PHYS 3220

Concept Tests Fall 2009

In classical mechanics, given the state (i.e. position and velocity) of a particle at a certain time instant, the state of the particle at a later time …

A) cannot be determined

B) is known more or less

C) is uniquely determined

Is Einstein's explanation of the photo effect enough to prove that energy quanta of radiation are *particles?*

A) Yes

B) No

Which *new* concept did Einstein use to explain the photo effect?

A) Light is a wave
B) Light consists of energy quanta
C) $E = mc^2$
D) Light consists of particles
E) I don't know

Compton did observe the effect using X-rays. Would he be able to detect the effect (change in wavelength) with visible light too?

A) Yes, and I know why
B) Yes, but I guess
C) No, and I know why not
D) No, but I guess

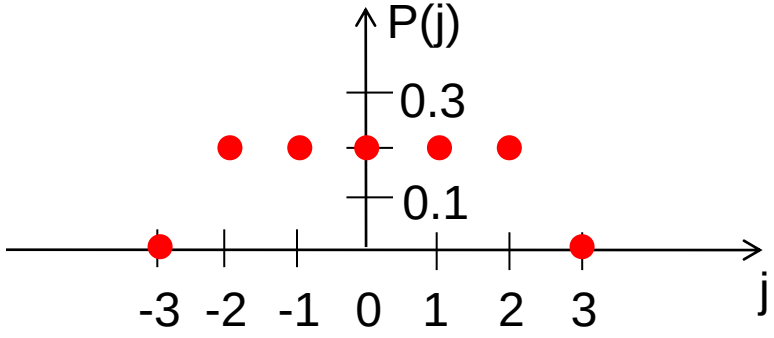
Compton effect: How many maxima will be detected if the detector is placed in forward direction?

A) 0
B) 1
C) 2
D) 3
E) More

For an electron with energy 25 eV, $\lambda \sim 2.5$ Angstrom. The slit separation in a standard double-slit experiment with a scintillation screen is $d \sim 0.25$ mm. What do you expect to see?

A) A nice interference pattern with minima and maxima
B) More or less a bright screen (no separation of minima and maxima)
C) A black screen, since the electrons are diffracted by large angles

Shown is a probability distribution *P(j)* for a discrete variable *j*. What is the average (mean) value <*j*>?



A) <*j*> = -2

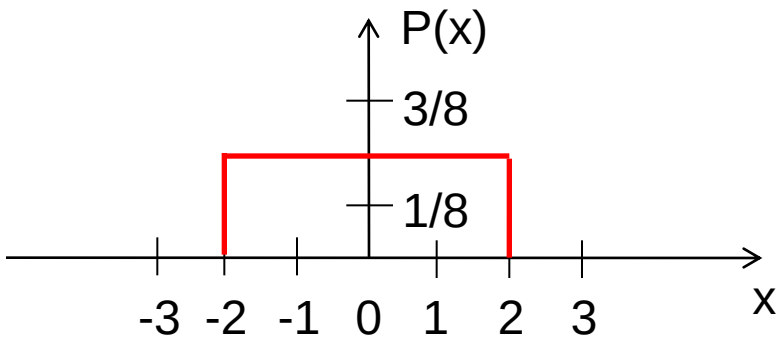
B) <*j*> = -1

C) <*j*> = 0

D) <*j*> = 1

E) <*j*> = 2

Shown is a probability distribution *P(x)* for a continous variable *x*. What is the average (mean) value <*x*>?



A) <*x*> = -2

B) <*x*> = -1

C) <*x*> = 0

D) <*x*> = 1

E) <*x*> = 2

How does one measure an expectation value <x> ?

A) Prepare an ensemble of many identical particles and measure the positions of each of the particle.

B) Measure the position of the same particle again and again many times.

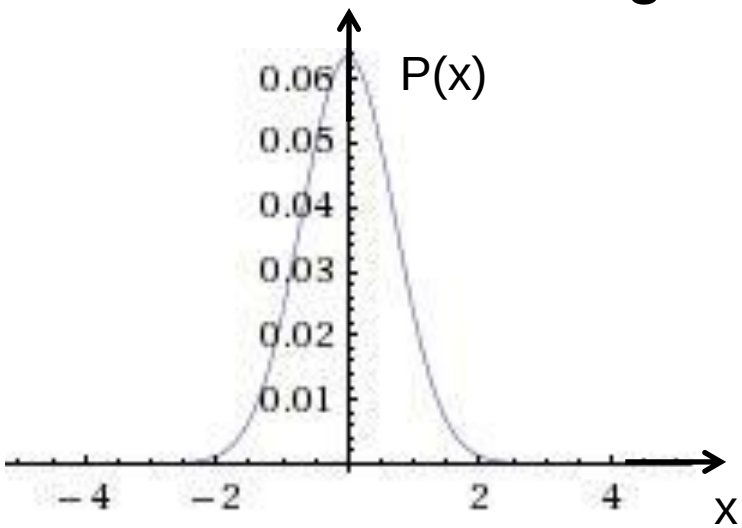
Requirements for the wave equation and its solution:

1. The equation has to be linear
2. The solution has to represent a frequency ω and wave vector $\boldsymbol{k}$ with $\omega = 2\pi E/h$ and $\boldsymbol{k} = 2\pi \boldsymbol{p}/h$
3. The solution must satisfy the dynamical equation $E = \boldsymbol{p}^2/2m + V(\boldsymbol{r},t)$

Which of these requirement has to be fulfilled?

A) None
B) 1 and 2
C) 1 and 3
D) 2 and 3
E) all of them

Below is shown a probability distribution. At which point x_0 is there equal probability for the particle to be either on the left or on the right of x_0



A) $x_0 = -2$

B) $x_0 = 0$

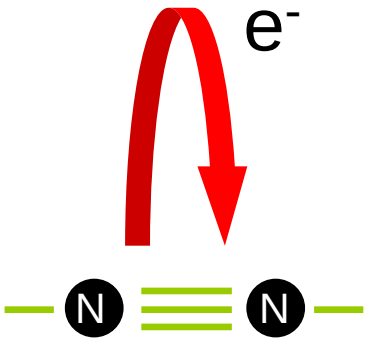
C) $x_0 = 1$

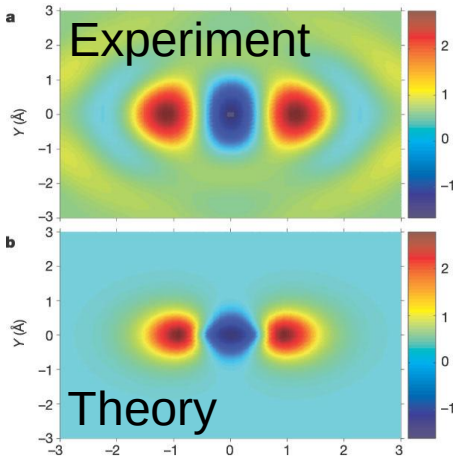
D) I cannot decide, but I know how to calculate it.

Ψ and $|\psi|^2$: Which of these quantities is measurable ?

A) None of them
B) ψ
C) $|\psi|^2$
D) Both of them

Orbital = wavefunction tomography





What is here wrong?

What do you read for PHYS3220?

A) Griffiths
B) Lecture notes
C) Griffiths and lecture notes
D) More than Griffiths and lecture notes
E) Nothing (besides notes taken in class)

What is $\hat{p}_x \Psi_k(x,t)$ with $\Psi_k(x,t) = \exp(i(kx - Et/\hbar))$

$\hat{p}_x \Psi_{-k}(x,t)$ with $\Psi_{-k}(x,t) = \exp(i(-kx - Et/\hbar))$?

A) $\hat{p}_x \Psi_k(x,t) = \hat{p}_x \Psi_{-k}(x,t) = 0$

B) $\hat{p}_x \Psi_k = +\hbar k \Psi_k$ and $\hat{p}_x \Psi_{-k} = +\hbar k \Psi_{-k}$

C) $\hat{p}_x \Psi_k = -\hbar k \Psi_k$ and $\hat{p}_x \Psi_{-k} = -\hbar k \Psi_{-k}$

D) $\hat{p}_x \Psi_k = +\hbar k \Psi_k$ and $\hat{p}_x \Psi_{-k} = -\hbar k \Psi_{-k}$

E) $\hat{p}_x \Psi_k = -\hbar k \Psi_k$ and $\hat{p}_x \Psi_{-k} = +\hbar k \Psi_{-k}$

$$\hat{x}\hat{p}_x\Psi(x,t) = \hat{p}_x\hat{x}\Psi(x,t) \ ?$$

A) Yes, that is correct !

B) No, something is wrong !

From the Ehrenfest theorem

$$\frac{d}{dt}\left\langle \vec{P} \right\rangle = -\left\langle \vec{\nabla} V \right\rangle$$

what can you conclude about $\hat{P}$ and $\hat{H}$

A) $\hat{P}$ and $\hat{H}$ commute

B) $\hat{P}$ and $\hat{H}$ do not commute

C) I cannot make a conclusion

A plane wave …
(I) is normalizable.
(II) is a solution of the free-particle Schrödinger equation.
(III) represents a state with a definite momentum.
Which one is/are true?

A) None B) I and II C) I and III

D) II and III E) All of them

The superposition of two plane waves

$$\Psi_{k_1}(x,t=0)+\Psi_{k_2}(x,t=0)$$

with $\Psi_{k_j}(x,t=0)=1/(2\pi)^{1/2}\exp\left[ik_j x\right]$ is …

A) always a periodic function

B) is not always a periodic function

Consider $\Psi(x,t=0) = 1/(2\pi)^{1/2} \int \Phi(k) \exp\left[ikx\right] \mathrm{dk}$
If $\Phi(k)$ is normalizable, what can you say about the limits of $\Psi(x,t)$ for $x \to \pm\infty$

A) Goes to zero for all t

B) Goes to zero for t=0, but for other t we do not know

C) Not enough information to decide.

Consider $\Psi(x,t=0) = 1/(2\pi)^{1/2} \int \Phi(k) \exp\left[ikx\right] \mathrm{dk}$

What is $\Psi(x,t)$?

A) $\Psi(x,t) = \dfrac{\exp\left[-iEt/\hbar\right]}{(2\pi)^{1/2}} \left[\int \Phi(k) \exp\left[ikx\right] \mathrm{dk}\right]$

B) $\Psi(x,t) = \dfrac{1}{(2\pi)^{1/2}} \left[\int \Phi(k) \exp\left[ikx - i\hbar k^2 t/2m\right] \mathrm{dk}\right]$

C) Something else.

Assume the state of a particle is described by

$$\Phi(x,t=0) = 1/(2\pi)^{1/2} \int \Phi(k) \exp\left[ikx\right] \mathrm{dk}$$

where $\Phi(k)$ is a Gaussian function about k_0 $(\Delta k \neq 0)$. What can you say about the position and momentum of the particle in this state?

A) Both are given with arbitrary precision
B) Momentum is given with arbitrary precision, but position has an approximately precise value.
C) Position is given with arbitrary precision, but momentum has an approximately precise value.
D) Both have approximately precise values.
E) Not enough information.

Even without measuring the position; in practice, no instrument can measure the momentum of a particle precisely. There is always an uncertainty in the measurement. Claim: This is due to Heisenberg's uncertainty relation.True or not?

A) TRUE

B) FALSE

Assume the state of a particle is described by

$$\Psi(x,t=0) = 1/(2\pi)^{1/2} \int \Phi(k) \exp\left[ikx\right] \mathrm{dk}$$

where $\Phi(k)$ is a Gaussian function about $k_0 \neq 0$ $(\Delta k \neq 0)$.

Does $\left|\Psi(x=0,t)\right|^2$ depend on time?

A) Yes

B) No

Assume the state of a particle is described by

$$\Psi(x,t) = 1/(2\pi)^{1/2} \int \Phi(k,t) \exp\left[ikx\right] \mathrm{dk}$$

Compared to $\Psi(x,t)$, $\Phi(k,t)$ contains

A) less information
B) more information
C) the same information
D) cannot be determined / depends

If $\Psi(x,t) = \chi(x)\Phi(t)$ is normalized, $\chi(x)$ is normalized too?

A) TRUE B) FALSE

C) Don't know / depends

Assume $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are stationary states of for different energies E_1 and E_2. Consider

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t)$$

Is $\Psi(x,t)$ a solution of the *time-dependent* Schrödinger equation?

A) Yes B) No

Assume $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are stationary states of for different energies E_1 and E_2. Consider

$$\Psi(x,t) = \Psi_1(x,t) + \Psi_2(x,t)$$

Is $\Psi(x,t)$ a stationary state?

A) Yes B) No

After assuming a product form solution $\Psi(x,t) = \chi(x)\Phi(t)$ the time-dependent Schrödinger equation becomes

$$i\hbar \frac{1}{\Phi}\frac{\partial \Phi}{\partial t} = -\frac{\hbar^2}{2m}\frac{1}{\chi}\frac{\partial^2 \chi}{\partial \chi^2} + V(x,t)$$

If the potential energy function V is a function of time as well as position, i.e. V=V(x,t), would separation of variables still work?

A) Yes, always B) No, never

C) Depends on the functional form of V on x and t

A general solution of the time-dependent Schrödinger equation can be expanded as

$$\Psi(x,t) = \sum_E c_E \Psi_E(x,t) + \int c_E \Psi_E(x,t) dE$$

where $\{\Psi_E\}$ are an orthonormal (basis) set of energy eigenfunctions (stationary states).

Statement: Because of the superposition principle $\Psi(x,t)$ is a stationary state too?

A) True B) False

Assume Ψ_1, Ψ_2 and Ψ_3 are energy eigenstates with E_1, E_2 and E_3 are the corresponding energies. A system is prepared in the following state:

$$\Psi = \frac{1}{\sqrt{3}}\Psi_1 + \sqrt{\frac{2}{3}}\Psi_2$$

Is Ψ normalized?

A) Yes B) No

Assume Ψ_1, Ψ_2 and Ψ_3 are energy eigenstates with E_1, E_2 and E_3 are the corresponding energies. A system is prepared in the following state:

$$\Psi = \frac{1}{\sqrt{3}}\Psi_1 + \sqrt{\frac{2}{3}}\Psi_2$$

What is the probability to measure E_1?

A) 0

B) $\sqrt{\frac{1}{3}}$

C) $\frac{1}{3}$

D) 1

E) Cannot be determined

Assume Ψ_1, Ψ_2 and Ψ_3 are energy eigenstates with E_1, E_2 and E_3 are the corresponding energies. A system is prepared in the following state:

$$\Psi = \frac{1}{\sqrt{3}}\Psi_1 + \sqrt{\frac{2}{3}}\Psi_2$$

What is the probability to measure E_3?

A) 0

B) $\sqrt{\frac{1}{3}}$

C) $\frac{1}{3}$

D) 1

E) Cannot be determined

The energy eigenstates χ_E form an orthonormal set, i.e. $\int \chi_{E'}^*(x)\chi_E(x)dx = \delta_{EE'}$

What is $\int \chi_{E'}^*(x)\left(\sum_E c_E \chi_E(x)\right)dx = ?$

A) $\sum_E c_E ???$ B) $c_{E'}$ C) c_E

D) $c_{E'}c_E$ E) None of these

Assume Ψ_1, Ψ_2 and Ψ_3 are energy eigenstates with E_1, E_2 and E_3 are the corresponding energies. A system is prepared in the following state:

$$\Psi = \frac{1}{\sqrt{3}}\Psi_1 + \sqrt{\frac{2}{3}}\Psi_2$$

We do a (first) measurement and measure E_2 . What is the probability to measure E_1 in a subsequent (second) measurement?

A) 0

B) $\sqrt{\frac{1}{3}}$

C) $\frac{1}{3}$

D) 1

E) Cannot be determined

What are the basic ideas in QM?

Write down some principles you have learned so far and whether they are derived or assumed/postulated!

A wave function has been expressed as sum/integral of energy eigenstates:

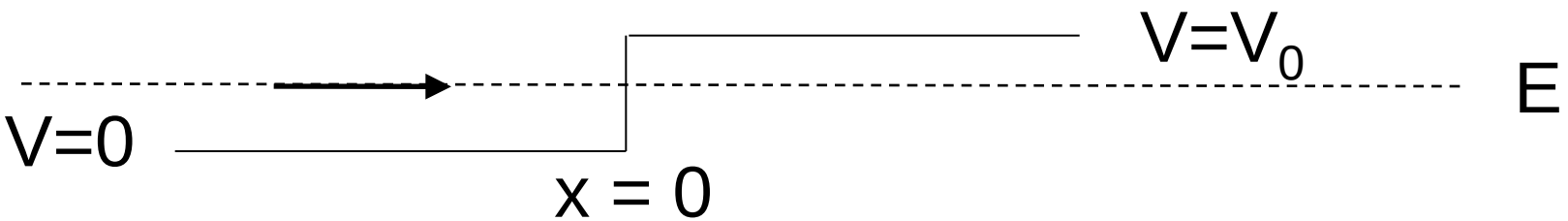
$$\Psi(x,t) = \sum_E c_E \Psi_E(x,t) + \int c_E \Psi_E(x,t) dE$$

Compared to the original Ψ the expansion contains:

A) More information
B) Less information
C) The same information
D) Cannot be determined/depends

A **classical** particle of energy E approaches a potential energy step of height V_0 ($E < V_0$).

What happens to the particle?



A) Always moves to the right at constant speed.

B) Always moves to the right, but its speed is different for x>0 and x<0.

C) Hits the barrier and reflects

D) Hits the barrier and has a chance of reflecting, but might also continue on.

E) None of these, or MORE than one of these!

Are $A_1 \exp(ikx)$ and $A_2 \exp(-ikx)$ solutions of the following equation?

$$\frac{d^2}{dx^2}\chi(x) + k^2\chi(x) = 0$$

A) None of them is a solution
B) $A_1 \exp(ikx)$ is a solution, but $A_2 \exp(-ikx)$ not
C) $A_2 \exp(-ikx)$ is a solution, but $A_1 \exp(ikx)$ not
D) Both are solutions.

How was yesterday's exam?

A) Too hard
B) Hard, but fair
C) OK or as expected
D) Easy
E) Too easy, make the next one harder.

How many values for *A* solve the following equation: $|A|^2 = 1$?

A) 1
B) 2
C) 4
D) Something else

Is $A\left(\exp(ikx) - \exp(-ikx)\right)$ an acceptable physical state?

A) Yes
B) No
C) Don't know

What is $\alpha\delta(x)$ ($\alpha>0$) at $x=0$?

A) 0
B) 1
C) α
D) ∞
E) Something else

For a delta-function potential barrier $\alpha\delta(x)$ at x=0: What are the general solutions for (I) x<0 and (II) x>0 ?

A) (I) $A_1\exp(ikx) + A_2\exp(-ikx)$ and

(II) $C_1\exp(ikx) + C_2\exp(-ikx)$

B) (I) $A_1\exp(ikx) + A_2\exp(-ikx)$ and

(II) $C_1\exp(ik'x) + C_2\exp(-ik'x)$

C) (I) $A_1\exp(ikx) + A_2\exp(-ikx)$ and

(II) $C_1\exp(\kappa x) + C_2\exp(-\kappa x)$

D) Something else

For a delta-function potential barrier: What are the correct boundary conditions?

A) $A_1 + A_2 = C_1 + C_2$ and $ik(A_1 - A_2) = ik(C_1 - C_2)$

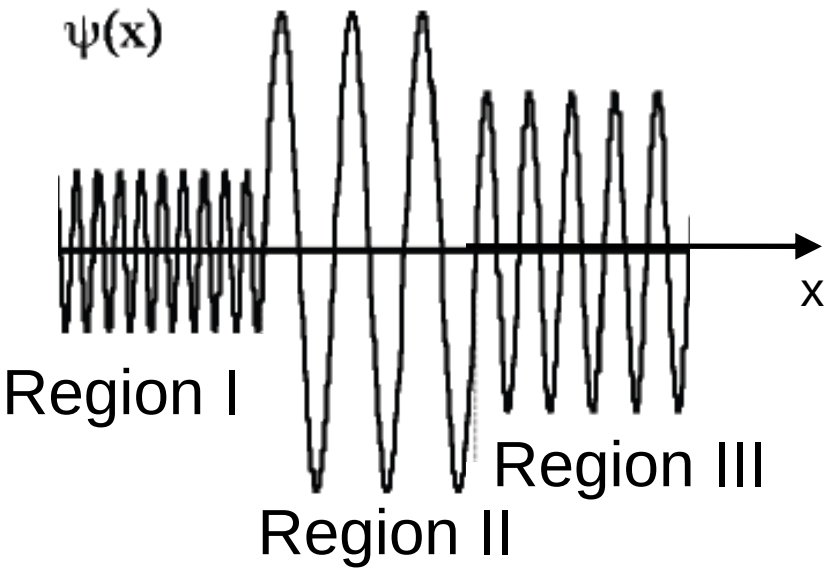
B) $A_1 + A_2 = C_1 + C_2$ and $ik(A_1 - A_2) = ik(C_1 + C_2)$

C) $A_1 + A_2 = C_1 - C_2$ and $ik(A_1 - A_2) = ik(C_1 - C_2)$

D) $A_1 + A_2 = C_1 - C_2$ and $ik(A_1 - A_2) = ik(C_1 + C_2)$

E) None of them

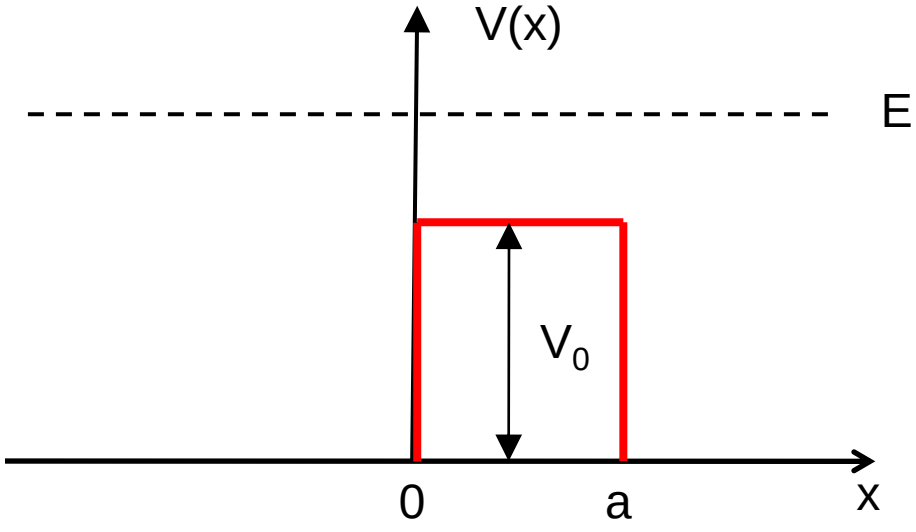
The real part of a stationary state wave function of a particle with energy E is shown below. Assume E > V(x) for all x.



In which region is the potential largest?

A)Region I B) Region II
C) Region III D) Everywhere the same
E) Cannot be determined

Consider the case of a potential barrier with the energy of the particle $E > V_0$. The energy spectrum will be continuous. True (A) or False (B)



For a particle with E > V_0, incident on a potential barrier, the transmission coefficient is given by:

$$T^{-1} = 1 + \frac{1}{4\varepsilon(\varepsilon - 1)} \sin^2\left(\delta\sqrt{\varepsilon - 1}\right)$$

with

$$\delta = 2a\sqrt{2mV_0/\hbar^2} \quad \text{and} \quad \varepsilon = E/V_0$$

T is equal to 1 for …

A) no E

B) E→∞ only

C) many values of E

For an infinite square well, the energy spectrum is discrete. What else can you say?

(I) The energy levels are equally spaced

(II) There is a finite number of energy levels

A) Both statements are true
B) Both statements are false
C) Statement (I) is true, but statement (II) is wrong
D) Statement (I) is wrong, but statement (II) is true
E) I don’t know

The expectation value of the momentum for a particle in the ground state of the infinite square well is ….

A) $< p >= -i\hbar$

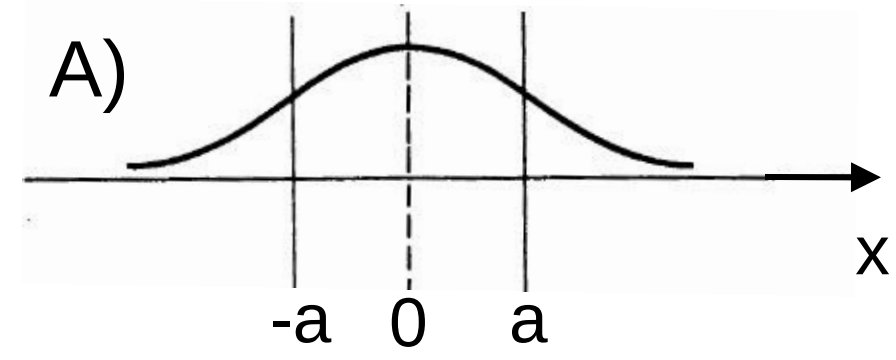
B) $< p >= -\hbar k_1$

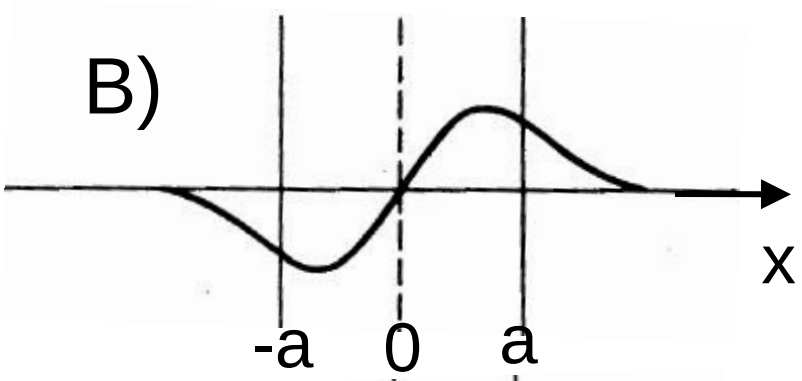
C) $< p >= 0$

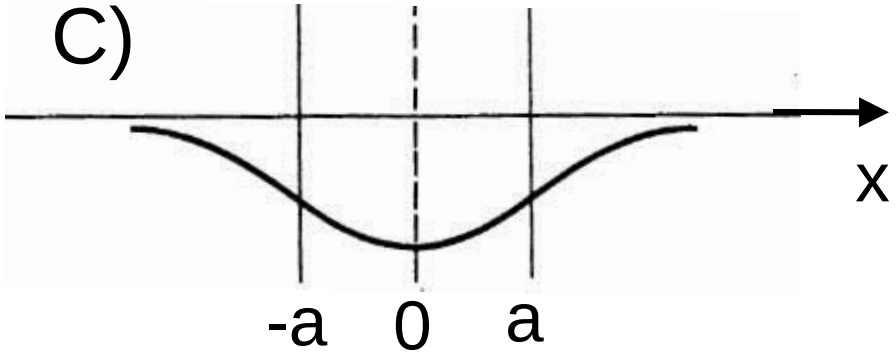
D) $< p >= \hbar k_1$

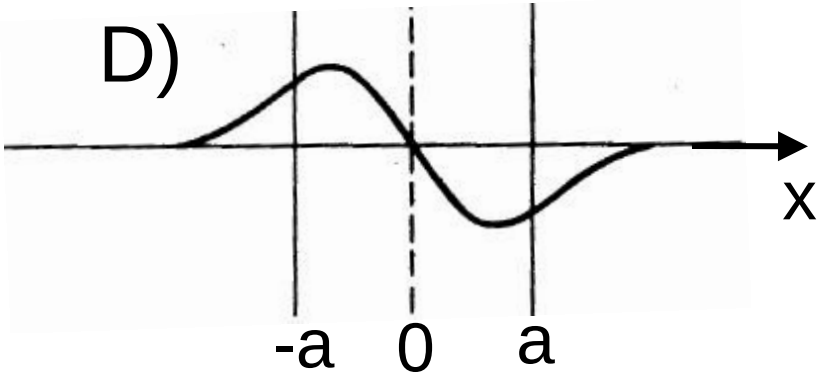
E) Something else

Which of the following does represent a ground state wave function of finite square well:









E) None or more than one of the above

The finite square well has a finite number of energy levels. TRUE (A) or FALSE (B)

The Hamiltonian for the harmonic oscillator is of the following form:

$$\hat{H} = a^2 \hat{x}^2 + b^2 \hat{p}^2, \ a \text{ and } b \text{ real numbers}$$

This can be rewritten as:

A) $\hat{H} = \left(a\hat{x} + b\hat{p}\right)\left(a\hat{x} - b\hat{p}\right)$

B) $\hat{H} = \left(a\hat{x} + ib\hat{p}\right)\left(a\hat{x} - ib\hat{p}\right)$

C) in none of the two forms above

We are about to find the stationary states (and their energies) for the harmonic oscillator.
These are the only possible states for a harmonic oscillator (state = normalizable wave function = normalizable solution of the time-dependent Schrödinger equation).

TRUE (A) or FALSE (B)

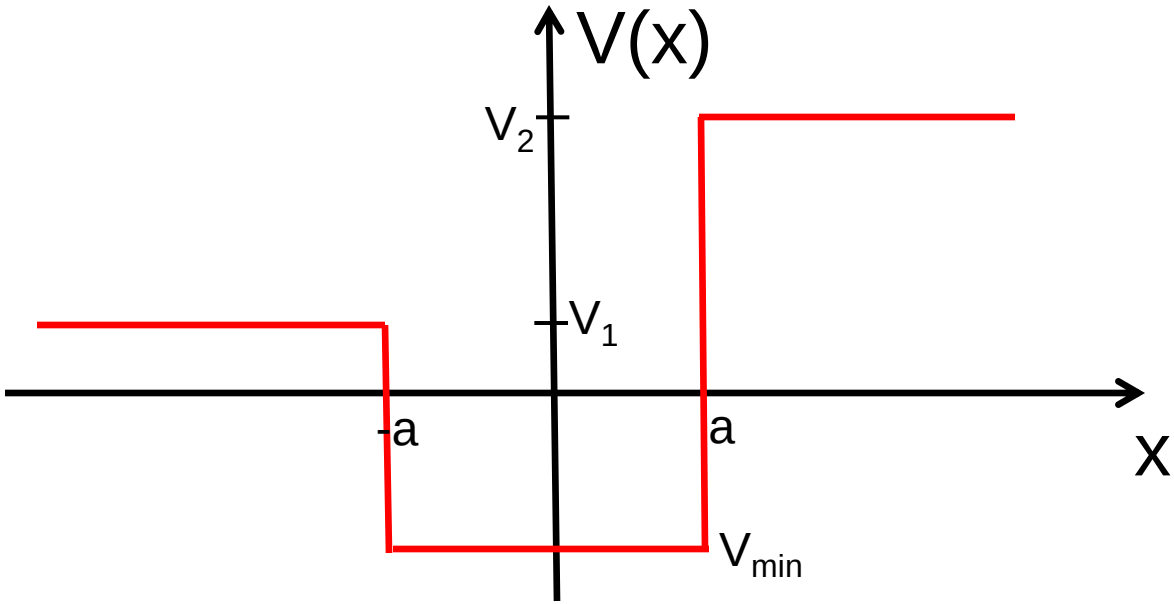
Starting with the ground state and applying successively the raising operator a_+, we have found ALL stationary states of the harmonic oscillator. Is this correct?

A) Yes, these are all stationary states.

B) Well, there might be some other stationary states, which we have missed.

C) I am sure, we have missed some stationary states.

Given the following 1-D potential:



Consider the cases:

(a) $E<V_{min}$, (b) $V_{min}<E<V_1$, (c) $V_1<E<V_2$, (d) $E>V_2$

Discrete/continuous energy spectrum? Bound or scattering states? General solution in each region? Boundary conditions? Transmission/reflection coefficients? …

Assume that Ψ_1 and Ψ_2 are solutions of time-dependent Schrödinger equation and α_1 and α_2 are complex numbers.
What does the principle of superposition tell you?

A) only that $\Psi_1 + \Psi_2$ is a solution too
B) only that $\alpha_1 \Psi_1$ is a solution too
C) $\alpha_1 \Psi_1 + \alpha_2 \Psi_2$ is a solution too
D) None of the above is correct.

If $\{|\Phi_n\rangle\}$ is a complete basis of a Hilbert space, then

$\sum_n |\Phi_n\rangle\langle\Phi_n|$ is an operator, TRUE (A) or FALSE(B)?

A state |Ψ> can be represented via its components in a given basis as a vector: $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ \vdots \end{pmatrix}$

How shall we represent an operator?

A) as a scalar B) as a vector C) as a matrix

D) Something else E) I have no idea

So far, we defined a Hermitian operator via a condition for the expectation value of the corresponding observable. What was the condition?

A) $<\Psi|\hat{A}\Psi>=a<\Psi|\Psi>$

B) $<\Psi|\hat{A}\Psi>=<\Psi|\hat{A}\Psi>^{*}$

C) $<\Psi|\hat{A}\Psi>=<\hat{A}\Psi|\Psi>^{*}$

D) $<\Psi|\hat{A}\Psi>=<\hat{A}\Psi|\Psi>$

E) None or more than one of the above are correct

The momentum operator $\hat{p} = -i\hbar \dfrac{\partial}{\partial x}$ is a Hermitian operator, meaning $\langle \Psi | \hat{p}\Phi \rangle = \langle \hat{p}\Psi | \Phi \rangle$

Is $\hat{p}^2$ a Hermitian operator ?

(A) Yes (B) No

Consider the Schrödinger equation of a harmonic oscillator :

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t), \quad \mathrm{V(x)} = \frac{1}{2}m\omega^2 x^2$$

If we change the potential V(x), do the eigenvectors of $\hat{P}$ change?

A) YES

B) NO

Suppose we have found an energy eigenstate $\Psi_E(x,t)$ for a given problem.
Can this state be written as:

$$\Psi_E(x,t) = \int f(x)\Phi(p,t)dp$$

where $\{\Phi(p,t)\}$ is a complete set of momentum eigenstates?

A) YES B) NO

Consider the following Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\Psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) \quad \text{with} \quad V(x) = x$$

Is it possible to find a set of simultaneous eigenfunctions of the operators $\hat{H}$ and $\hat{p}$?

A) YES

B) NO

How do the components of the operator for the orbital angular momentum look like?

A) $\hat{L}_x = \hat{x}\hat{p}_x,\ \hat{L}_y = \hat{y}\hat{p}_y,\ \hat{L}_z = \hat{z}\hat{p}_z$

B) $\hat{L}_x = \hat{y}\hat{p}_y + \hat{z}\hat{p}_z,\ \hat{L}_y = \hat{x}\hat{p}_x + \hat{z}\hat{p}_z,\ \hat{L}_z = \hat{x}\hat{p}_x + \hat{y}\hat{p}_y$

C) $\hat{L}_x = \hat{y}\hat{p}_y - \hat{z}\hat{p}_z,\ \hat{L}_y = \hat{x}\hat{p}_x - \hat{z}\hat{p}_z,\ \hat{L}_z = \hat{x}\hat{p}_x - \hat{y}\hat{p}_y$

D) $\hat{L}_x = \hat{y}\hat{p}_z - \hat{z}\hat{p}_y,\ \hat{L}_y = \hat{z}\hat{p}_x - \hat{x}\hat{p}_z,\ \hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$

E) None of the above.

Is $\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$ a Hermitian operator?

A) Yes, and I can show it (using formulas)

B) Yes, and I have a good argument.

C) No, for sure not.

D) Probably not.

E) Are you joking? This is a HW problem, but not a clicker question.

Is $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ a Hermitian operator?

A) Yes

B) No

Are the commutators zero or non-zero?

$$(\mathrm{i}) : [\hat{y}\hat{p}_z, \hat{z}\hat{p}_x]$$

$$(\mathrm{ii}) : [\hat{y}\hat{p}_z, \hat{x}\hat{p}_z]$$

A) Both zero
B) (i): zero, (ii): non-zero
C) (i) : non-zero, (ii): zero
D) Both non-zero

What is the unit of the orbital angular momentum?

A) J / s
B) J s
C) J s / m
D) J m / s
E) None of the above.

For a given λ does the $\hat{L}_+ / \hat{L}_-$ ladder stop somewhere?

A) Yes, it must stop in both directions
B) It will stop in downwards direction, but not in upwards direction
C) It will stop in upwards direction, but but not in downwards direction
D) In both directions it will not stop

Does the following relation hold for a state of the (orbital) angular momentum of a particle:

$$\Delta L_x = \Delta L_y = 0$$

A) Yes, always

B) No, never

C) Depends on the state

Does the following relation hold for a state of the (orbital) angular momentum of a particle:

$$\Delta L_z = \Delta L^2 = 0$$

A) Yes, always

B) No, never

C) Depends on the state

For a time-independent potential V(r) we have found solutions to the TDSE given by:

$$\Psi_E(\mathbf{r},t) = \chi_E(\mathbf{r})\exp(-iEt/\hbar)$$

$$\text{with} \quad H\chi_E(\mathbf{r}) = E\chi_E(\mathbf{r})$$

How can the general solution to the TDSE be written (assume a discrete energy spectrum)?

A) $\Psi(\mathbf{r},t) = \Psi_E(\mathbf{r},t) = \chi_E(\mathbf{r})\exp(-iEt/\hbar)$

B) $\Psi(\mathbf{r},t) = \left(\sum_n c_n \chi_{E_n}(\mathbf{r})\right)\exp(-iEt/\hbar)$

C) $\Psi(\mathbf{r},t) = \sum_n c_n \chi_{E_n}(\mathbf{r})\exp(-iE_n t/\hbar)$

D) None of the above

For a central potential V(r), H can be written as:

$$H = -\frac{\hbar^2}{2M}\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{2Mr}\hat{L}^2 + V(r)$$

Does H commute with L^2 and/or with L_z?

A) No, H does not commute with L^2 and not with L_z ?

B) H commutes with L^2 but not with L_z

C) H commutes with L_z but not with L^2

D) H commutes with both operators