University of Colorado, Department of Physics
PHYS3220, Spring 11, HW#2
due Wed, Jan 19, 9AM at start of class

1. Compton effect

   a) Consider a scattering between a massless light particle (photon) with energy $E_1 = h\nu$ and momentum $\mathbf{p}_1 = \hbar\mathbf{k}$ and a particle at rest ($E_2 = mc^2$ and $\mathbf{p}_2 = 0$). Using the formulae for four-momentum conservation

   $$E_1 + mc^2 = E_1' + E_2' \tag{1}$$
   $$\mathbf{p}_1 = \mathbf{p}_1' + \mathbf{p}_2' \tag{2}$$

   derive the following dependence of the wavelength of the scattered radiation, λ', on the scattering angle θ:

   $$\lambda' - \lambda = \frac{h}{mc}\,(1 - \cos\theta) \tag{3}$$

   (This theoretical result for the shift of the wavelength as a function of the scattering angle was in agreement with the experimental observations by A.H. Compton and gave evidence that light must behave as if it consists of particles.)

   b) A.H. Compton observed the so-called Compton shift $\Delta\lambda = \lambda' - \lambda$ in scattering experiments, in which a beam of X-rays was scattered at graphite. Would he be able to observe the shift with visible light too? Justify your answer.

   c) For scattering of X-ray light at an atom the scattered radiation as a function of wavelength shows two peaks at any scattering angle $\theta \neq 0$. One peak appears at about the same wavelength as the incident radiation ($\lambda' \approx \lambda$) while the other peak appears at a larger wavelength ($\lambda' > \lambda$). Give a physical interpretation for both components.

2. Interference

   a) In a two-slit interference experiment the intensity distribution $I(\theta)$ on the screen is proportional to:

   $$I(\theta) \propto 4\cos^2\left(\frac{k}{2} d \sin\theta\right) \tag{4}$$

   Assume that you perform an experiment with electrons having a kinetic energy of 25 eV and the separation of the two slits is $d = 0.25$ mm. What do you expect to see on the screen? Justify your answer.

   b) Assume that you perform a two-slit experiment with electrons, in which you are able to determine in some way (e.g. by using a sensor) through which slit each electron passes (Such an experiment can be performed). Do you expect to observe an interference pattern on the screen? Why or why not.

   c) Consider a two-slit experiment, in which the wave function (or, field) at slit 1 acquires an arbitrary *random* phase ϕ that is to be averaged over, so that the total wave function at the screen is $\Psi(x,t) = \exp(i\phi)\Psi_1(x,t) + \Psi_2(x,t)$ (or, $E(x,t) = \exp(i\phi)E_1(x,t) + E_2(x,t)$). Calculate $|\Psi(x,t)^2|$ (or, $|E(x,t)|^2$) and average over the phase. What will you see on the screen according to your result?

- There are more problems on the back -

3. Probabilities
   Determine A, the average $< x >$ and the variance σ_x^2 for the following probability distributions:

   a)
$$\rho(x) = \begin{cases} A, & \text{if} \quad a \le x \le b \\ 0, & \text{else} \end{cases}$$

   b)
$$\rho(x) = A\exp(-x^2/2a^2) \qquad -\infty < x < \infty \tag{5}$$

4. Wave equation
   The second-order differential equation,
$$\frac{d^2 f(x)}{dx^2} = -k^2 f(x) \tag{6}$$
   has two linearly independent solutions. These can be written in more than one way, and two convenient forms are
$$f(x) = A\exp(ikx) + B\exp(-ikx) \tag{7}$$
$$f(x) = a\sin(kx) + b\cos(kx) \tag{8}$$
   Verify that both are solutions of the equation (6) above. Since both are equally good solutions, we must be able to determine a and b in terms of A and B; do so.