University of Colorado, Department of Physics
PHYS3220, Spring 11, HW#6
due Fri, Feb 25, in class
- Please note: Late Homeworks will not be accepted -

1. Some quantitative problems for a potential step

   a) An electron with a kinetic energy of 10 eV at large negative values of x is moving from left to right along the x-axis. The potential energy is given by:

   $$V(x) = \begin{cases} 0 & x < 0 \\ 20\text{eV}, & x \geq 0 \end{cases}$$

   Let's define a penetration depth $\Delta x > 0$ such that $|\chi(x = \Delta x)|^2 = |\chi(x = 0)|^2/e$. Calculate the penetration depth for the electron.

   b) Repeat part a) for a 70 kg person initially moving at 4 ms^{-1} and running into a wall which can be represented by a potential step of height equal to four times this person's kinetic energy before reaching the step.

2. Scattering by a potential well
   Consider the case of a particle beam which comes in from the left with $E > 0$ and scatters of the potential

   $$V(x) = \begin{cases} 0, & x < 0 \\ -V_0 & 0 < x < a \\ 0, & a < x \end{cases}$$

   with $V_0 > 0$ (note the minus sign in front of V_0 in the formula).

   a) Write down the wave function in each region and give the wave numbers in your general solutions as functions of E and V_0.

   b) Write down the appropriate boundary conditions using the wave functions you have determined and justify any terms in your general solutions you set to zero.

   c) The transmission coefficient for particles coming from the left to be transmitted to $x \to \infty$ is given by

   $$T^{-1} = 1 + \frac{1}{4\frac{E}{V_0}\left(1 + \frac{E}{V_0}\right)} \sin^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 + E)}\right)$$

   Determine T for the limiting cases $E \to 0$ and $E \to \infty$.

   d) Consider the case where $E = V_0$. Determine T at three different values of the parameters: $V_0 \to 0$, $a^2 m V_0/\hbar^2 = \pi^2/16$, $a^2 m V_0/\hbar^2 = \pi^2/4$. How does this compare to your classical expectation?

- More problems on the back -

3. Scattering by a potential barrier
   Consider now a potential with a barrier of height V_0

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0 & 0 < x < a \\ 0, & a < x \end{cases}$$

   a) Adapt the results from the previous problem to demonstrate that for $0 < E < V_0$ the transmission coefficient is given by

$$T^{-1} = 1 + \frac{1}{4(E/V_0)(1 - E/V_0)} \sinh^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 - E)}\right)$$

      To do this, turn the barrier downside up by replacing V_0 with $-V_0$ in that formula, and obtain an expression for T^{-1} where everything is real. It may help to know that $\sinh(ix) = i \sin x$.

   b) Consider this system as a model of a baseball being thrown at a wall. A baseball has a mass of about 150 g, and we take it to be thrown at 40 m/s (near 90 mph). Assume that the wall is 0.1 m thick, and let's make the approximation that the ball would have to be 5 times as energetic to punch through the wall classically, so $V_0 = 5E$ with E determined by the quantities above.
      What is the order of magnitude for T? Let's put this into perspective: if you keep trying, tossing a baseball at the wall once per second, roughly how long do you have to wait until it "pops through" the wall quantum mechanically? Give your answer in seconds, and also in ages of the Universe (current models show the Universe to be about 13.7 billion years old), and comment on your results.

4. Combinations of two potentials
   Write down the form of the wave function in each region that represents the following physical system, give the wave numbers in your general solutions as functions of E and V_0, and justify any terms you set to zero assuming a particle with $0 < E < V_0$ is incident from the *right*. Write down the appropriate boundary conditions using the wave functions you have determined. You do *not* have to solve for the parameters.