University of Colorado, Department of Physics
PHYS3220, Fall 09, Some final review problems

1. At time t=0, a particle is represented by the wave function:

$$\Psi(x,t=0) = \begin{cases} A\frac{x}{a}, & \text{if } 0 \le x \le a \\ A\frac{b-x}{b-a}, & \text{if } a \le x \le b \\ 0, & \text{else} \end{cases}$$

   where a and b are constants. At which x does the probability density peak? Calculate the probability to find the particle to the left of a (i.e. for $x \le a$).

2. Consider the following wave function for a particle of mass m at time $t = 0$, characterized by a positive constant k_0

$$\Psi(x,t=0) = A\left[\exp(ik_0x) + \exp(-ik_0x)\right]$$

   Find the potential $V(x)$ for which $\Psi(x,t=0)$ solves the time-dependent Schrödinger equation? Does $\Psi(x,t=0)$ represent an acceptable physical state? Justify your answer.

3. How does the probability current density $J(x,t)$ change with time, if the system is in a stationary state (energy eigenstate)? Explain your answer.

4. A normalized wave function of a particle is written as:

$$\Psi(x,t=0) = \frac{1}{\sqrt{3}}\chi_1(x) + \frac{1}{\sqrt{6}}\chi_2(x) + \frac{1}{\sqrt{2}}\chi_3(x)$$

   where χ_1, χ_2 and χ_3 are energy eigenstates with the corresponding *unequal* energies $E_1 \ne E_2 \ne E_3$. Write down $\Psi(x,t)$. Is $\Psi(x,t)$ a stationary state? Justify your answer.

5. The potential energy V(x) for a particle is given by:

$$V(x) = \begin{cases} V_0, & x < 0 \\ 0, & 0 < x < a \\ V_0/2, & a < x < 2a \\ V_0, & 2a < x \end{cases}$$

   Sketch this potential. Assume V_0 and a have been chosen so that $0 < E_1 < V_0/2 < E_2 < V_0$. Draw two separate sketches, one for the ground state (with energy E_1) and another one for the first excited state (with energy E_2). Comment on the features of the curves, in particular at $x = 0$, a, and $2a$.

6. Consider the potential step defined by

$$V(x) = \begin{cases} V_0, & x < 0 \\ 0, & x > 0 \end{cases}$$

where $0 < V_0$. Consider states with $E > V_0$. Write down the transmission coefficient T for particles moving from the far right ($x \gg 0$) to the far left ($x \ll 0$) in terms of the amplitudes of the general solutions in the different regions and any other constants (Reminder: If $\Psi(x) = A\exp(ikx)$, then A is called the amplitude).

7. Why do we care about commutators of operators in quantum mechanics? Give more than one reason.

8. We have shown that $(\hat{A}\hat{B})^\dagger = \hat{B}^\dagger\hat{A}^\dagger$ for any operators A and B (even ones which are not Hermitian operators). Use this result to answer: When is the product of two Hermitian operators itself also a Hermitian operator? Is the product $\hat{x}\hat{p}$ a Hermitian operator?

9. An operator $\hat{A}$ (representing an observable A) has two normalized eigenfunctions Ψ_1 and Ψ_2 with eigenvalues a_1 and a_2. Operator $\hat{B}$ (representing an observable B) has two normalized eigenfunctions Φ_1 and Φ_2 with eigenvalues b_1 and b_2. Suppose that the two sets of eigenfunctions are related by:

$$|\Psi_1 >= \frac{2}{3}|\Phi_1 > + \frac{\sqrt{5}}{3}|\Phi_2 > \qquad |\Psi_2 >= -\frac{\sqrt{5}}{3}|\Phi_1 > + \frac{2}{3}|\Phi_2 >$$

   a) Let's start in some unspecified state, and then observable A is measured. Further, assume that you do measure the particular value a_1. What is the state of the system immediately after this measurement?

   b) Immediately after the measurement of A (which, recall, happened to yield a_1), the observable B is then measured. What are the possible results of the B measurement, and what are their probabilities?

   c) Suppose that the outcome of the B measurement is b_1. And immediately after that measurement, we measure A again. What is the probability of getting a_1 again?

10. Starting with the definitions of raising and lowering operators $\hat{L}_+$ and $\hat{L}_-$ in terms of $\hat{L}_x$ and $\hat{L}_y$, prove that $[\hat{L}_z, \hat{L}_+] = \hbar L_+$. Use the result to show that: If $Y_{l,m}$ is an eigenfunction of $\hat{L}_z$ with eigenvalue $m\hbar$ (and $m < l$) then $\hat{L}^+ Y_{l,m}$ is an eigenfunction of L_z with eigenvalue $(m+1)\hbar$.