Review:

The time-dependent SE: $i\hbar \frac{\partial}{\partial t} \Psi(x,t) = \hat{H} \Psi(x,t)$

Separation of variables $\Rightarrow$ special sol'ns = energy eigenstates:

$\Psi_n(x,t) = \psi_n(x)\, e^{-iE_n t/\hbar}$ where

ψ_n's, E_n's are solutions of time-independent SE:

$$\hat{H}\psi_n = E_n \psi_n$$

Most general sol'n to TDSE is

$$\Psi(x,t) = \sum_n c_n \Psi_n(x,t) = \sum_n c_n \psi_n(x)\, e^{-iE_n t/\hbar}$$

c_n's any complex constants

- Ψ_n's are "stationary" states: $|\Psi_n(x,t)|^2 = |\psi_n(x)|^2$
- Ψ_n's are orthonormal $\int \Psi_m^* \Psi_n\, dx = \int \psi_m^* \psi_n\, dx = \delta_{mn}$
- $\Psi = \sum_n c_n \Psi_n$ are not stationary states, are not orthogonal states

To normalize $\Psi(x,t) = \sum_n c_n \Psi_n$, must have

$$\int \Psi^* \Psi\, dx = \int \left(\sum_m c_m^* \Psi_m^*\right)\left(\sum_n c_n \Psi_n\right) dx$$

$$= \sum_{m,n} c_m^* c_n \underbrace{\int \Psi_m^* \Psi_n\, dx}_{\delta_{mn}} = \sum_n |c_n|^2 = 1$$

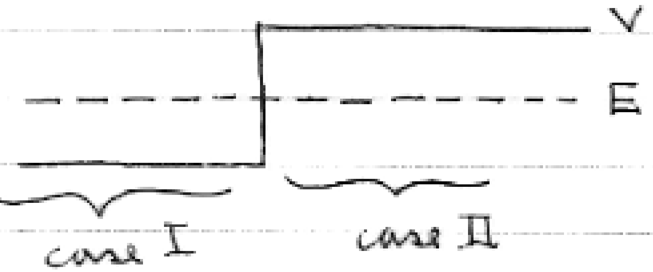
Qualitative Sol'ns to TISE $\hat{H}\Psi_n = E_n\Psi_n$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi(x) + V(x)\Psi(x) = E\,\Psi(x)$$

$$\Psi'' = -\frac{2m}{\hbar^2}(E-V)\cdot\Psi$$ $\Rightarrow \Psi, \Psi'$ continuous (unless E or $V=\infty$)

Case I $E > V$

Case II $E < V$



I. $E>V$ $(+KE) \Rightarrow -\frac{\hbar^2}{2m}(E-V) = -k^2 =$ negative nbr

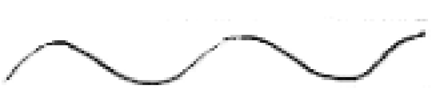
$E - V = KE = \frac{\hbar^2 k^2}{2m} \overset{d.B.}{=} \frac{p^2}{2m} > 0$ (like always in classical Mech.)

$\Psi'' = -k^2\Psi \Rightarrow$ sinusoidal solutions

$\Psi(x) = A\sin kx + B\cos kx$ or $\alpha e^{ikx} + \beta e^{-ikx}$

$KE = E - V = \frac{\hbar^2 k^2}{2m}$, $\sin(kx) = \sin\left(2\pi\frac{x}{\lambda}\right)$

$\Rightarrow$ big KE $\Leftrightarrow$ big k $\Leftrightarrow$ small λ



smaller $(E-V)$

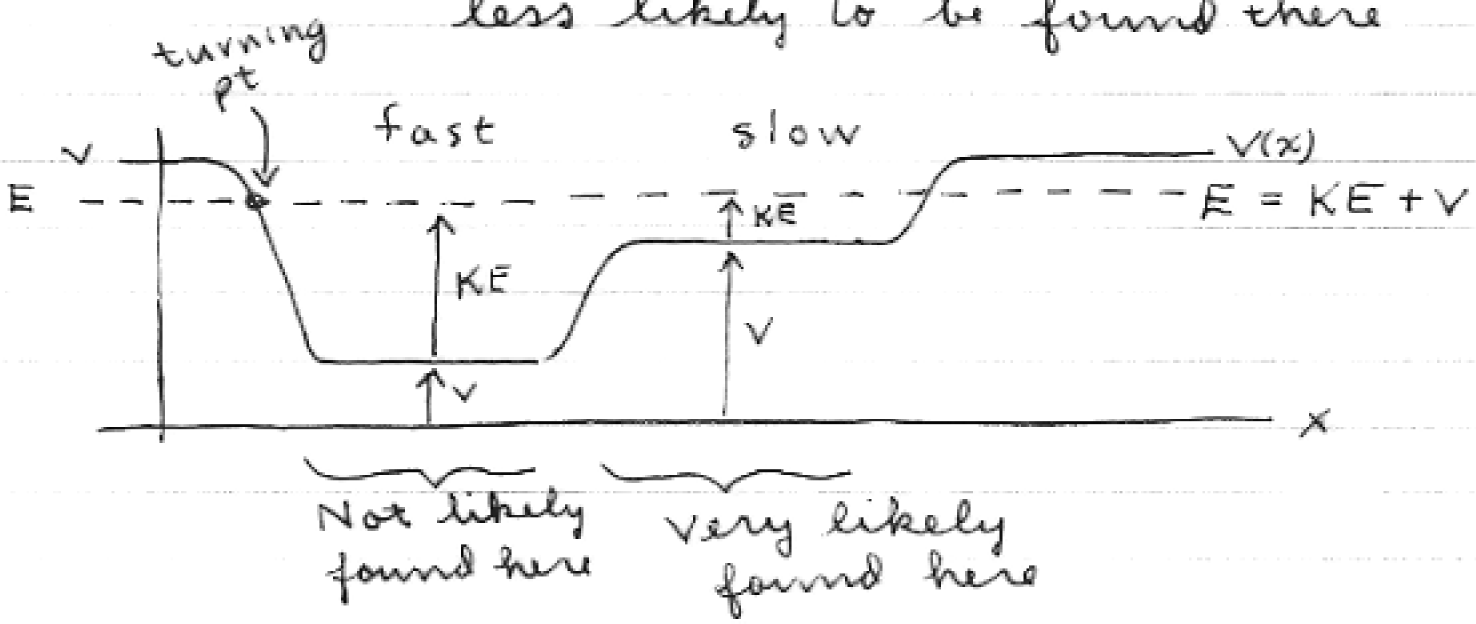
bigger $(E-V)$

Bigger KE = faster wiggles

What about amplitude of wiggles?

Larger Ψ $\Longleftrightarrow$ larger probability of finding there

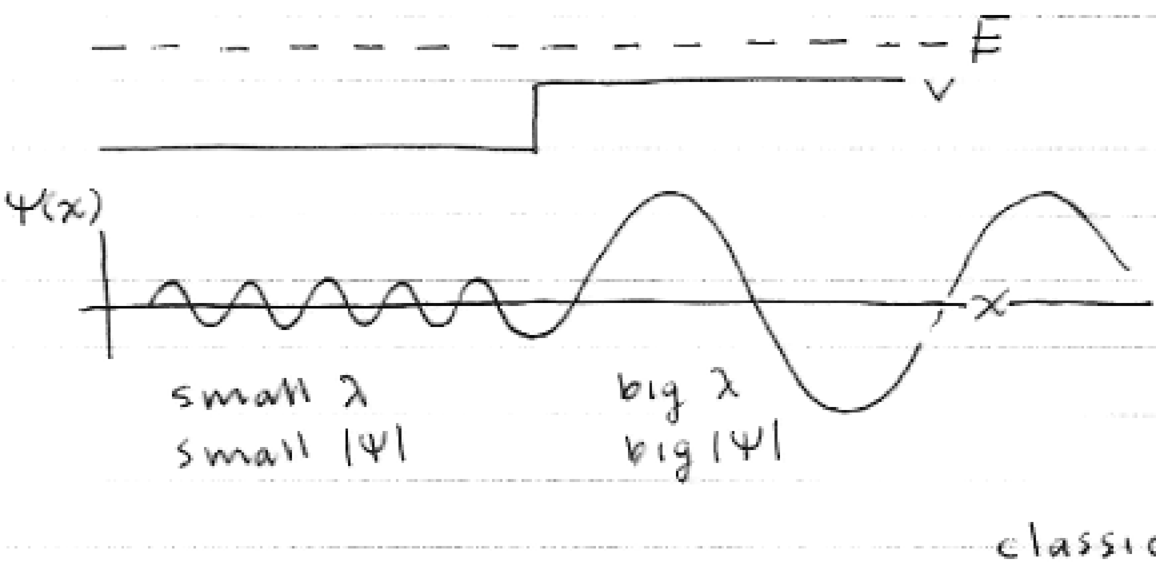
<u>Classically</u>, big KE $\rightarrow$ higher speed v $\rightarrow$ moves quickly thru region $\rightarrow$ less likely to be found there



Vestige of this classical behavior found in QM:

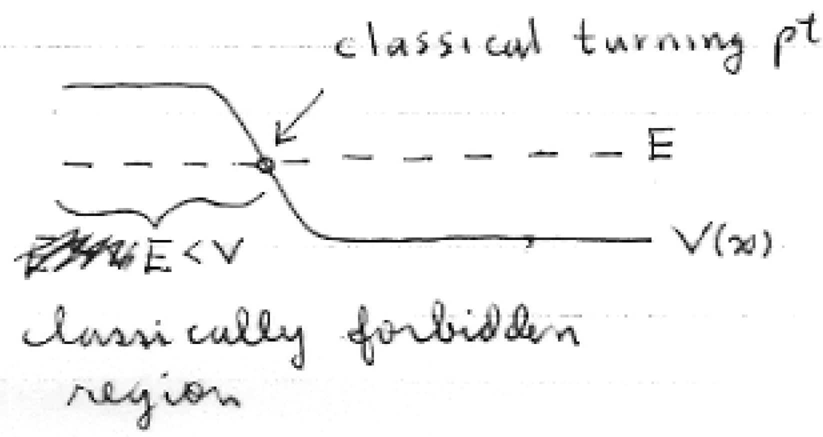
Big KE $\Rightarrow$ small λ $\Rightarrow$ small $|\Psi|$ (small prob.)

Small KE $\Rightarrow$ big λ $\Rightarrow$ big $|\Psi|$ (high prob.)



Case II: $E < V$

$\Rightarrow$ negative KE !

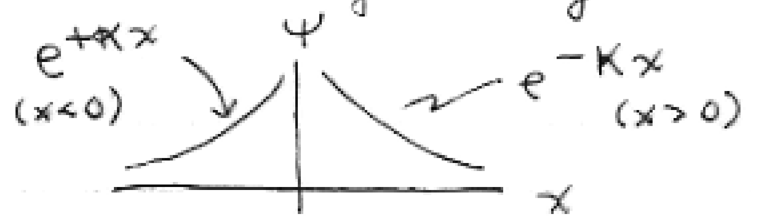


$$E < V \Rightarrow \quad \psi'' = \underbrace{+\frac{2m}{\hbar^2}(V-E)}_{(+)}\psi = +\kappa^2\psi$$

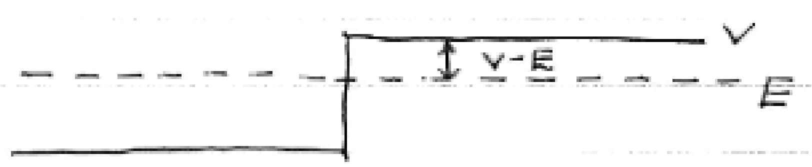
$$\kappa \equiv +\frac{\sqrt{2m(V-E)}}{\hbar}, \quad \psi'' = +\kappa^2\psi \Rightarrow$$

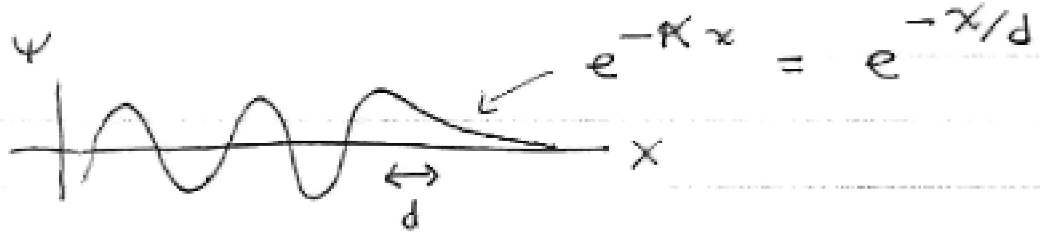
$$\psi = C\,e^{-\kappa x} + D\,e^{+\kappa x}$$

~ exponentially decaying and exponentially diverging sol'ns
Usually, diverging sol'n unphysical due to unnormalizability



In classically forbidden region, $\psi \neq 0$ <u>allowed</u> but ψ must decay to zero (to be normalizable)





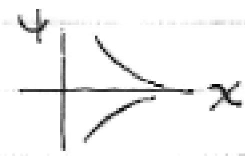
$$d = \text{penetration depth} = \frac{1}{\kappa} = \frac{\hbar}{\sqrt{2m(V-E)}}$$

Large $(V-E)$ = "energy depth" $\Rightarrow$ small d

$\Rightarrow$ small penetration, rapid decay

Infinite square well: $(V-E) = \infty$, $d = 0 \Rightarrow$ no penetration

Recap:

- Ψ, Ψ' continuous (unless V or $E = \infty$)
- $E > V \Rightarrow \Psi$ sinusoidal 
- bigger $KE = (E - V) \Rightarrow$ smaller λ AND smaller $|\Psi|$
- $E < V \Rightarrow \Psi$ exponential decay 

- bigger $(V - E)$ = bigger "energy depth"

  $\Rightarrow$ faster decay, smaller penetration

Finite Square Well