University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#4
due Wed, Sep 16, 2PM at start of class

1. (Griffiths, problem 1.7, 15 pts) Prove the Ehrenfest theorem

$$\frac{d < p_x >}{dt} = \left\langle -\frac{\partial V}{\partial x} \right\rangle \tag{1}$$

where the potential V is a real quantity. (This theorem tells us that expectation values obey classical laws.)

2. The three expressions xp_x, $p_x x$ and $(xp_x + p_x x)/2$ are equivalent in classical mechanics. The corresponding quantum mechanical operators are $\hat{X}\hat{P}_x$, $\hat{P}_x\hat{X}$ and $(\hat{X}\hat{P}_x + \hat{P}_x\hat{X})/2$. Show that $\hat{X}\hat{P}_x$ and $\hat{P}_x\hat{X}$ are not Hermitian operators, but $(\hat{X}\hat{P}_x + \hat{P}_x\hat{X})/2$ is a Hermitian operator. (15 pts).
Hint: To show that an operator $\hat{A}$ is Hermitian, check if $< A >=< A >^*$, where $\hat{A}$ is the operator assigned to the observable A.

3. In classical mechanics all quantities obey the rules of ordinary algebra, e.g. the commutation rule. The previous problem has shown that operators in quantum mechanics in general do not commute with each other. Thus, if $\hat{A}$ and $\hat{B}$ are two operators, the product $\hat{A}\hat{B}$ is not necessarily equal to the product $\hat{B}\hat{A}$. We define the *commutator* of two operators $\hat{A}$ and $\hat{B}$ as $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$. Show the following useful relations for commutators (total: 15 pts):

    a) Antisymmetry: $[\hat{A}, \hat{B}]$ = -$[\hat{B}, \hat{A}]$
    b) Linearity: $[\hat{A}, \hat{B} + \hat{C}] = [\hat{A}, \hat{B}] + [\hat{A}, \hat{C}]$
    c) Distributivity: $[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + B[\hat{A}, \hat{C}]$
    d) Jacoby identity: $[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0$

4. (Griffiths, problem 1.8, total: 15 pts) Let $\Psi(x, t)$ be a solution of the (one-dimensional) Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\Psi(x, t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x, t)\right]\Psi(x, t). \tag{2}$$

Suppose you add a constant V_0, which is independent of x and t, to the potential energy term $V(x, t)$.

    a) Show that the solution of the new Schrödinger equation is given by $\Psi(x, t)\exp(-iV_0 t/\hbar)$.
    b) What effect does this time-dependent phase factor have on the expectation value of an observable? Consider the general case, not a special observable.