University of Colorado, Department of Physics
PHYS3220, Fall 09, HW#2
due Wed, Sep 2, 2PM at start of class

1. Using the formula for four-momentum conservation (special relativity) to derive the dependence of the wavelength of the scattered radiation on the scattering angle in the Compton effect. How would you interpret the scattering contribution, which appears at the wavelength of the incident radiation at all scattering angles? (10 pts)

2. Derive the Bragg relation for the maxima in the scattering of monochromatic waves (wavelength λ) at a chrystal: $\sin(\Theta_{max}) = n\lambda/2d$ with $n = 1, 2, 3, ,$ and d is the lattice spacing. Assume that the wave penetrates up to the second plane of the lattice only. (10 pts)

3. Calculate the de-Broglie wavelength for

   a) a neutron with thermal energy $E_{kin} = 1/40$ eV,
   b) an electron with kinetic energies of $E_{kin} = 10$ eV, 1 keV, 1 MeV,
   c) a proton in the particle accelerator at CERN ($E_{kin} = 270$ GeV),
   d) a C_{60} molecule with kinetic energy $E_{kin} = 100$ eV.

   In which of the cases do you have to consider a relativistic motion of the particle? (10 pts)

4. Some more practice with complex numbers (20 pts)

   a) Prove the following relations that hold for any complex numbers z, z_1 and z_2.

$$\begin{aligned} Re(z) &= (z + z^*)/2 && (1)\\ Im(z) &= (z - z^*)/(2i) && (2)\\ Re(z_1 z_2) &= Re(z_1)Re(z_2) - Im(z_1)Im(z_2) && (3)\\ Im(z_1 z_2) &= Re(z_1)Im(z_2) + Im(z_1)Re(z_2) && (4) \end{aligned}$$

   b) In doing quantum mechanics confusing z^2 and $|z|^2 = z^* z$ is very common. What is $Im(|z|^2)$, and what is $Im(z^2)$?

   c) The second-order differential equation,

$$\frac{d^2 f(x)}{dx^2} = -k^2 f(x) \tag{5}$$

   has two linearly independent solutions. These can be written in more than one way, and two convenient forms are

$$\begin{aligned} f(x) &= A\exp(ikx) + B\exp(-ikx) && (6)\\ f(x) &= a\sin(kx) + b\cos(kx) && (7) \end{aligned}$$

   Verify that both are solutions of the equation (5) above. Since both are equally good solutions, we must be able to determine a and b in terms of A and B; do so.

5. Consider the following example of a quantum mechanical wavefunction for a particle of mass m moving in one dimension (10 pts),

$$\Psi(x,t) = A\,exp(-\alpha(x^2 + i\hbar t/m)) \tag{8}$$

where α and A are constants.

a) Normalize the wavefunction, that is find a value for A such that $\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$. Hint: A useful integral is $\int_{-\infty}^{\infty} exp(-z^2)dz = \sqrt{\pi}$.

b) At what value of x is there equal probability for the particle to be either to the left or to the right of that x? Explain.