PHYS 3220 – Spring 2009
Lecture, Materials and Activities Schedule

Session #1 – 01/12/2009

Oliver introduced the class and associated practicum, PHYS 3221. We passed out whiteboards and asked "What is Quantum Mechanics?" Some of the interesting and/or widely repeated answers are listed here:

Oliver discussed the boards and then did a quick review of classical mechanics from the point of view of state variables (position, velocity) and the defining differential equation of motion (F=ma as suggested by a student). He then moved on to introduce the quantum state variable (*i.e.*, the wave function) and discuss probability density and total probability.

Only 31 clickers were used in class so it is probably not a good time to look at the class as a whole. However, there were still many wrong votes on the simple classical concept questions.

Session #2 – 01/14/2009

Oliver recapped the wave function, probability and probability density and then began discussion of the other postulates. He talked about the measurement postulates and finished with the Schrödinger equation.

Measurement brought up several questions. One student asked whether the 'two measurements in a row' postulate had actually been tested and how one might do that (Oliver answered yes but that the details were very dependent on the experimental set up). Another asked if we could really measure a delta function (no, every real apparatus has a finite width). Finally, a student asked if it was possible to measure a particle in two places by collapsing it into two delta functions (no, Oliver mentioned that special relativity causes some issues and I mentioned that placing multiple detectors doesn't cause multiple hits, the particle always lands in at most one).

Session #3 – 01/16/2009

Oliver covered the mathematics of probability and statistics from discrete through continuous (at very end). There were four concept test questions. The opening question had no issues.

There was a CT question on the probability of measuring the expectation value in a small distribution of ages. While the question was not an issue, there was the question: "Why is it called the expectation value" (Oliver just stated some displeasure with name).

The next CT question was about the expectation value of $(j - \langle j \rangle)$ with about 70% choosing zero and 23% with "it depends on the distribution". A couple of students said something about needing a Gaussian distribution in order to get zero (not clearly stated and put off for later discussion about Gaussian distributions), however, Oliver showed a simple proof of the correct answer. I noticed that students seemed to be guessing rather than either trying to use the formulas or even looking at the sample distributions on the board to see if their answer makes sense. I had a discussion with Oliver after class about encouraging more sense making during the open question period.

The final CT question was after introduction of the standard deviation (squared) with all but one choosing $<x^2> >= <x>^2$. Some were influenced by the formula for the standard deviation (not squared) which Oliver had placed on the board during the question. One student thought that $<j^2>$ should be calculated with $j^2P(j^2)$ inside the sum. Oliver showed the general formula for calculating discrete expectation values and then finished with the continuous version.

Session #4 – 01/21/2009

Today's class was mostly review covering traveling waves (both periodic and a Gaussian wave packet), a review of complex numbers and an introduction to linear operators.

The opening CT question was on traveling waves and a bunch of the class seemed to not remember how to answer it although there were still a large number of correct answers. After the review, there was a higher comfort level but I still observed students who were reluctant to simply work out the answer or who were still having trouble applying the terms to find out the wave with a higher speed. There were four different explanations given by students on how to arrive at the correct answer which was nice.

The last part of class was an introduction to linear operators. The CT question went fairly well although the $\hat{O}[f] = A \bullet f + B$ tripped up several students.

We need to get more of the class talking to their neighbors and keep encouraging them to work out the problems on paper.

Session #5 – 01/23/2009

The lesson started with a review of the review material from Wednesday. Most of the time was spent on linear operators. The remainder of the lesson was on the history of quantum mechanics up through de Broglie. *It would be interesting to change this lesson into one which probed the student's thinking and prior knowledge of QM through a series of CT questions.*

The opening CT question on whether a linear combination of solutions to a linear partial differential equation (the wave equation) is also a solution. 100% got this correct.

The second CT question on wave superposition did not raise any issues. One student was confused by the baseline being positive instead of zero. *redraw?*

One student asked why there was a UV catastrophe. Oliver explained that even though contributions from higher frequencies were small, the integral diverged but that having minimum energy caused the integral to truncate above some frequency which kept the integral finite.

Session #6 – 01/26/2009

The lecture covered a `motivation' of the Schrödinger equation using a plane wave. Oliver then talked about normalization and the linearity of the equation leading to the superposition principle applying to solutions of the SE. He showed that normalization was independent of time, motivated the momentum operator using the time derivative of the expectation value of x and then stated the general formula for the expectation value of an operator.

The opening CT question had to do with plane waves and momentum (test of de Broglie relations). There did not seem to be any issues with this question.

The second CT question was about the superposition principle and posed no problems (nearly 100% correct).

The third CT question asked if a superposition of normalized solutions of the SE was normalized. This prompted some discussion. One group I spoke with thought that it depended on the solution since if one of them was zero, the superposition remained normalized. I asked about the zero solution and one member immediately realized that that is not normalizable.

Later, after Oliver had derived the time-independence of the total probability, a student asked: “Do we have to worry about the derivative terms going to infinity?” She was referring to the boundary terms which have a wave function (or complex conjugate) multiplied by a spatial derivative, evaluated at infinity. Oliver explained that if the derivative did not also go to zero then the wave function could not end up at a constant (never mind zero).

Session #7 – 01/28/2009

This lecture covered a wide range of introductory quantum mechanical concepts:

- Review of operators concentrating on x and p and how classical quantities become quantum operators.
- Introduction of the expectation value of an operator (with the formula declared to be a postulate of quantum mechanics).
- Review of the full SE. Explanation that while the situation may change (*e.g.*, more dimensions, different potential functions), the form of the SE does not.
- Introduction of the commutator using x and p as examples.
- Connection between the Heisenberg Uncertainty Principle (HUP) and the idea of incompatible operators. Description of connection between HUP and restrictions on what can be known about the quantum state.

Using separation of variables (SoV) to create the TISE. Description of solution to the time equation and application to stationary states.

Issues with upcoming homework:

For the first problem where students are asked to calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$ and $\langle p^2 \rangle$ for the n^{th} stationary state, many students had math fatigue. Should we add a hint for this section?

For the expanding square well problem, students have several conceptual issues which need to be addressed. Steve suggested that the problem be changed from size 1 to size a to make it easier to line up with problem 1 (and more familiar). Do you agree with this change? Will it make it more difficult to put the numerical part into matlab (I’m pretty sure mathematica won’t mind)?

- Students need to remember that the wave function is zero when the potential is infinity and also that the initial wave function is zero in the right half of the expanded well. Not remembering this causes them to do the incorrect integral for the coefficients (*i.e.*, by integrating the product $\psi^*(x,0)u_n(x)$ from 0 to 1 (or a) and then invoking orthogonality.
- Students do not really understand the expansion of the wave function in a basis at this point. Could there be a simple class activity which could help them work through this issue?

Session #8 – 01/30/2009: Attendance 45

This lecture covered a review of the creation of the TISE and worked up through the basic solution of the infinite square well:

- Review of creation of the TISE using SoV.
- Review of the stationary state
- `Proof' that in a stationary state, $<\hat{H}> = E$ and that $\sigma_H = 0$
- Statement that the set of all stationary states forms a basis
- Brief introduction to the idea of a basis for f(x), no mention of Hilbert space
- Formula for the expansion of $\Psi(x,t=0)=\psi(x)$ as a sum of energy eigenstates, u_n
- Quick review of terms, eigenvalue equation, eigenvalue, eigenvector (eigenstate)
- Introduction to infinite square well including: finding solution to TISE in each region, using boundary conditions to limit solutions, finding basis functions, finding energy of basis functions.

Close of lecture noting that the quantization of the states and energy levels came about because of the boundary conditions.

Instructor tips:

Be sure to draw wave functions on their own graph, clearly showing where nodes are. Avoid drawing a wave function on a potential energy graph, this has been shown to be confusing to students.

Be careful when talking about potential energy functions to talk about where they are applicable and to explain that the metaphors often used to describe common potential energy functions (*e.g.*, potential well, potential barrier) are not meant to imply that the potential energy function is a physical object. For instance, when describing the harmonic oscillator, try to describe an actual physical system where that function is a good approximation of the potential energy experienced by the particle under study.

Possible student conceptual difficulties:

At one point, Oliver showed the expansion of $\Psi(x,t=0)$ and stated: "I want $\Psi(x,t)$ so I need to know how the u_n functions depend on time. How do they depend on time?" One student replied that they do not depend on time. While I saw a few students agreeing with this statement and a couple offered verbal support, not one student offered up any time dependence. Thinking back, I am not sure this is because they did not see that the time dependent term should be included, or because they felt that time canceled out for stationary states, or simply that the set of u_n functions were introduced explicitly as a function of x only.

Session #9 – 02/02/2009: Attendance 42

This lecture covered several topics which allowed Oliver to complete the initial discussion of the infinite square well. Topics included:

- Review of the infinite square well
- A sample superposition state (in conjunction with the opening CT question)
- Properties of stationary states (*e.g.*, quantized energy, symmetry)
- Use of the square well PhET sim to demonstrate time evolution of a stationary state wave function as well as the time evolution of the probability density in a superposition state

- Review/expansion on the concepts of orthogonality and orthonormality
- Review of the concept of a complete basis using the ISW basis as an example
- Fourier's trick (in conjunction with CT question #2)
- The norm of a superposition state (*i.e.*, $\Sigma |c_n|^2=1$)
- The expectation value of the energy, $\langle H \rangle$
- The idea that $P(E_n) = |c_n|^2$

While the students seemed to be struggling (as seen by the scattered CT votes and by a couple of explanations), it appeared to me that this was more students struggling with new material rather than having serious difficulties after spending a significant amount of time on the subject.

<u>Issues with upcoming homework:</u>

Math issues seemed to be common in problem set #5. For instance, in 5.1.c, students did not see that they had to rewrite $a_\pm$ in terms of x and p before doing the problem.

In 5.2.b, students needed to write x or p in terms of $a_\pm$ but then did not see that some terms (*e.g.*, $\langle u_n|a_+a_+u_n\rangle$) had to be zero. Possibly, emphasizing that $a_\pm u_n$ is a state with a different energy and/or is a state which is orthogonal to the original state could help.

The wording of 5.3.a seemed to confuse some students. Perhaps changing `Demonstrate that a coherent state …' to `Demonstrate that the coherent state (7) …' would help clarify what is to be done.

Problem 5.5 presented issues with Mathematica as students tried to cut and paste. The problem character seems to be the apostrophe (which represents a derivative in Mathematica). Also, there is a z squared but the superscript does not come through so we should substitute a more simple syntax.

Some students were confused by the boundary conditions for part 5.5.b. Perhaps a hint to sketch the ground state and first excited state would be appropriate (that hint worked in help session last Fall).