University of Colorado, Department of Physics
PHYS3220, Fall 09, Some review problems - will not be graded -

1. The state of a particle in an infinite square well with $V(x) = 0$ for $0 < x < a$ and $V(x) = \infty$ elsewhere is given by the wave function

$$\Psi(x, t = 0) = \begin{cases} \sqrt{\frac{30}{a^5}} x(a - x), & 0 \leq x \leq a \\ 0, & \text{else} \end{cases}$$

The wave function $\Psi(x, t = 0)$ can be expanded in terms of a superposition of the stationary states of the infinite square well as:

$$\Psi(x, t = 0) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right)$$

Which c_n do you expect has the largest magnitude? Explain briefly. Then determine the nth coefficient of the expansion.

2. The state of a particle in an infinite square well is given by the wave function:

$$\Psi(x, t = 0) = A\left(\chi_1(x) + \exp(i\phi)\chi_2(x)\right)$$

where $\chi_n(x)$ $(n = 1, 2)$ are the two lowest stationary states of the infinite square well.

   a) Find $\Psi(x, t)$, $|\Psi(x, t)|^2$, and $< x >$. Study the special cases $\phi = 0$, $\phi = \pi/2$ and $\phi = \pi$.

   b) At time $t = 0$ is the particle more likely to be found in the left half, the right half or equally likely to be in either half of the well? Does your answer depend on the relative phase ϕ?

   c) Then find the next moment in time (expressed in given or fundamental constants) after which the particle is more likely to be found in the right half of the well.

   (This problem shows you that although the overall phase of the wave function is of no physical significance, the relative phase of the coefficients in the expansion matters.)

3. Consider a finite potential well of width $2a$ given by

$$V(x) = \begin{cases} -V_0, & -a < x < a \\ 0, & \text{elsewhere} \end{cases}$$

and a semi-infinite potential well of width a given by

$$V(x) = \begin{cases} \infty, & x < 0 \\ -V_0, & 0 < x < a \\ 0, & \text{elsewhere} \end{cases}$$

with $V_0 > 0$. Which ground-state energy is lower - that to the finite potential well (width $2a$) or that of the semi-infinite potential well (width a)? Argue without elaborate calculations.

4. Is the uncertainty principle satisfied for the stationary states of the harmonic oscillator? Which state comes closest to the uncertainty limit? (You may use the results for the expectation values derived in problem 3 of HW11)

5. Consider the potential step

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x > 0 \end{cases}$$

with particles incident from the right with $E > V_0$

   a) Write down the general solution in each region, indicate the meaning of each term (incident, reflected, transmitted) and justify any term that you set to zero.

   b) In the case of $E = 2V_0$ how do the wavelengths of the plane waves in the two regions $x < 0$ and $x > 0$ compare?