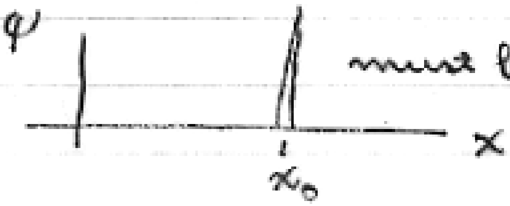
Eigenstates of $\hat{H}$ = states of definite energy
Eigenstates of $\hat{p}$ = states of definite momentum
Eigenstates of $\hat{x}$ = states of definite position

Position eigenstates $g_{x_0}(x)$



must be delta function

$$\hat{x}\, g_{x_0}(x) = x_0\, g_{x_0}(x)$$

variable x particular $x = x_0$

$$\hat{X} = x\cdot(\ ) \Rightarrow x \cdot g(x) = x_0\, g(x) \Rightarrow (x - x_0)\, g(x) = 0$$

$\Rightarrow g(x)$ is zero everwhere, except at $x = x_0$

$$\Rightarrow g_{x_0}(x) = \delta(x - x_0)$$

Notation: $g_{x_0}(x)$

Postulate 3 says $\text{Prob}\,(x_0 \to x_0 + dx) = |\langle x_0 | \Psi(x,t)\rangle|^2 dx$

$$= \left| \int dx\, \delta(x - x_0)\, \Psi(x,t) \right|^2 dx = |\Psi(x_0, t)|^2 dx \quad ✓$$

(agrees w/ previous version of P3)

<u>Postulate 4</u> (Wave function collapse)

If measurement of observable Q gives result q_n, then wavefn instantly collapses into corresponding eigenfunction $f_n(x)$.

Discrete spectrum example:

eigenstates of $\hat{H}$

$$\Psi(x,t) = \sum_n c_n(t)\, \psi_n(x) = \sum_n c_n\, e^{-iE_n t/\hbar}\, \psi_n(x)$$

Measure energy, find $E = E_{n_0} \Rightarrow$

$$\Psi \xrightarrow[\text{collapse}]{} \psi_n(x) \quad \left(\text{New } \Psi(x,t) = e^{-iE_{n_0}t/\hbar}\, \psi_{n_0}(x)\right)$$

Continuous Spectrum example:

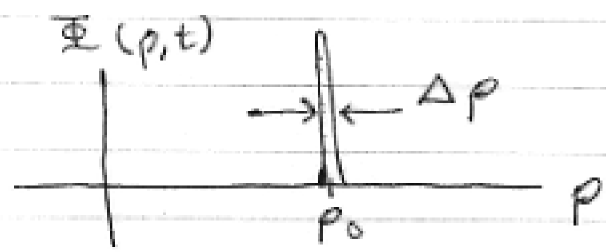
$$\Psi(x,t) = \int dp \; \Phi(p,t) \underbrace{\frac{e^{ipx/\hbar}}{\sqrt{2\pi\hbar}}}_{f_p(x) \,=\, \text{momentum eigenstates}} = \int dp \; \Phi(p,t) \, f_p(x)$$

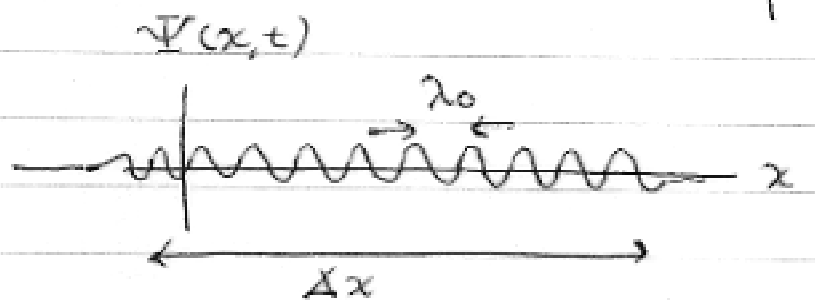
Measure momentum, get p within $p_0 \to p_0 + \Delta p$

No measurement of continuous variable has infinite precision. Precision Δp depends on measurement. $\Rightarrow$

Collapse is to normalizable Ψ that is almost an eigenstate $f_{p_0}(x)$

After collapse:





$p_0 = h/\lambda_0$

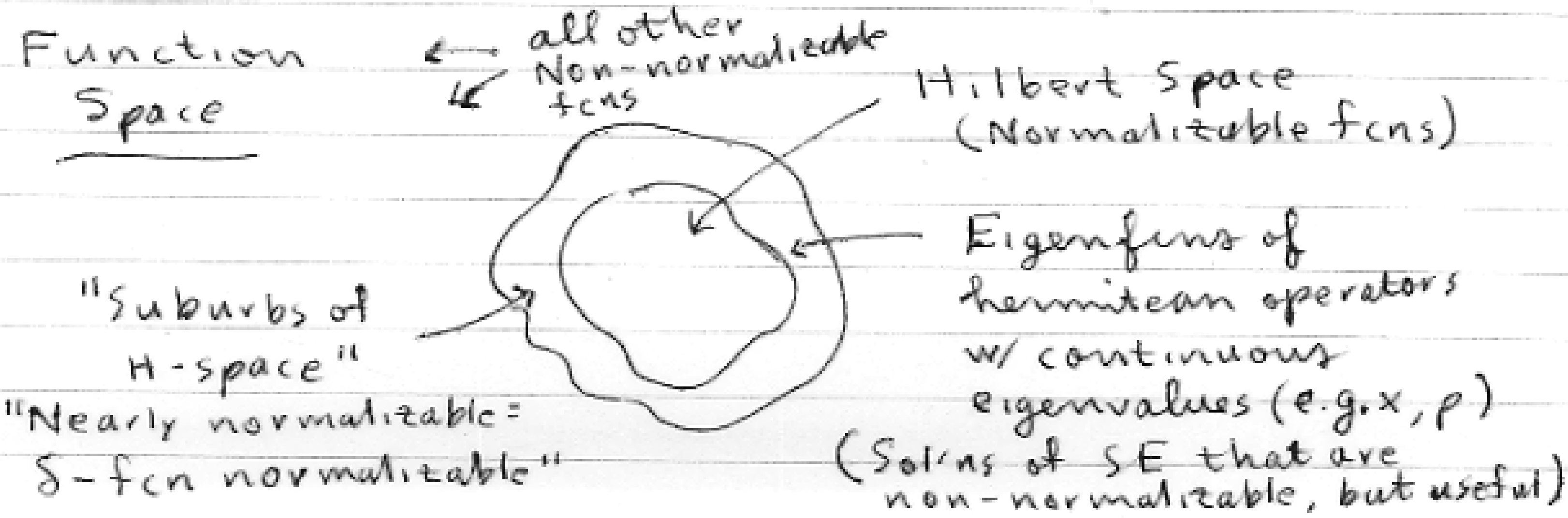
$\Delta x \cdot \Delta p \simeq \hbar$

(uncertainty principle)

$$\text{Prob}\,(p_0 \to p_0 + dp) = \left| \langle f_{p_0}(x) \,|\, \Psi(x,t) \rangle \right|^2 dp$$

$$= \left| \Phi(p_0, t) \right|^2 dp$$

Function Space



To make our list of postulates complete:

<u>Postulate 5</u> (Schrödinger Eqn) The time evolution of the wave fcn $\Psi(x,t)$ determined by TDSE

$$\hat{H}\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

To solve TDSE: Separation of variables ⇒ special sol'ns $\Psi_n(x,t) = e^{-iE_n t/\hbar} \cdot \psi_n(x)$

E_n's, $\psi_n(x)$'s from TISE $\hat{H}\psi_n(x) = E_n \psi_n(x)$

General Sol'n to TDSE:

$$\Psi(x,t) = \sum_n c_n(t)\, \psi_n(x) = \sum_n c_n\, e^{-iE_n t/\hbar} \cdot \psi_n(x)$$

$$c_n = c_n(t=0) = \int \psi_n^* \Psi(x,0)\, dx = \langle \psi_n | \Psi(x,0) \rangle$$

(ψ_n's form complete orthonormal set, since $\hat{H}$ hermitean!)

<u>Any</u> hermitean operator associated w/ observable has a complete orthonormal set of eigenfunctions, but the energy eigenfunctions are <u>special</u> in that they provide the time-dependence of $\Psi(x,t)$.

Comment about probabilities and normalization: Consider normalized basis states $\psi_n(x)$ and <u>un</u> normalized $\Psi = c_1\psi_1 + c_2\psi_2$

$\langle \Psi | \Psi \rangle = |c_1|^2 + |c_2|^2 \neq 1$. In this case, postulate 3 should read:

$$\text{Prob (find } q_1) = \frac{|c_1|^2}{|c_1|^2 + |c_2|^2} = \frac{|\langle \psi_1 | \Psi \rangle|^2}{\langle \Psi | \Psi \rangle}$$

(Must divide by $\langle \Psi | \Psi \rangle$ for probabilities to add up to 1)

$$\sum_n \text{Prob (find } q_n) = \frac{\sum_n |\langle \psi_n | \Psi \rangle|^2}{\langle \Psi | \Psi \rangle} = \frac{\sum_n |c_n|^2}{\sum_n |c_n|^2} = 1$$

Review:

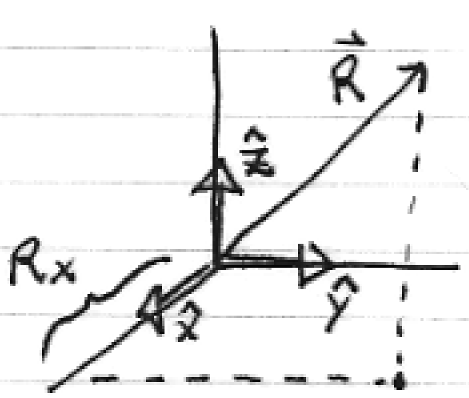
System $\Psi(x,t)$. Measure Q. $\hat{Q} f_n(x) = q_n f_n(x)$

P3: Find q_n w/ Prob $= |\langle f_n(x) | \Psi \rangle|^2$

P4: $\Psi(x,t) \xrightarrow[\text{collapse!}]{} f_n(x)$

$\langle f_n(x) | \Psi \rangle$ is "projection of Ψ onto $f_n(x)$"

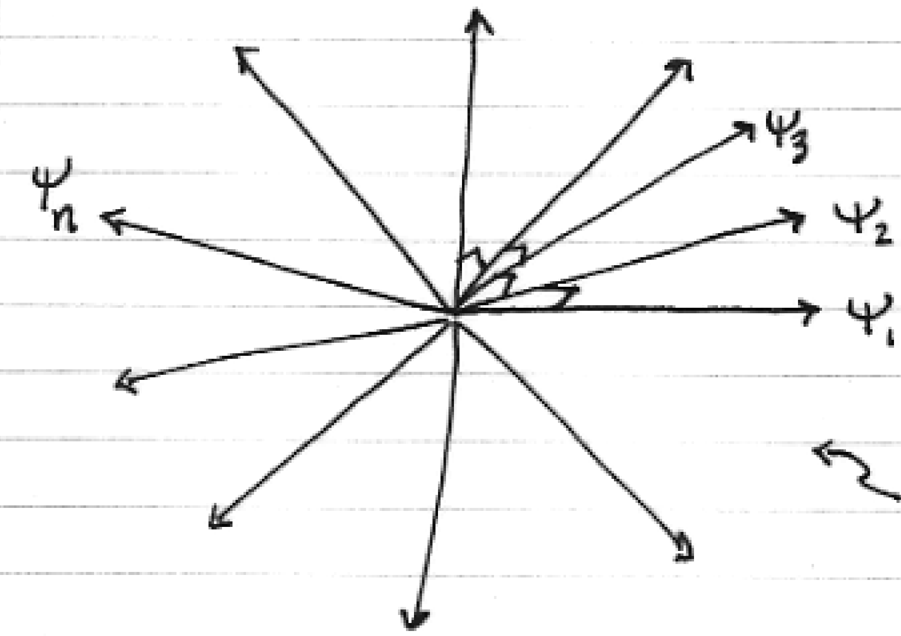
Euclidean space



$R_x = \hat{x} \cdot \vec{R}$ = projection of $\vec{R}$ along $\hat{x}$

Hilbert Space is a complex, infinite-dimensional vector space.

basis states: ψ_n from $\hat{H}\psi_n = E_n \psi_n$

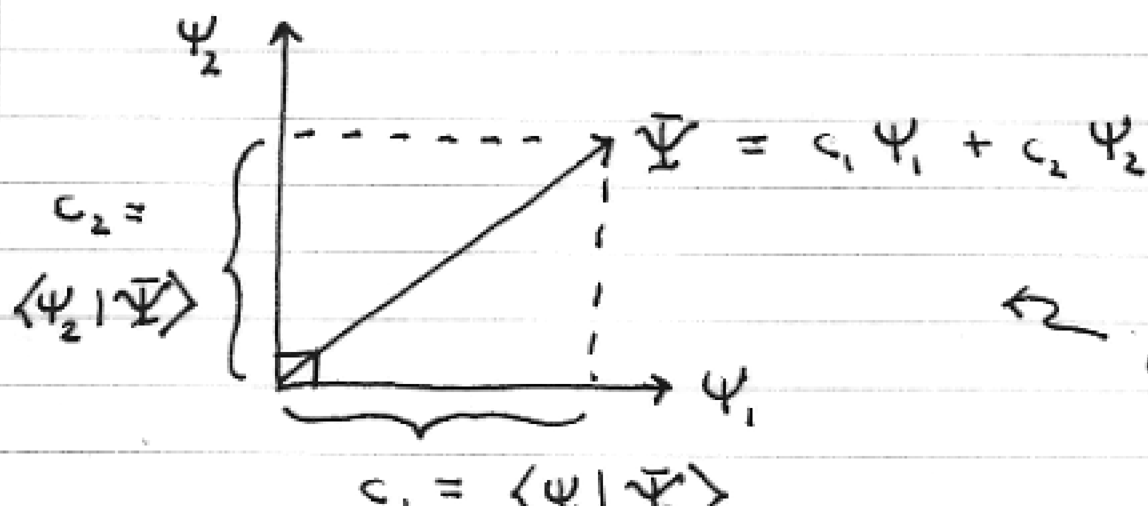


A̲ny wavefunction

$$\Psi = \sum_n c_n \psi_n$$

complex nbrs

all "perpendicular": $\langle \psi_n | \psi_m \rangle = \delta_{nm}$



a "poetic representation"

$$\Psi = \sum_n c_n \psi_n(x)$$

$$= \sum_n \langle \psi_n | \Psi \rangle \psi_n(x)$$

: c_n tells how much of Ψ is along ψ_n axis in H-space