis de Broglie, wave-particle duality
1932 Werner Heisenberg, Uncertainty Principle and Matrix formulation of QM
1933 Erin Schrödinger and Paul Dirac, Formulation of QM
1937 Clinton Davisson and George Thomson, experimental discovery of electron diffraction
1945 Wolfgang Pauli, exclusion principle
1954 Max Born, interpretation of the wave function

- Ernest Rutherford received the 1908 Nobel prize in chemistry for experimental investigations of radioactive decay, but never received the prize for discovery of the nuclear atom.
- Albert Einstein never received the prize for either Special or General Relativity.

Max Planck, German, 1858-1947

Albert Einstein, German-Swiss-American, 1879-1955

Ernest Rutherford, New Zealand-British, 1871-1937

Niels Bohr, Danish, 1885-1962

Louis de Broglie, French, 1892-1987

Erwin Rudolf Josef Alexander Schrödinger, Austrian, 1887-1961