I: One-dimensional wave functions

A. Free particle with momentum p:

1. Write down the de Broglie relations relating momentum ($\vec{p}$) to wavelength (λ) and energy to frequency. Momentum is a vector but wavelength is not. How do you reconcile this discrepancy?

2. Physicists tend to introduce new terms in order to simplify working with equations. Today, we will use the following terms:

$\vec{k} = (2\pi/\lambda)\hat{r}$:	wave vector where $\hat{r}$ is the direction of propagation of the wave
$\omega = 2\pi f$:	angular frequency (f is the frequency of a wave)
$\hbar = h/(2pi)$:	A convenient term (pronounced 'h bar') which helps keep track of many of the factors of 2π which appear in quantum mechanics

   Write the deBroglie relations using the wave vector, the angular frequency, and $\hbar$.

3. What is the general form in one dimension of the wave function for a free particle of mass m and momentum p? Recall that the wave function has to satisfy the one-dimensional Schrödinger equation:

   $$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}$$

   If you have trouble coming up with a solution, try a general solution and let the Schrödinger equation guide you to find out the correct specific solution.

4. Can this wave function ever be entirely real? If so, show how this is possible. If not, explain why not.

5. What can you say about the integral of $|\Psi(x,t)|^2$ from $-\infty$ to ∞?

6. Is this a possible wave function for a real, physical particle? Why or why not?

✓ Check your results with a tutorial instructor.

B. Free particle with energy E:

1. What is the general form of the wave function for a free particle of mass m and energy E? How (if at all) does this wave function differ from the wave function you obtained in part A?

2. Can this wave function ever be entirely real? If so, show how this is possible. If not, explain why not.

3. What can you say about the integral of $|\Psi(x,t)|^2$ from $-\infty$ to ∞?

4. Is this a possible wave function for a real, physical particle? Why or why not?

✓ Check your results with a tutorial instructor.