Quantum I (PHYS 3220)

concept questions

Operators

A wavefunction $\psi(x)$ has been expressed as a sum of energy eigenfunctions ($u_n(x)$'s):

$$\psi(x) = \sum_n c_n \, u_n(x)$$

Compared to the original $\psi(x)$, the set of numbers $\{c_1, c_2, c_3, \ldots.\}$ contains:

A) more information.

B) less information.

C) the same information

D) cannot be determined/depends.

Consider a complex vector **V**:

$$|\mathbf{V}\rangle \Leftrightarrow (V_1, V_2)$$

Where V_1 and V_2 are complex numbers (they are the "components of **V**")

If we want the inner product of V with itself, $\langle\mathbf{V}|\mathbf{V}\rangle$, to be positive (for nonzero **V**), what should $\langle\mathbf{A}|\mathbf{B}\rangle$ be?

A) $A_1 B_1 + A_2 B_2$

B) $A^*_1 B_1 + A^*_2 B_2$

C) $|A_1 B_1 + A_2 B_2|$

D) More than one of these options

E) *NONE* of these makes sense.

If $f(x)$ and $g(x)$ are wave functions, and c is a constant, then $\langle c \cdot f | g \rangle = ?$

A) $c \langle f | g \rangle$

B) $c^* \langle f | g \rangle$

C) $|c| \langle f | g \rangle$

D) $c \langle f^* | g \rangle$

E) None of these

A vector can be written as a column of its components; likewise a vector in Hilbert space (a wave function) can be written as an infinite column of its components in a basis of the u_ns:

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \qquad \Psi = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \end{pmatrix}$$

The dot product of two vectors **A** and **B** is:

$$\vec{A} \cdot \vec{B} = \sum_{i=x,y,z} A_i B_i$$

The inner product of two wavefunctions,

$$\Psi = \sum_n c_n \psi_n \text{ and } \Phi = \sum_n d_n \psi_n, \text{ is } \int \mathrm{d}x\, \Psi^* \Phi = \ldots$$

A) $\sum_n c_n d_n$

B) $\sum_n |c_n|\,|d_n|$

C) $\sum_n |c_n|^2 |d_n|^2$

D) $\sum_n \left(|c_n|^2 + |d_n|^2\right)$

E) something else!

A vector can be written as a column of its components in a basis $(\hat{x},\hat{y},\hat{z})$; likewise a vector in Hilbert space (a wave function) can be written as an infinite column of its components in a basis of the ψ_ns:

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \qquad \Psi = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \end{pmatrix}$$

The dot product of two vectors **A** and **B** is:

$$\vec{A}\cdot\vec{B} = \sum_{i=x,y,z} A_i B_i$$

The inner product $\int dx\Psi^*\Phi$ of wavefunctions $\Psi = \sum_n d_n\psi_n$ and $\Phi = \sum_n c_n\psi_n$ is given by

A) $\sum_n d_n^* c_n$

B) $\sum_n |d_n|\,|c_n|$

C) $\sum_n |d_n|^2 |c_n|^2$

D) $\sum_n \left(|d_n|^2 + |c_n|^2\right)$

E) zero

Do you plan to attend today's Tutorial (on "functions as vectors")

A) Yes, at 3 pm
B) Yes, at 4 pm
C) Perhaps, more likely at 3
D) Perhaps, more likely at 4
E) No, can't come/not planning on it.

Do the set of all normalized wave functions form a vector space?

A) Yes

B) No

A wavefunction $\psi(x)$ has been expressed as a sum of energy eigenfunctions ($u_n(x)$'s):

$$\psi(x) = \sum_n c_n \, u_n(x)$$

Viewing $\psi(x)$ as a *vector* in Hilbert space, what role do the c_n's and u_n's play?:

A) u_n's are basis vectors, c_n's are norms of vectors
B) u_n's are components, c_n's are norms of vectors
C) u_n's are basis vectors, c_n's are components
D) u_n's are components, c_n's are basis vectors
E) Something else/I don't understand

A wavefunction $\psi(x)$ has been expressed as a sum of energy eigenfunctions ($u_n(x)$'s):

$$|\psi\rangle = \sum_n c_n \, |u_n\rangle$$

Viewing $|\psi>$ as a *vector* in Hilbert space, what role do the c_n's and $|u_n>$'s play?:

A) u_n's are basis vectors, c_n's are norms of vectors
B) u_n's are components, c_n's are norms of vectors
C) u_n's are basis vectors, c_n's are components
D) u_n's are components, c_n's are basis vectors
E) Something else/I don't understand

If a wave function, *f*(*x*) is square-integrable

$$\int_{-\infty}^{\infty} |f(x)|^2 \mathrm{d}x < \infty$$

does that mean that *f*(*x*) is always normalizable? That is, can we always find a number, *A*, such that)

$$\int_{-\infty}^{\infty} |A \cdot f(x)|^2 \mathrm{d}x = 1$$

A) Yes B) No

True (A) or False (B):

The operator i (i.e. multiplying by the constant $i = \sqrt{-1}$) is a Hermitian operator.

The momentum operator $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ is hermitian,

meaning $\langle f \mid \hat{p} g \rangle = \langle \hat{p} f \mid g \rangle$.

Is $\hat{p}^2$ hermitian?

A) Yes

B) No

True (A) or False (B):
If f(x) is a wave function, then

$$\left(\frac{1}{i}\frac{\mathrm{d}f}{\mathrm{d}x}\right)^{*} = -\frac{1}{i}\frac{\mathrm{d}f^{*}}{\mathrm{d}x}$$

True Always (A), False Always (B) or True Sometimes (C):

$$\int_{\text{limits}} \mathrm{d}x \; f(x)\frac{\mathrm{d}g(x)}{\mathrm{d}x} = f(x)g(x)\Big|_{\text{limits}} - \int_{\text{limits}} \mathrm{d}x \; \frac{\mathrm{d}f(x)}{\mathrm{d}x} g(x)$$

Given that Q is a Hermitian operator, what can you say about $<Q^2>$, i.e. $\langle \Psi | Q^2 | \Psi \rangle$?

A) It will be real *if and only if* Ψ is a stationary state (eigenstate of H)
B) It will be real *if and only if* Ψ is an eigenstate of Q.
C) It *must* be real, and $= <Q>^2$
D) It *must* be real, but cannot $= <Q>^2$
E) It *must* be real, and may or may not $= <Q>^2$

Suppose $|f_1\rangle$ and $|f_2\rangle$ are eigenvectors of operator Q, with eigenvalues q_1 and q_2 respectively.

Is $a|f_1\rangle+b|f_2\rangle$ an eigenvector of Q?

A) Yes, always

B) No, never

C) Only if $a=b$

D) Only if $q_1=q_2$

E) Not sure/something else/???

In the simple harmonic oscillator, the eigenvalues of H are $E_n=\hbar\omega(n+1/2)$, and a measurement of energy will always observe one of these values.

What can we say about $\langle H \rangle$?

A) It must always be one of the E_n

B) It will never be one of the E_n

C) It is sometimes one of the E_n, but only for a stationary state

D) It is sometimes one of the $E_{n,}$ not necessarily for a stationary state

E) Something else!

Postulate 3: A measurement of observable "O" can only result in one of the eigenvalues of Ô

If we measure the momentum of a free particle in 1D, what outcomes are possible?

A) Any real value is possible

B) Any positive value is possible

C) Any value is possible (including complex values)

D) Only integer values are possible

E) For a given particle, there is only ONE possible value (or perhaps 2, $\pm p_0$)

To think about: What if we measure x instead of p?

WHITEBOARDS:

Come up with *two different* normalized states, $\psi(x)$, for a particle in a harmonic oscillator potential, such that the probability of measuring E_0 (ground state energy) is exactly 1/3.

For each state you come up with, what is <H>?

(*To think about if you have time:*

- If I give you the value of <H>, is your $\psi(x)$ uniquely determined?
- How does your state evolve with time, $\psi(x,t)$?
- Given only that Prob(E_0)=1/3, what is the range of all possible <H>'s?

Suppose we take the particle in the state you came up with, measure H, and happen to get E_0, (the energy of the ground state.)

Sketch $|\psi(x)|^2$ immediately after measurement. What happens as time goes by?

If you have time, resketch if…

- … you had been given a particle, measured H, and happened to get E_1 (the energy of the first excited state?)
- … you had been given a particle, measured *x*, and happened to get $x=0$ (to high precision!)

Observable A: $\hat{A}\psi = a\psi$

normalized eigenstates ψ_1, ψ_2, eigenvalues a_1, a_2.

Observable B: $\hat{B}\phi = b\phi$

normalized eigenstates ϕ_1, ϕ_2, eigenvalues b_1, b_2.

The eigenstates are related by

$$\psi_1 = (2\phi_1 + 3\phi_2)\Big/\sqrt{13} \qquad \psi_2 = (3\phi_1 - 2\phi_2)\Big/\sqrt{13}$$

Observable A is measured, and the value a_1 is found. What is the system's state immediately after measurement?

A) ψ_1 B) ψ_2 C) $c_1\psi_1 + c_2\psi_2$ (c_1 & c_2 non-zero)

D) ϕ_1 E) ϕ_2

Observable A: $\hat{A}\psi = a\psi$

normalized eigenstates ψ_1, ψ_2, eigenvalues a_1, a_2.

Observable B: $\hat{B}\phi = b\phi$

normalized eigenstates ϕ_1, ϕ_2, eigenvalues b_1, b_2.

The eigenstates are related by

$$\psi_1 = {(2\phi_1 + 3\phi_2)}\Big/{\sqrt{13}} \qquad \psi_2 = {(3\phi_1 - 2\phi_2)}\Big/{\sqrt{13}}$$

Immediately after the measurement of A, the observable B is measured. What is the probability that b_1 will be found?

A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

Observable A: $\hat{A}\psi = a\psi$

normalized eigenstates ψ_1, ψ_2, eigenvalues a_1, a_2.

Observable B: $\hat{B}\phi = b\phi$

normalized eigenstates ϕ_1, ϕ_2, eigenvalues b_1, b_2.

The eigenstates are related by

$$\psi_1 = {(2\phi_1 + 3\phi_2)}\Big/{\sqrt{13}} \qquad \psi_2 = {(3\phi_1 - 2\phi_2)}\Big/{\sqrt{13}}$$

If the grad student failed to measure B, but instead measured A for a second time, what is the probability that the second measurement will yield a_1?

A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

A system is in a state which is a linear combination of the n=1 and n=2 energy eigenstates

$$\Psi(x,t) = \frac{1}{\sqrt{2}} e^{-i\omega_1 t}\psi_1(x) + \frac{1}{\sqrt{2}} e^{-i\omega_2 t}\psi_2(x)$$

What is the probability that a measurement of energy will yield energy E_1?

A) $\frac{1}{2}\exp(-i2\omega_1 t)$ B) $1/\sqrt{2}$ C) $1/2$ D) $1/4$

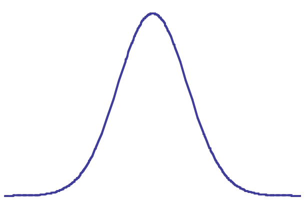
E) Something else!

You measure the energy of a particle in a simple harmonic oscillator, and find E_1 (i.e. the first excited energy, $3/2\ \hbar\omega$)

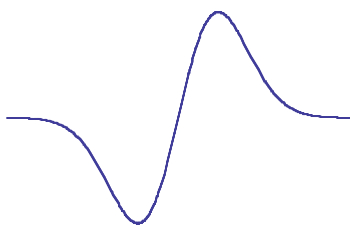
Which graph best represents $|\psi(x)|^2$ immediately after the measurement?

To think about: how does the plot evolve in time?

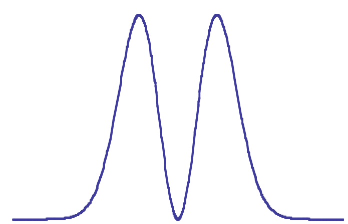
A)



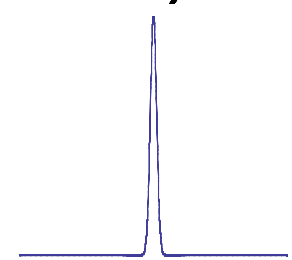
B)



C)



D)



Or E) None of these is remotely correct.

If we change the potential $V(x)$, do the eigenvectors of $\mathbf{x}$ change? (*i.e.*, $g_{x0}(x)$)
How about the eigenvectors of $\mathbf{p}$ (*i.e.*, $f_{p0}(x)$)?

A) g_{x0} will change, and so will f_{p0}
B) g_{x0} will change, but f_{p0} will NOT
C) g_{x0} will NOT change, but f_{p0} will
D) Neither g_{x0} nor f_{p0} will change
E) It depends!!

Suppose Ψ(x,t) is known to be an energy eigenstate (state n):

$$\Psi(x,t) = u_n(x)\exp(-iE_n t/\hbar) \ .$$

Can that energy eigenstate be written as

$$\Psi(x,t) = \int \mathrm{d}p\,\Phi(p,t)\, f_p(x)$$

$$\text{where } f_p(\mathrm{x}) = \left(1 \big/ \sqrt{2\pi\hbar} \right) \exp\left(\mathrm{ipx} \big/ \hbar \right) \ ?$$

A) Yes B) No C) Maybe

Do the set of delta-functions, $\delta(x\text{-}x_0)$ (for all values of x_0), form a complete set? That is, can any function $f(x)$ in the Hilbert Space be written as a linear combination of the delta function like so:

$$f(x) = \int_{-\infty}^{\infty} F(x_0)\delta(x - x_0)\mathrm{d}x_0$$

A) Yes B) No

(If you answer Yes, you should be able to construct the function F(x0).)

An isolated system evolves with time according to the TDSE with $V = V(x)$. The wave function $\Psi = \Psi(x,t)$ depends on time.

Does the expectation value of the energy $\langle \hat{H} \rangle$ depend on time?

A) Yes, always

B) No, never

C) Sometimes, depending on initial conditions

A system (described by PE = $V(x)$) is in state $\Psi(x,t)$ when a measure of the energy is made. The probability that the measured energy will be the nth eigenvalue E_n is

$$\left|\langle u_n \mid \Psi(x,t)\rangle\right|^2 = \left|c_n \exp\left(\frac{-iE_n t}{\hbar^2}\right)\right|^2 .$$

Does the probability of finding the energy = E_n when the system is in state $\Psi(x,t)$ depend on the time t of the measurement?

A) Yes

B) No

Can the wave function $\Psi(x,t)$ describing an **arbitrary** physical state **always** be written in the form

$$\Psi(x,t) = u_n(x)e^{-iE_nt/\hbar}$$

where $u_n(x)$ and E_n are solutions of

$$\hat{H}u_n(x) = E_nu_n(x)\ ?$$

A) Yes B) No

A system (described by PE = $V(x)$) is in state $\Psi(x,t)$ when a measurement of the energy is made. Does the probability of finding the energy = E_n depend on the time t of the measurement?

A) Yes B) No

C) Depends on $\Psi(x,0)$

D) Depends on V(x)

Given two quantum states labeled $|f\rangle$ and $|g\rangle$, which relation below must be true?

A) $\langle f|f\rangle \langle g|g\rangle \geq |\langle f|g\rangle|^2$

B) $\langle f|f\rangle \langle g|g\rangle \leq |\langle f|g\rangle|^2$

C) $\langle f|f\rangle \langle g|g\rangle = |\langle f|g\rangle|^2$

D) None of the above is guaranteed, it depends on the states f and g.

In general, given Hermitian operators A and B, and a state ψ, (and with the usual notation $\langle A \rangle = \langle \psi | A | \psi \rangle$ what can you say about

$$\langle \psi | \langle A \rangle B | \psi \rangle = ?$$

A) $\langle AB \rangle$

B) $\langle BA \rangle$

C) $\langle B \rangle \langle A \rangle$

D) MORE than one of these is correct!

E) NONE of these is, in general, correct!

The proof of the generalized uncertainty principle involves inner - products like $\left\langle \left\langle \hat{\text{A}} \right\rangle \Psi \middle| \hat{B}\Psi \right\rangle$

Does this equal $\left\langle \hat{\text{A}} \right\rangle \left\langle \hat{\text{B}} \right\rangle$?

$$\text{Hint: } \left\langle c\Psi_1 \middle| \Psi_2 \right\rangle = c^* \left\langle \Psi_1 \middle| \Psi_2 \right\rangle$$

A) Yes

B) No

Suppose the state function, Ψ, is known to be the eigenstate Ψ_1 of operator $\hat{A}$ with eigenvalue a_1 : What is the standard deviation

$$\sigma_A = \sqrt{\left\langle \Psi_1 \left| \left(\hat{A} - \langle \hat{A} \rangle \right)^2 \Psi_1 \right. \right\rangle}?$$

A) Zero always

B) Non-zero always

C) Zero or non-zero depending on the details of the eigenfunction, Ψ_1

Suppose two observables commute,

$$\left[\hat{A},\hat{B}\right]=0.$$

Is $\sigma_A\sigma_B$ zero or non-zero?

A) Zero

B) Non-zero

C) Zero or non-zero depending on details of the state function Ψ used to compute $\sigma_A\sigma_B$

Consider a Hamiltonian such as,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

What is the value of?

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle E \rangle = \frac{\mathrm{d}}{\mathrm{d}t}\langle \Psi | \hat{H}\Psi \rangle$$

A) Zero always

B) Non-zero always

C) Zero or non-zero depending on Ψ.

Consider a Hamiltonian such as,

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x})$$

What is the value of?

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle E\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\left\langle \Psi \middle| \hat{H}\Psi \right\rangle$$

A) Zero always

B) Non-zero always

C) Zero or non-zero depending on Ψ.

What is the value of,

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\Psi|1\Psi\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\left\langle\Psi\middle|\hat{1}\Psi\right\rangle = \frac{\mathrm{d}}{\mathrm{d}t}\left\langle\hat{1}\right\rangle\ ?$$

where the operator is defined as

$\hat{1}\Psi = \Psi$ for any wave function, Ψ.

A) Zero always

B) Non-zero always

C) Zero or non-zero depending on Ψ.

If $\left[\hat{H},\hat{Q}\right]=0$, then $\left\langle\left[\hat{H},\hat{Q}\right]\right\rangle=\left\langle\Psi\middle|\left[\hat{H},\hat{Q}\right]\Psi\right\rangle=0$ for any Ψ (can you see why?). If $\left[\hat{H},\hat{Q}\right]\neq 0$, then does it follow that $\left\langle\left[\hat{H},\hat{Q}\right]\right\rangle\neq 0$ for any Ψ?

A) Yes
B) No

If you have a *single* physical system with an unknown wave function, Ψ, can you determine $\langle E \rangle = \langle \Psi | \hat{H} \Psi \rangle$ experimentally?

A) Yes

B) No

If you have a system initially with some state function Ψ, and then you make a measurement of the energy and find energy E, how long will it take, after the energy measurement, for the expectation value of the position to change significantly?

A) Forever, <x> is a constant

B) $\hbar/E$

C) neither of these

Complex number $z = a + ib$.

What is the value of $\left|\frac{z}{z^*}\right| = \frac{|z|}{|z^*|}$

A) $a^2 + b^2$

B) a

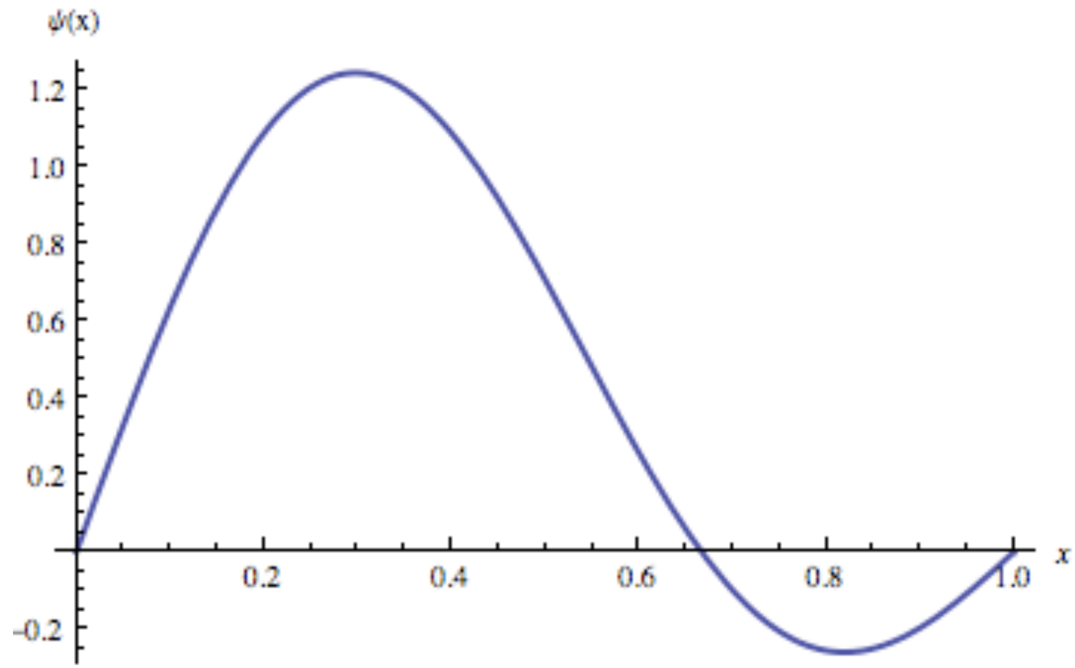
C) b

D) 1

E) 0

The wave function $\Psi(x,t)$ below is a solution to the time-independent Schrödinger equation for an infinite square well that goes from 0 to 1.

How many energy eigenstates of the system have non-zero amplitude?



A) 1 B) 2 C) 3 or more

D) Not enough info

Do the set of bras $\langle f \,|$ corresponding to the kets $|\, f \rangle$ form a vector space?

A) Yes

B) No

Consider the object formed by placing a ket to the left of a bra like so: $|f\rangle\langle g|$.

This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number).
C) a function (transforms a number into a number).
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Consider the object formed by placing a bra to the left of a operator like so: $\langle g | \hat{Q}$.

This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number).
C) a function (transforms a number into a number).
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

The **Hermitian conjugate** or **adjoint** of an operator $\hat{A}$, written $\hat{A}^\dagger$ ("A-dagger") is defined by $\langle f \mid \hat{A}^\dagger g \rangle = \langle \hat{A} f \mid g \rangle$

An operator $\hat{Q}$ is **Hermitian** or **self-adjoint** if $\langle f \mid \hat{Q} g \rangle = \langle \hat{Q} f \mid g \rangle$, that is, if $\hat{Q}^\dagger = \hat{Q}$.

Consider $\hat{R} = i\hat{Q}$ where $\hat{Q}$ is Hermitian. What is $\hat{R}^\dagger$, the adjoint of $\hat{R}$?

A) $\hat{R} = i\hat{Q}$ (R is Hermitian) B) $-i\hat{Q}$

C) $\hat{Q}$ D) None of these

Hint: we are looking for the operator $\hat{R}^\dagger$ such that

$$\langle f \mid \hat{R}^\dagger g \rangle = \langle \hat{R} f \mid g \rangle = \langle i\hat{Q} f \mid g \rangle = -i\langle \hat{Q} f \mid g \rangle = -i\langle f \mid \hat{Q} g \rangle$$

Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_2|\psi\rangle$, where $\hat{P}_2 = |2\rangle\langle 2|$

is the projection operator for the state $|2\rangle$?

A) c_2

B) $|2\rangle$

C) $c_2|2\rangle$

D) $c_2{}^*\langle 2|$

E) 0

Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_{12}|\psi\rangle$, where $\hat{P}_{12} = |1\rangle\langle 1| + |2\rangle\langle 2|$?

A) $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$

B) $|1\rangle + |2\rangle$

C) 0

D) $\langle\psi| = c_1^*\langle 1| + c_2^*\langle 2|$

E) None of these

If the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$ as well as the basis states $|1\rangle$ and $|2\rangle$ are normalized, then the state $\hat{P}_1|\psi\rangle = |1\rangle\langle 1|\psi\rangle = c_1|1\rangle$ is

A) normalized.

B) not normalized.

A particle in a 1D Harmonic oscillator is in the state $\Psi(x) = \sum_n c_n u_n(x)$ where $u_n(x)$ is the n^{th} energy eigenstate $\hat{H} u_n = E_n u_n$.

A measurement of the energy is made.

What is the probability that result of the measurement is the value E_m ?

A) $\langle c_m | \Psi(x) \rangle$

B) $\left| \langle c_m | \Psi(x) \rangle \right|^2$

C) $\left| \langle u_m | \Psi(x) \rangle \right|^2$

D) $\langle u_m | \Psi(x) \rangle$

E) c_m