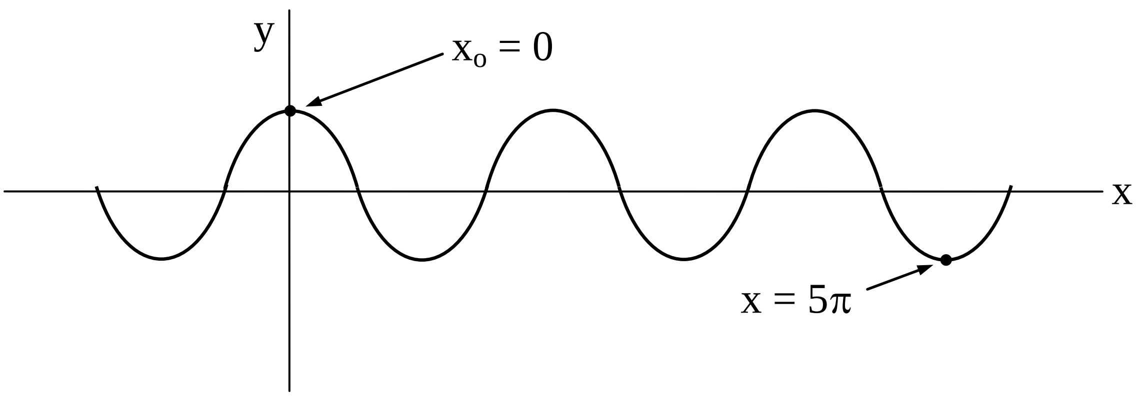
CT1. Recall the Taylor Series expansion of a function f(x):

$$f(x) = f(x_o) + \left.\frac{df}{dx}\right|_{x_o} \cdot (x - x_0) + \frac{1}{2}\left.\frac{d^2f}{dx^2}\right|_{x_o} \cdot (x - x_0)^2 + \ldots$$

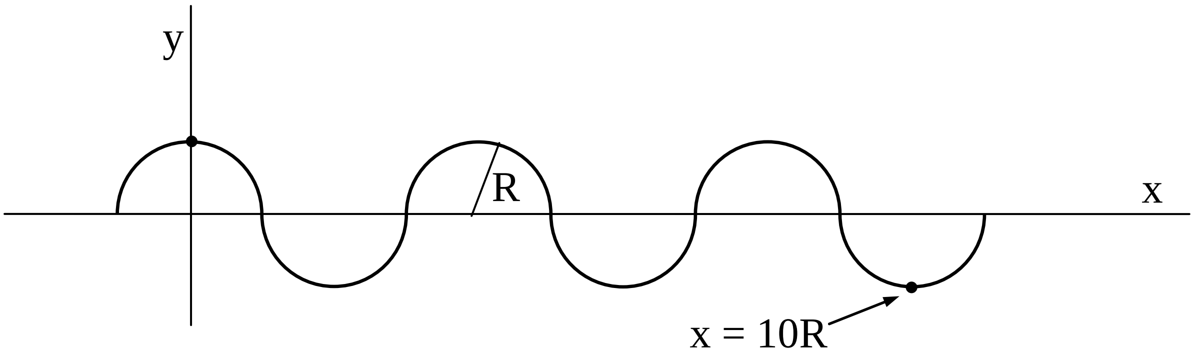
Consider the function f(x) = cos(x). If $x_o = 0$, will the Taylor Series expansion produce the correct value of cos(5π)? That is, is the following equation correct?

$$\cos(5\pi) \overset{??}{=} \cos(0) + \left.\frac{d\cos(x)}{dx}\right|_0 \cdot (5\pi) + \frac{1}{2}\left.\frac{d^2\cos(x)}{dx^2}\right|_0 \cdot (5\pi)^2 + \ldots$$



A) Yes, the series expansion will give the exact answer cos(5π)= –1.
B) No, the series will not produce an answer even close.
C) The series will produce an answer that is close, but not exactly equal to –1.

CT2. The function f(x) consists of a infinite series of semi-circles each of radius R, as shown.



If $x_o = 0$, will the Taylor Series expansion produce the correct value of f (x = 10R) = –R?

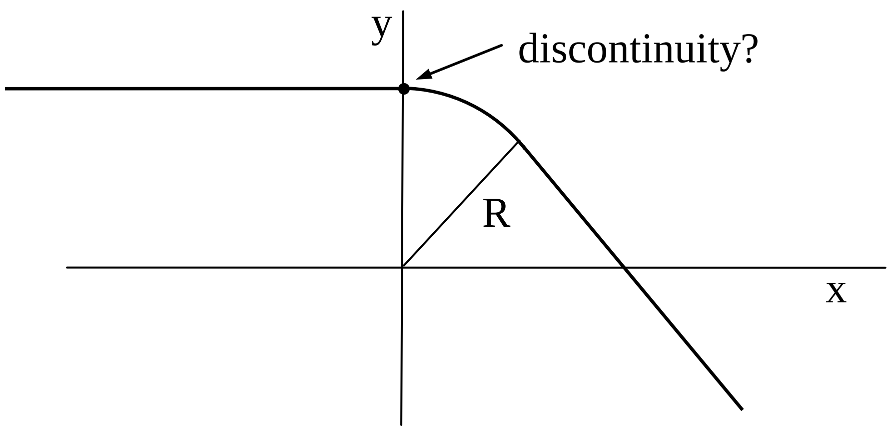
A) Yes, the series expansion will yield f = –R , exactly.

B) No, the series will not produce an answer even close.

C) The series will produce an answer that is close, but not exactly equal to f = –R.

CT3. Consider the function f(x) which is a quarter-circle joining two straight lines, as shown. Near x = 0, the function has the form

$$f(x) = \begin{cases} R, & x < 0 \\ \sqrt{R^2 - x^2}, & x > 0 \end{cases}$$



Is there a discontinuity in f(x) or any of its derivatives at x = 0?

A) f(x) is discontinuous at x = 0.

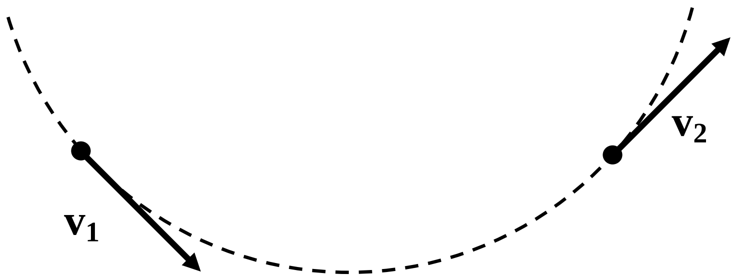
B) f(x) is continuous, but df/dx is discontinuous at x = 0.

C) f(x) and df/dx are continuous , but d^2f/dx^2 is discontinuous at x = 0.

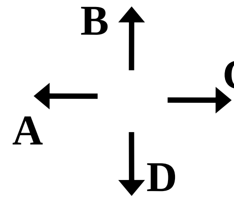
D) f(x), df/dx, and d^2f/dx^2 are all continuous, but d^3f/dx^3 is discontinuous at x = 0.

E) f(x) and all its higher derivatives are continuous at x = 0.

CT4. An object has velocity $\vec{v}_1$ at an earlier time and velocity $\vec{v}_2$ at a later time, as shown. What is the direction of $\vec{\Delta v} = \vec{v}_2 - \vec{v}_1$?

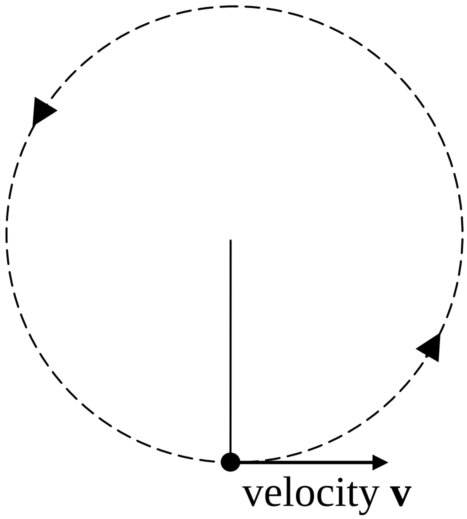


direction of **Δv**?

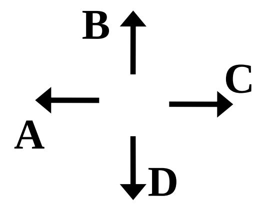


E) None of these

CT5. A rock is twirled on a string at constant speed by an astronaut in inter-galactic space (where gravitational effects are negligible). At the moment shown, what is the direction of the acceleration **a** of the rock?

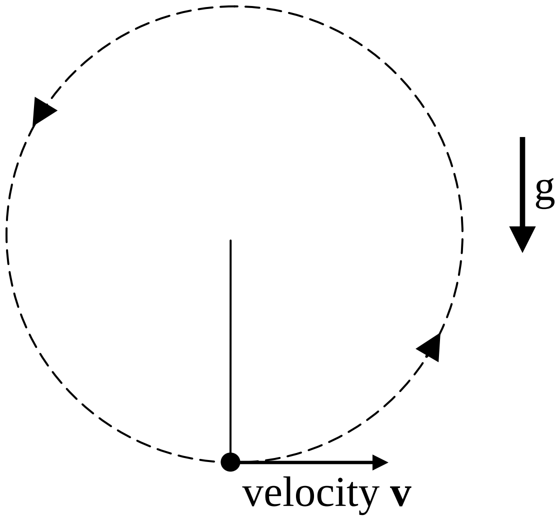


direction of **a**?

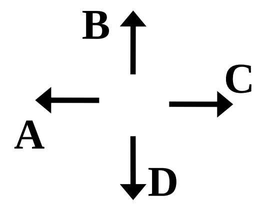


E) None of these

CT6. Near the surface of the earth, a rock is twirled on a string in a vertical plane at constant speed. At the moment shown, what is the direction of the acceleration **a** of the rock?

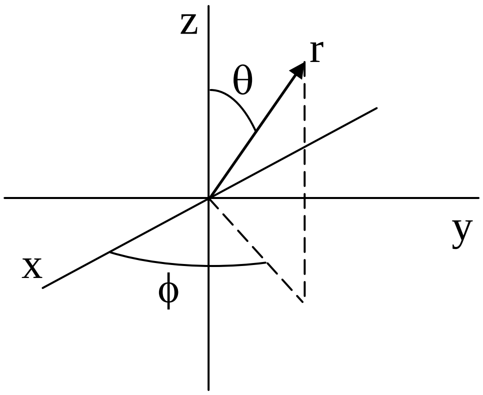


direction of **a**?



E) None of these

CT7.If we use spherical coordinates (r, θ, ϕ) to integrate over the volume of a sphere of radius R, centered on the origin, what are the correct limits of integration?



A) r = 0 to R, θ = 0 to π, ϕ = 0 to 2π

B) r = –R to R, θ = 0 to 2π, ϕ = 0 to π

C) r = 0 to R, θ = 0 to 2π, ϕ = 0 to π

D) r = –R to R, θ = 0 to 2π, ϕ = 0 to 2π

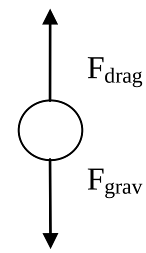
CT8.For which of the following integrals can we usefully apply integration by parts?

$$1.\quad \int dx \frac{\ln x}{x^2} \qquad 2.\quad \int dx\, x\, e^{x^2} \qquad 3.\quad \int dx\, x^2\, e^{2x}$$

A) 1 and 2
B) 2 and 3
C) 1 and 3
D) 1, 2, and 3
E) None of the integrals can be integrated by parts.

CT9. Consider the equation $\frac{dv}{dt} = -k\,v$, where v is the velocity, t is the time, and k is a constant. This equation describes the motion of
A) a mass on a spring
B) a mass in free-fall (no air resistance)
C) a moving mass experiencing a drag force
D) a moving mass with no net force
E) a mass moving in a circle at constant speed

CT10. Consider a mass falling under the influence of gravity while experiencing a drag force that is proportional to the square of its velocity. (Down corresponds to increasing position and velocity.) What is the equation describing the motion of the mass?



A) $\frac{dv}{dt} = -g + k\,v$

B) $\frac{dx}{dt} = g - k\,x^2$

C) $\frac{dv}{dt} = g - k\,v^2$

D) $\frac{dx}{dt} = g - k\,x$

E) $\frac{dv}{dt} = mg - v^2$

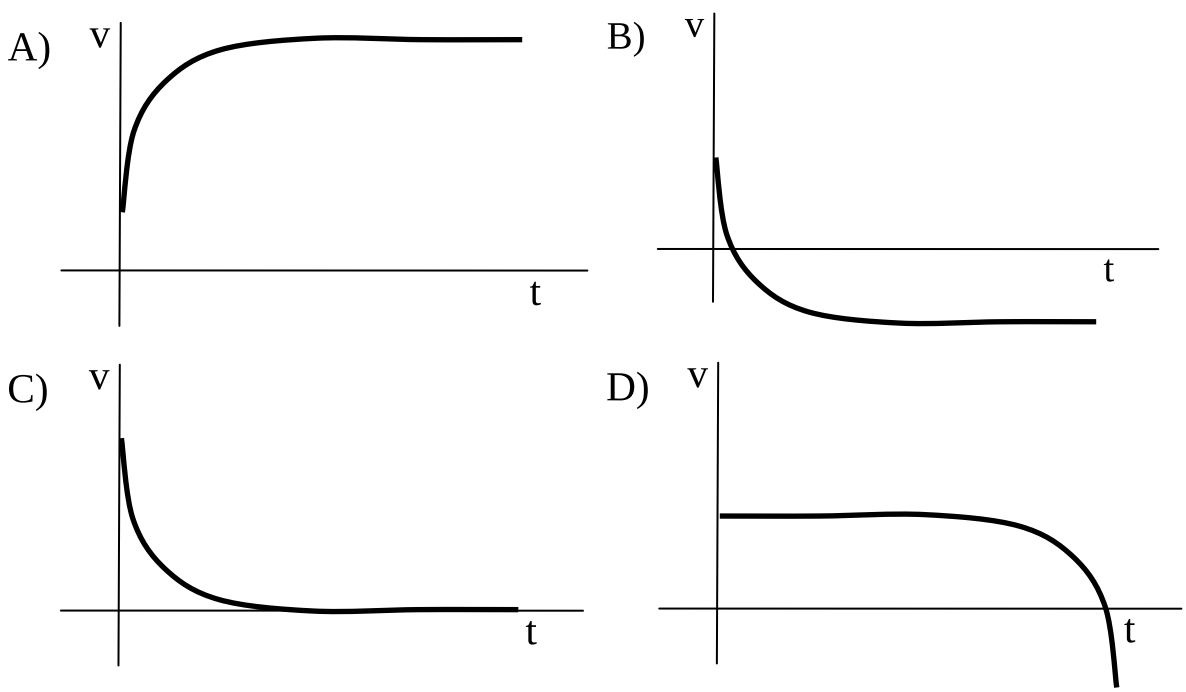
CT11. We found that the solution to the equation describing a mass moving with a drag force

$$\frac{dv}{dt} = -kv; \qquad v(t=0) = v_o > 0,$$

is given by

$$v(t) = v_o e^{-kt}$$

Which of these figures shows the velocity of the mass as a function of time?



CT12. We found that the solution to the equation for a mass falling under the influence of gravity, but with a drag force

$$\frac{dv}{dt} = -g - kv; \quad v(t=0) = v_o > 0, \; z(t=0) = h > 0$$

is given by

$$v(t) = \frac{-g}{k} + \left(\frac{kv_o + g}{k}\right)e^{-kt}$$

Which of these figures shows the height of the mass as a function of time?