

Monday 8/25/08

On board:

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Exam dates: Tuesday 7-9:30 pm, 9/30 & 11/11

HW due on Wed!!

**Office hours:** Fri, Mon, Tues (figure out good times)

**Reading: Griffiths Preface and 1.1-1.3, and/or lecture notes pp. 1-10**

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**Topics:** [Introduction, syllabus: Ask questions!]

Prerequisites are 2170, 2210 and 3210, plus differential equations and linear algebra.

Talk to me if you have any possible deficiencies.

White boards: What is QM

Must cover: text, 1st hw logistics, exams, choice & importance of text (and QM as "sea"), clickers & discussion in class, office hours (come!), lecture is about sensemaking.

Notes: Discussion of "where QM fits" (speed and scale), and essence of classical mechanics (particle trajectories)

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Wed 8/27/08

On board:

[www.colorado.edu/physics/phys3220](http://www.colorado.edu/physics/phys3220)

Reading: Griffiths 1.3 & 1.4, and/or lecture notes p. 11-19 (and through p. 26 if that freshman physics material is not familiar!)

(ON board)

**Topics:** Postulates of QM,

ACTIVITY : Clicker questions

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Fri 8/29/08

Check in with SG about permission forms - maybe remind/ask for last ones

On board: Monday is a holiday!

Tut today @4 (basement), (Announce!)

Office hr Tues@3-5 11th floor

Reading (for Wed): Griffiths 1.4, lecture notes "Ch 1 part 2"

(ON Board): Prob (finding particle between  $x_1$  and  $x_2$ ) =  $\int_{x_1}^{x_2} \psi^2(x) dx$

**Topics: Statistics:** Prob,  $\langle x \rangle$ ,  $\sigma$ , for discrete and continuous distributions.

Activities:

Monday 9/2/08  
HOLIDAY

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Wed 9/4/08

MUST FIGURE OUT NEW TUTORIAL TIME

Reading: Griffiths 1.5, 2.1 (??), and/or lecture notes Ch 2 part 1, p. xxx

ON BOARD:

**Topics:** Complex #'s, classical waves, superposition

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Fri 9/6/08

On board:

Tut today @ 3 PM, or 4 PM (basement), (

Reading (for Mon): Griffiths 2.1-2.2, lecture notes “Ch 2 part 1, and start of part 2”

(ON Board): traveling wave:  $f(x-vt)$ , wave equation  $d^2y/dt^2=v^2d^2y/dx^2$

**Topics:** Linear operators, then some history:, deBroglie. Intro to SE, motivate it, talk about pieces.

Activity: WHITE BOARDS: First, do the “deBroglie” trick to get  $n\lambda = 2\pi r$  for a wave on a circle, thus  $L = n\hbar$

Second, come up with an operator to “extract” the energy from a plane wave.

L6: Monday 9/8/08

Reading For Wed: Griffiths 2.1 (could start 2.2) and/or lecture notes Ch 2, part 1 (up to p2. 7)

(ON Board):

EXAM dates!

$$E = \hbar \omega = h f$$

$$P = \hbar k = h/\lambda$$

Covered notes 1.C.1-1.C.7

Topics: Operators/eigenvalues, and introduce the Hamiltonian, KE, and momentum as operators. (So, first time seeing  $p = -\hbar d/dx$ : motivate it from deBroglie, but also from  $\langle p \rangle = m d\langle x \rangle/dt$ ) Talk about normalizing the wavefunction (and that this is true at further times – don't work it out, just discuss, they do it on hw) I also talk about the expectation value of Classical operators (the game in which you replace  $Q(x,p)$  with  $Q(x, \hbar/i d/dx)$ )

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L7: Wed 9/10/08

Reading: Griffiths 2.2, and/or lecture notes Ch 2 part 2 (up to page 2.16)

ON BOARD:  $\langle x \rangle = \int \psi^* x \psi dx$

$\langle p \rangle = \int \psi^* \hbar/i d/dx \psi dx$

write “physical observables are associated with OPERATORS in QM”

$\langle O \rangle = \int \psi^* O \psi dx$

Start with:

$$\sigma_x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}, \text{ (also write } \sigma_p)$$

$$\sigma_x * \sigma_p \geq \hbar/2$$

Covered my Notes 2.1 – 2.5)

**Topics:** Start with review from last time, discuss INTERPRETING  $\langle x \rangle$  as “average of many measurements”, and  $\sigma_x$  as “experimental uncertainty”. So, talk about QM as tool to calculate, but these quantities are related to experiment. (Today was LHC turn-on, so it fit nicely to talk about “the beam” and its  $\Delta x$ ,  $\Delta p$ , etc) Mention that HUP is about nature, **not** just our “errors” or “poor measurements” or even “we don't know it, but it's out there”. After this, today is "separation of variables day", This will cover the idea of Separation of variables, specialize to case  $V(x)$  (not  $V(x,t)$ ), derive the two ODE's, solve the "time equation", discuss the TISE, and the idea of stationary states. Write TISE as an e-value equation. Discuss/be clear about stationary states as special, and general states can be formed by linearly combining them. Show them that stationary states are stationary, both for  $\psi^2$  and for  $\langle O \rangle$ .

ACTIVITY : During clicker question on whether  $V(x,t)$  still allowed separation, and we decided  $V(x,t) = V_x(x) + V_t(t)$  works, I asked for a physical example. Several came up

with one, including my favorite “a piston with a tilted ramp on it, rising steadily, with something rolling on the ramp”.

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L8 Fri 9/12/08

On board:

Tut today @ 3 PM, or 4 PM (basement),

Reading (for Mon): Griffiths 2.3 (start), lecture notes (to 2.16 or so)

TDSE (write it out)

Special solutions, “stationary states”,  $\psi(x,t) = u(x) \exp[-iEt/\hbar]$

With TISE (write it out) in diff eq form, also  $Hu = Eu$

Write  $\Psi_{\text{general}} = \sum(c_n u_n \exp[-iE_n t/\hbar])$

Cover notes: 2.6-2.11

**Topics:** Start with idea that  $\langle H \rangle = \text{energy}$  for stationary states (and  $\sigma_H = 0$ ) and **interpret** this (eigenfunctions of  $H$ , stationary states, have a definite energy!) and end with a reminder of constructing general states by linear combinations.

But mostly, today is infinite square well. Motivate it (physical example, like short superconducting wire), but also “canonical”. Think about classical box, make sense of the infinities, picture what this  $V(x)$  means.

Write out SE, discuss  $u(x)=0$  outside box

Rewrite SE inside box, define  $k$ .

Use BC's to get  $\sin(n \pi x/a)$

Discuss spectrum, plots,

Do normalization, discuss physics of phase/sign.

*Comment: Today, on the concept test about the superposition of two states, the “dam broke”. We just had a string of questions from many people, especially interpreting “uncertain energy” (what do you MEAN it doesn't have a definite energy?!), and time dependence. (This has continued for next two lectures – many questions throughout class!!)*

L9: Monday 9/15/08

Reading For Wed: Griffiths 2.3 (start) or lecture notes up to about 2.18

(ON Board):

TDSE, and TISE, also in form  $Hu_n = E_n u_n$ ,

$\Psi_n(x,t) = u_n(x) \exp[-iE_n t/\hbar]$

Write out  $E_n$  formula, and  $u_n$  formula, and  $E_n = \hbar^2 k_n^2/2m$

Covered notes 2.12 – 2.14 only

Topics: Started with CT about “how does width affect  $E$ ”, and also talked about the funny graph where the wave function is shown drawn “around” the  $E$  line, (which is strange). Let this lead to discussion of CURVATURE of wave function, spent some time discussing this and answering questions about it (1<sup>st</sup> pass at qualitative sketching based on TISE)

Next CT is about whether a linear combo of states is a stationary state –(and then, what IS the time dependence?) Good discussion, many questions.

Then general discussion about eigenstates of square well (and many problems):

Even/odd story (spent time on this)

Oscillatory nature (discussed above), and # of crossings

Orthogonality (write it out, discuss the “ $u^*$ ”, which we don’t need yet), and Kronecker delta.

Completeness (write it out, discuss Fourier series)

Fourier’s Trick (work it out)

General time dependent solution (show it)

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L10: Wed 9/10/08

Reading: Griffiths 2.3 (finish) and/or lecture notes Ch 2 up to 2.

ON BOARD:

“Solve TISE,  $Hu_n = E_n u_n$ , to find  $u_n$ ’s and  $E_n$ ’s.

$\Psi(x,t=0) = \text{Sum}(c_n u_n)$

Then  $\Psi(x,t) = \text{Sum}(c_n u_n \exp[-iE_n t/\hbar])$

With  $c_n = \text{Integral}[u_n \Psi(x,t=0)]$  – Fourier’s trick!

Covered my Notes 2.15-2.18 only

**Topics:** Start with fact that normalization and  $\langle H \rangle$  lead you to INTERPRET  $|C_n|^2$  as “probability of measuring  $E_n$ .” (Took time on this, worked out the algebra including the “double sum” and the “collapse of the sum with the Kronecker delta)

Introduced harmonic oscillator, discussed classical results, then did the activity.

ACTIVITY : Got them up to blackboards in groups of 4.

Group 1: Sketch ground state of HO ( $n=0$ ), and  $\psi^2$ . Think about time dependence

Group 2:  $n=1$  state

Group 3:  $n=2$

Group 4: large  $n$

Group 5: classical limit

Rest of class: check their work, look for mistakes and help, and think about time evolution of  $\psi_0 + \psi_2$  (didn't get to this, though)

Also, showed PHET sim for plot of ground state wave function, and larger  $n$ .

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L11 Fri 9/19/08

Read 2.4 (start) or notes to 2.33 (or so)

On board:

Write out TISE for harmonic oscillator

Tut today @ 3 PM, or 4 PM (basement),

Cover notes: 2.18-2.21

**Topics:** Motivate the Harmonic Oscillator (common, all potentials look like it near a minimum, and it's theoretical basis for QFT) then 1) sketch (we did this last time, quick review), 2) Numerical (spend some time discussing "shooting", use even/odd to help, talk about normalization, talk about "stepping"... 3) Method of Frobenius - including shifting to dimensionless variable, pulling out asymptotic solution, then power series method, THEN truncation, leading to energy quantization and Hermite Polynomials.

End with ~1 min lead-in to method of operators.

L12: Monday 9/22/08

Reading For Wed: Start Griffiths 2.4 or notes to page 32

(ON Board):

TISE written as  $(\frac{1}{2m})(p^2 + (m\omega\hbar)^2 x^2)u = E u$

Covered notes 2.22 to 2.26

Topics: This was the operator method of the Harmonic Oscillator. Introduced  $a+$  and  $a-$ , discussed the commutator (worked out  $[x,p]$ , found  $[a+,a-]$ ) worked out  $H$  on  $(a+u)$ , discovered it's a "ladder" operator. Talked about lowering operator, argued that there has to be a bottom rung, ended with the equation  $a-(u_0) = E_0(u_0)$

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L13: Wed 9/24/08

Oliver takes over!

Reading: Griffiths 2.4

ON BOARD: He had the TISE for HO in operator form (both  $a+a-$  and  $a-a+$  versions), the definition of  $a+$  and  $a-$ , and what  $H(a+/-)u$  does.

Covered my notes 2.26 - 2.30

**Topics:** Wrapped up HO (discussion of the tower of states, brief nod to normalization, the uniqueness/completeness of these states.) Then, 2nd half, moved to "free particle" - what's  $H$ , differences between classical and quantum, solution as plane wave, interpretation of plane wave, role of momentum operator.

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L14 Fri 9/26/08

Read 2.4

On board: Midterm #1

Free Particle:  $V = \text{const}$  (0 for now)

Stationary states:  $\Psi_k = A \exp[i k x] \exp[-i E t/\hbar]$

$E = \hbar^2 k^2 / 2m$

Phat  $\Psi_k = \hbar k \Psi_k$ : eigenstate of  $E$  and  $p$ .

Tut today @ 3 PM, or 4 PM (basement),

Cover notes: 2.31 – 2.33 (Plane waves and Fourier transform intro)

**Topics:** Review from last time re plane waves -  $\langle p \rangle$  for plane wave,  $\sigma_p \Rightarrow$  conclude it's an eigenstate. Then discussion of normalization issue. Introduce purpose of plane wave – as idealization, but also as basis set for packets. The Fourier transform (introduced by strong/close analogy to Fourier series) Time dependence of plane waves and packets. Introduction to "momentum space wave function".

L15: Monday 9/29/08

Reading For Wed: (Still in 2.4!)

(ON Board): Fourier Transforms:  $\Psi(x,0) = 1/\sqrt{2\pi} \int dk \phi(k) \exp[ikx]$

Inverse:  $\phi(k) = \dots \exp[-ikx]$

Exam Thursday, (Office hours reminder)

Covered notes 2.33 - 2.34 (which is multiple pages, 2.33b, 2.33c...)

Topics: More on Fourier transform . Whiteboard activity/sketch (see below) . Then Gaussian, and repeat of the story with different math. Discussion of Heisenberg Uncertainty. Introduction to delta function, and delta as FT of plane wave. Ended with "orthogonality" integral for plane waves.

**Activity:** Whiteboards: We gave them a "square"  $\psi(x)$ , gave formula for  $\phi(k)$ , asked them to sketch limit as width  $\rightarrow 0$ , and then width  $\rightarrow$  infinity. Took about 5 minutes for EACH of those 2 sketches, with ~5 minutes between, so 15 minutes total to get at the "wide  $\leftrightarrow$  Small" idea

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L16: Wed Oct 1/08

Test tomorrow.

ON BOARD: Free Particle: wide  $\leftrightarrow$  Narrow,  $1/\sqrt{2\pi} \leftrightarrow \delta(x)$  ,

$1/\sqrt{2\pi} \exp[ikx] \leftrightarrow \delta(x-x_0)$

AND, on other board

$U_k(x) = 1/\sqrt{2\pi} \exp[ikx]$

$\int U_k U_{k'} = 1/2\pi \int \exp[ix(k-k')] dx = \delta(k-k')$ , Plane wave orthog.

Covered notes 34-38 (sort of, didn't really do the group/phase velocity story, this material is in Oliver's notes but not so directly in mine)

**Topics:** Momentum space wave functions – normalization, matrix elements, new operators in momentum space, interpretation time development, spreading of packets.

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L17 Fri 10/03/08

Read : Starting 2.5

On board: Momentum space  $\leftrightarrow$  Position space

$\Phi(p) \leftrightarrow \psi(x)$

Free particle time development (sketch wave packet moving and spreading, like runners in a race)

No Tut today

Cover notes: (Oliver is diverging from my online notes in order and focus a little)

**Topics:** Review/wrapup of Fourier transforms, emphasis on time development. Move on to 1-D scattering, introduction to the basic idea, goal is to find R or T. Probability current, and its interpretation, including the special case of plane waves. Setting up TISE for particle which is free "far left" and "far right", as preliminary for finding scattering results.

L18: Monday 10/6/08  
Reading For Wed: 2.6

ON Board "Scattering" (picture of incident from left)  $d|\psi|^2/dt = -dJ/dx$ , with  $J = \hbar/m \text{Im}(\psi^* d\psi/dx)$   
 $\Psi = A \exp(i(kx - \omega t)) \Rightarrow J = |A|^2 \hbar k/m$ .

Covered notes (not quite matching notes here, but R, T, J, continuity, step potential)  
Topics: Interpretation of J for plane waves, conservation of probability, introduction to R and T coefficients, the step up potential, boundary conditions (u continuous, u' continuous)

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L19: Wed Oct 8/08

ON BOARD: (Picture of Step up, with Solutions for u, k's, R and T, when  $E > V_0$ )

**Topics:** Step up potential: limits (as  $E \gg V$ , or  $E \rightarrow V$ ), and then case  $E < V$ , including current = 0 in classically forbidden region. Intro to tunneling (set up Solutions, formally)

Activities: Showed Phet sim, plane waves with step up potential. Started in  $E < V$  region, talked through features, moved E up until it crossed the  $E = V$  line. Generated at least a dozen questions, took about 10 minutes.

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L20 Fri 10/10/08

Read : (Finish Chapter 2 on Monday, starting Ch 3 on Wed)

On board:

Tut today! ON qualitative wave functions/sketches.

Cover notes: (Still in Ch. 2, doing tunneling and intro to finite well)

**Topics: Tunneling.**

Activities:

No time: We had planned to do a variation on the OSU "finite square well" activity. 1/3 of the class is "Al-Gallium-Arsenide", middle is Ga-Arsenide, remaining 1/3 is "Al-Ga-As", which constitutes a practical finite square well (used e.g. to tune laser frequencies). Each 1/3, on whiteboards, had to work out form of solutions to TISE, assuming we're looking for an even wave function.

"to think about" - From OSU: "When each group has finished, we will have a group discussion to decide how to make sure that the total wave function over the entire space is appropriately continuous and otherwise well-behaved. If you finish early, consider whether or not your answer would be different if  $E > V_0$ , and then consider a different region, swapping roles of taskmaster, cynic, and recorder.



L21: Monday 10/13/08  
Reading For Wed: 3.1 and 3.2

ON Board : (2 kinds of states, scattering and bound, with lists under each: can escape/trapped, classically allowed/forbidden off at infinity, any energy/discrete)

Covered notes : Finished Chapter 2 today.  
Topics: Finite square well, including form of solution, transcendental solution, interpretation sensemaking of limits, ended with qualitative behaviour of wave functions.

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L22: Wed Oct /15

ON BOARD: (just HW and reading stuff)

Covered notes: Entering Ch 3, Dubson's 1-pager on vector space. (After this Oliver is likely to diverge from order, but cover, my or Mike's Ch 3 notes)

**Topics:** Vectors in vector spaces. Functions as vectors. Bra-ket notation at its simplest. Inner product for functions. Wave functions live in Hilbert space.

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L23 Fri 10/17/08

Read : (Nothing on board)

On board: 1st postulate of QM, state is  $|\psi\rangle$  Hilbert space; complex vector functions with inner product,  $\langle B|A\rangle = \langle A|B\rangle^*$ , and  $\langle A|A\rangle < \infty$

**Topics:**  $\Psi$  as a vector, the inner product, Fourier's trick in Dirac notation. Linear Transformations - matrices in vector space, but operators in our Hilbert space (so far).. Discussion of Dirac Notation, equivalence/naming of  $|Q\psi\rangle$  and  $Q|\psi\rangle$ , integral (position space) representation of inner product for expectation values. Definition of adjoint, and Hermitian operators. Proof that "i" is not hermitian, but x and p are. First two postulates of QM: State is  $|\psi\rangle$  and Observables correspond to Hermitian operators.

Tut today : On Functions as Vectors (we modified the UW Tutorial a bit)

L24: Monday 10/20/08

Reading For Wed:

ON Board : Hermitian:  $\langle f|Qg\rangle = \langle Qf|g\rangle$ , which implies  $\langle Q\rangle$  is real. E.g.  $x, p$

On other board, 1<sup>st</sup> 2 postulates.

Covered notes:

Topics: Determinate states, basic theorems regarding real e-values and orthogonality of e-functions of Hermitian operators, basic introduction to degeneracy, Third postulate (if you measure  $Q$ , you get one of the  $\{q\}$ 's).

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L25: Wed Oct 22

ON BOARD: (Honors announcement), Postulates 1-3 filled in, with 4-6 blank. (3 is that observables for  $Q$  must be e-values of  $Q$ )

**Topics:** Review of Postulate 3 (for continuous eigenvalue case), Postulate 4 – expansion and probability =  $|\langle \psi | f_n \rangle|^2$ . Reminder of examples from earlier when expanding in energy e-functions. Rewriting it using “fourier’s trick”. (Whiteboard activity) 5<sup>th</sup> postulate (collapse) Sequential measurements.

**ACTIVITY:** After postulate 4, whiteboards: Make up states with  $P(E_0)=1/3$ , and calculate  $\langle H \rangle$ .

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L26 Fri 10/24/08

Read : Finish Chapter 3, starting Ch 4 next week.

On board: All the postulates (with a blank for #6/

Tut today (#8) From UW, on measuring  $x, p$ , and  $E$ .

**Topics:** Review postulates 4 and 5. Continuous variables, and eigenvectors of  $x$  and  $p$ . Discussed delta function orthogonality, and how collapse works with degeneracy. Ended with Postulate #6, the TDSE.

L27: Monday 10/27/08

Reading For Wed: (Finish Ch 3, starting CH 4 next)

ON Board : 6 postulates. On other board: E-values of position, momentum, and energy are labeled  $|x_0\rangle$   $|p_0\rangle$  and  $|u_n\rangle$ , then  $|\psi\rangle = \int dx_0 \psi(x_0) |x_0\rangle = \int dp_0 \phi(p_0) |p_0\rangle = \sum c_n |u_n\rangle$  and thus  $\psi(x) = \langle x|\psi\rangle$   $\phi(p) = \langle p|\psi\rangle$   $c_n = \langle u_n | \psi\rangle$

Covered notes:

Topics: Review of postulates, and discussion of "x, p, and E representations". Review of time independence of energy probabilities, and the special role of H in QM. Generalized uncertainty principle: proof/derivation in detail. Compatible observables.

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L28: Wed Oct 29

ON BOARD: (review of generalized uncertainty principle)

**Topics:** Review of uncertainty principle, proof/discussion that compatible variables (can) have simultaneous eigenvectors, time dependence of expectation values, and time-energy uncertainty principle.

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L29 Fri 10/31/08

On board: Review of :uncertainty principle formula,  $d\langle Q\rangle/dt$  formula, and energy time uncertainty formula.

Tut today (#9) On Quantum operator methods and the "quantum mouse".

**Topics: Review**, then 3D. Emphasis on "what's still the same". 1<sup>st</sup> postulate gives us  $\Psi(\mathbf{r}\text{-vector}, t)$ , need 3 labels. 2<sup>nd</sup> postulate gives us operator/observables in each dimension ( $p_x, p_y, p_z$ ) and new Shrod Eq is nearly the same. As is TDSE. Separation of variables for time still works. Units of wave function modified. Commutation relations give "kronecker deltas". Separation of variables in 3D might work. Example: 3D infinite square well. Energy eigenstates and eigenvalues, and degeneracy.

(Students did their own spontaneous "kinesthetic activity" today – particle in a box, and expectation value sloshing back and forth).

L30: Monday 11/03/08

Reading For Wed: (Finish Ch 3, starting CH 4 next)

ON Board :  $\psi(\mathbf{r}, t)$ , and  $H$  in 3-D. Commutation relations for  $r$ ,  $p$ . TISE in 3D. Solutions for infinite square well.

Topics: Degeneracy of infinity square well (review). Spherical (central) potentials.  $\Delta^2$  in spherical, Separation of variables in spherical. Equation for  $Y$ , then second separation. Solutions for  $\phi$  ( $e^{i m \phi}$ ) and  $P_l^m$ . (Not derived, but discussed, including generating functions)

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L31: Wed 11/5

ON BOARD: (Formulas for separation of v'bles into  $r$ ,  $\theta$ , and  $\phi$ , including the Diff Eq's, and the generating formulas for the  $P_l$  and  $P_l^m$ 's, as well as general  $Y_l^m = A_m P_l^m \exp[i m \phi]$ , with  $m, l$  integers,  $l \geq 0$ , and  $|m| \leq l$ .)

**Topics:  $Y_l^m$ 's today.** Mostly just generating functions, plots and visualization. Started radial equation, including defining  $u = r R$ , and getting 1-D like SE.

**Activities:** Whiteboards. After generating  $P_0, P_1$  and  $P_2$ , we had people compute the associated legendre functions that go with these, and had them called out. Then we split the class up and had people plot the "parametric" plot of  $|P_l^m(\theta)|$ , for  $l=1$  and  $2$ , and all  $m$ 's. (Except  $P_{00}$  and  $P_{10}$  which Oliver did for them first) Took about 5-10 minutes, very productive. At end, representatives came to the board to draw their sketches. Then, Oliver showed MMA plots (of  $|Y_l^m|^2$ ) and "chemistry" plots (which are related but slightly different, these show the real linear combos of  $Y_{l \pm m}$ 's. )

<http://web.uniovi.es/qcg/harmonics/harmonics.html>

<http://mathworld.wolfram.com/SphericalHarmonic.html>

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L32 Fri 11/07/08

On board: Exam. Central potentials: TISE, separation, angular equation yields  $Y_l^m$ 's, with  $l \geq 0$ , and  $|m| \leq l$ .

Tut today (#10)

**Topics:** Radial equation,  $u(r)=rR(r)$ , resulting effective 1D TISE, and discussion of centrifugal term (classical and quantum) Stationary states. General superpositions. Normalization. Then leave  $R$ 's for now, move to Angular momentum: define  $L$ , classically and then QM, Hermiticity proof, cyclic commutation relations,  $L^2$  and its commutation relations. Consequence - incompatible observables.

L33: Monday 11/10/08

ON Board : Exam, Angular momentum (def, and commutators), Central potentials (separation of variables).

Topics: Review that stationary states are not ALL states. Generalized uncertainty principle for components of  $L$ .  $L$  in spherical components (as differential operators). Separation of laplacian, seeing that angular equation is the  $L^2$  operator. So, our  $Y_{lm}$ 's are also eigenfunctions of  $L^2$ , and  $L_z$ . (with e-values  $\hbar^2 l(l+1)$ , and  $\hbar m$ ) Idea of "compatible operators yield simultaneous eigenfunctions". Review of classical (torque, conservation of  $L$ ) Then quantum conservation of  $L$ , using  $d\langle L \rangle/dt$  formula. Complete set of commuting observables, " $n, l, m$ " corresponding to " $H, L^2$ , and  $L_z$ ".

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L34: Wed 11/12

ON BOARD: Exam tomorrow Ang momentum,  $L = r \times p$ ,  $L^2$  operator written out,  $L_z = i\hbar d/d\phi$

Hamiltonian written out,  $L^2 Y_{lm} = \hbar^2 l(l+1)$

**Topics:** Review how  $[H, a_-] = -\hbar \omega a_-$  gave us that  $a_-$  is a "lowering operator". Use that as analogy to motivate an  $L_+$  and  $L_-$  operator. Define  $L_{\pm} = L_x \pm iL_y$ . Reproduce the old method to prove  $L_+$  is a raising operator. Proof that  $m$  is bounded, then use that and the expression (derived)  $L_+L_- = L^2 - L_z^2 + \hbar L_z$  to prove that  $M(\max) = l$ . End with explicit formula for  $L_{\pm} Y_{lm}$  (including coefficient)

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L35 Fri 11/14/08

On board: Reading: 4.1 and 4.3 is old, 4.2 for Today and Mon, 4.4 for next week and beyond.

NO TUTORIAL

**Topics: Review:**  $Y_{lm}$ 's, (picture, with  $l = 0, 1, 2$ , to right, and then "towers" of  $m$  above) Griffiths "L is a cone" example for visualizing. Reminder of radial equation, normalization condition, boundary condition. Set up hydrogen, discuss potential, sketch  $l=0$  states. Discussed  $E > 0$  and  $E < 0$  solutions.

**Activity:** Build a "quiz" (quiz\_  $Y_{lm}$ ), spent about 5 minutes on their own, then another 5 minutes in groups. (While they worked, I added formula  $L^2 Y_{lm}$  and  $L_z Y_{lm}$  on board.)  
2<sup>nd</sup> activity: Brought a top, just to have a prop to show angular momentum.

2<sup>nd</sup> activity: On back of quiz, sketch  $-A/r + B/r^2$ .

L36: Monday 11/17/08

ON Board : HW due Wed. (Or, Thursday by noon is ok)

Read: 4.2 for today. 3.6 (Example 3.8) for review. 4.4 (through 4.4.1) coming up next

Topics: Hydrogen radial wave functions. Effective potential, relation  $l \leq n-1$ , Energies, method of Frobenius, asymptotic behavior (both large  $l$  and small  $l$ ), showed (but not yet discussed) a few associated Laguerre polynomials. Radial probability density.

**Activity:** Whiteboards: Given the lowest few  $R_{nl}$ 's (on board), sketch the wave functions. (Little open ended - can focus on  $R(r)$ , or  $u(r)$ , or the whole thing including  $\theta/\phi$  dependence if they wanted. Just wanted them to wrestle with "how to represent". They did well!)

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L37: Wed 11/19

ON BOARD: Reading: Example 3.8, read 4.4.1 HW due tomorrow non.

1-D effective TISE,  $\psi = R_{nl} Y_{lm}$ ,  $R_{nl} = \text{constant} * r^l * \exp[-r/na] * L(2r/na)$

**Topics:** Associated Laguerre polynomials: notation, behavior. Also, notation of "s,p,d,f" states, degeneracy. Transitions, and  $\Delta l = 1$  selection rule. Balmer and Lyman lines. (Start of class was a clicker question on whether any stationary state can be written as  $R_{nl}(r)Y_{lm}$ . Turns out to be subtle and generated long discussion and debate, took about 10 minutes!)

**Activity:** We had a brief activity – just asked them to come up with a formula for the number of nodes in an  $R_{nl}$  state, and then think about what that says about the corresponding associated Laguerre polynomial.

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L38 Fri 11/21/08

On board:

NO TUTORIAL, HW 12 online, Read -> 4.4.1

**Topics:** Review/finish hydrogen (3D wavefunctions, and difference between spherical probability density and radial probability density) Then, intro to matrix mechanics: including bra, and ket and matrices in a basis, and diagonal nature of operator in a basis.

Activity: I pulled up the website ,

[http://webphysics.davidson.edu/faculty/dmb/hydrogen/intro\\_hyd.html](http://webphysics.davidson.edu/faculty/dmb/hydrogen/intro_hyd.html)

we went through for 20 minutes, predicting what the pictures would look like, then looking at them. Got up through about  $\psi(3,2,m)$ 's.

L39: Monday 12/1/08

ON Board : Read -> 4.4.2

Topics: Review: Ylm's, then operator methods and "spectrum" of angular momentum, then matrix methods in qm. Then spend about 15 minutes on the quiz, and another 10 discussing it (matrix form of  $L^2$  and  $L_z$ , and time dependence) Ended with motivation for spin (but not yet a discussion of it in detail)

**Activity: We spend about 6 minutes alone, then ~10 minutes in groups, on my small quiz "quiz\_matrix\_operators.doc"**

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L40: Wed 12/3/08

ON BOARD: Read -> 4.4.2, Last HW Due next Wed.

**Topics:** Spin, start with magnetic moment, Stern-Gerlach (as motivation for spin), intro to spin  $1/2$ , parallels to angular momentum, commutators, intrinsic nature, not "spacial" in origin.

This was FCQ day, so missed ~15 minutes.

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L41 Fri 12/5/08

On board: Finish Ch 4 for next week.

**Topics:** Spin (parallels to Angular momentum), particles of various spin, matrices for spin, raising, lowering,  $S_x$ ,  $S_y$ , Pauli Matrices. Statistical interpretation of a spinor.

TUTORIAL 11: Spin and Angular Momentum

L42: Monday 12/8/08

ON Board : Final exam, last HW, survey.

Topics: Spin 1/2: Eigenvectors of  $S_x$  (and  $S_y$ ) in z-basis, probabilities, Stern-Gerlach story (sequential measurements). Computed and discussed expectation values of operators for various states. Quickly went through "Larmor Precession" - big ideas.

**Activity: Stern Gerlach phet sim - talked them through it, let them guess a couple of outcomes.**

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L43: Wed 12/10/08

ON BOARD: Final exam notice

**Topics: None - posttest**

**ACTIVITY: Post-test today**

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L44 Fri 12/2/08

Last lecture: On board (final info)

**Topics: Review :** The postulates of QM, some review clicker questions from Mike, and ended with about 20 minutes of discussion - first, what Quantum II will entail, then a little bit about Feynman diagrams and gyromagnetic ratio of the electron.

Tutorial: We held a "review of the posttest" today