CT13-1. The general azimuthally symmetric solution to $\nabla^2\Phi=0$ has the form:

$$\Phi(r,\theta)=\sum_{\ell=0}^{\infty}\left[A_\ell r^\ell+\frac{B_\ell}{r^{\ell+1}}\right]P_\ell(cos\theta)\,.$$

If the boundary conditions require that $\Phi$ approaches zero for large r, what can you conclude about the coefficients A and B?

A) $A_\ell=0 \quad \text{for } \ell>0$

B) $B_\ell=0 \quad \text{for } \ell>0$

C) $A_\ell=0 \quad \text{for all } \ell$

D) $B_\ell=0 \quad \text{for all } \ell$

CT13-2. If $\Phi$ has the form

$$\Phi(\mathrm{r},\theta)=\sum_{\ell=0}^{\infty}\frac{B_\ell}{r^{\ell+1}}P_\ell(\cos\theta),$$

which term is largest in the limit of large r?

A) $\dfrac{B_0}{r}$

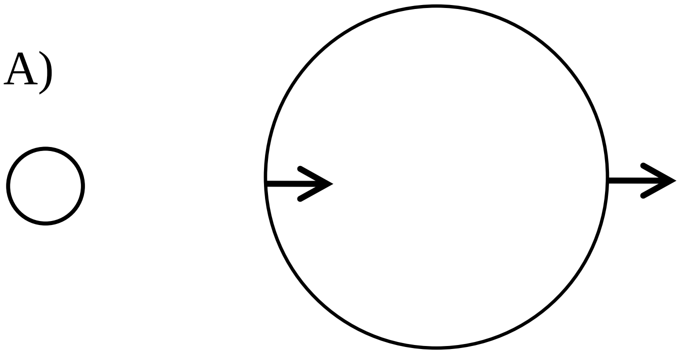
B) $\dfrac{B_1}{r^2}P_1(\cos\theta)$

C) $\dfrac{B_2}{r^3}P_2(\cos\theta)$

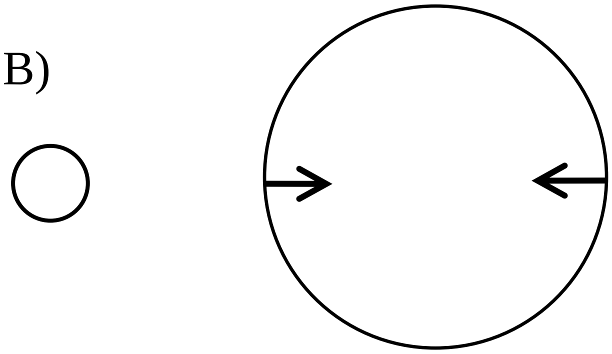
D) $\dfrac{B_3}{r^4}P_3(\cos\theta)$

CT13-3. Which figure shows the correct direction of the tidal force on the earth due to the moon, at the points on the earth's surface closest to and farthest from the moon?

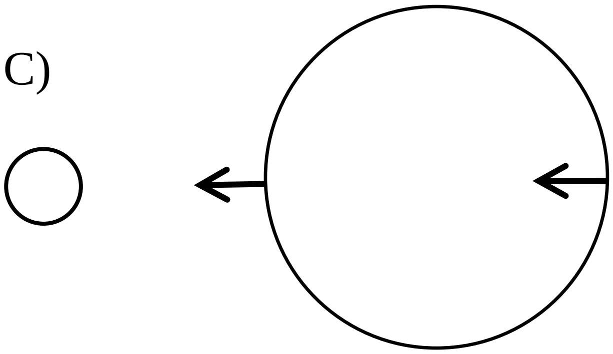
A)



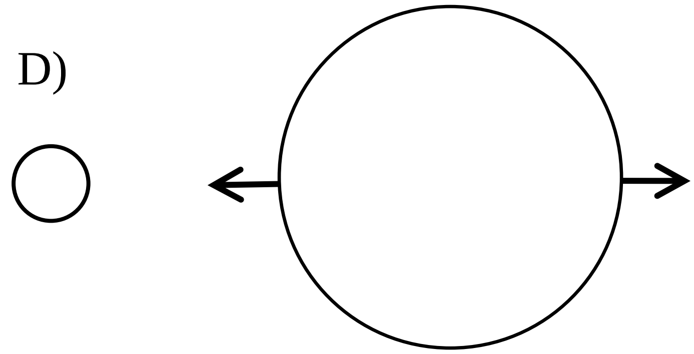
B)



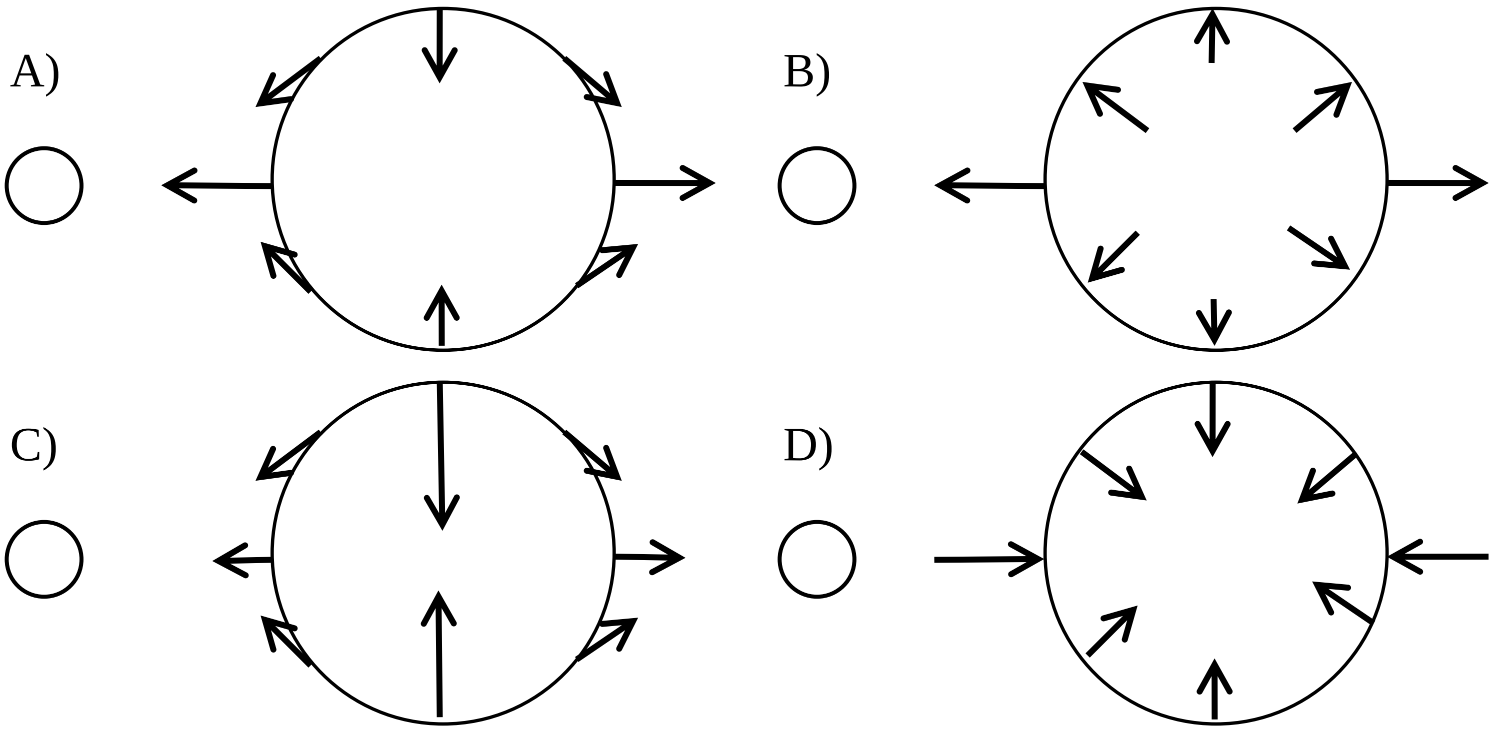
C)



D)



CT13-4. Which figure shows the correct direction of the tidal force on the earth due to the moon?

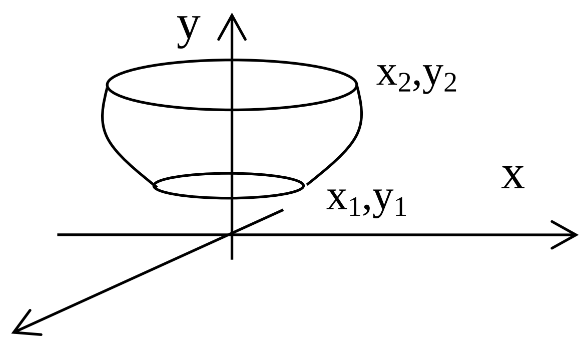


CT13-5. The mass of the sun is $3\times10^7$ the mass of the moon. The sun is 400 times farther from the earth than the moon. What are the relative magnitudes of the tidal forces on earth due to the sun ($F_{ts}$) and the moon ($F_{tm}$)?

A) $F_{ts}$ is much greater than $F_{tm}$.
B) $F_{ts}$ is slightly greater than $F_{tm}$.
C) $F_{ts}$ is the same as $F_{tm}$.
D) $F_{ts}$ is slightly less than $F_{tm}$.
E) $F_{ts}$ is much less than $F_{tm}$.

CT13-6. Soap films form **minimal surfaces** (surfaces with minimum surface area). Suppose a surface is described by a curve from $(x_1,y_1)$ to $(x_2,y_2)$ revolved about the y axis. Which is the correct expression for the area of the surface?




A) $A = 2\pi \int_{x_1}^{x_2} y' \sqrt{1+x^2}\, dx$

B) $A = \pi \int_{x_1}^{x_2} y'^2 \sqrt{1+x^2}\, dx$

C) $A = 2\pi \int_{x_1}^{x_2} y \sqrt{1+y'^2}\, dx$

D) $A = \pi \int_{x_1}^{x_2} x^2 \sqrt{1+y'^2}\, dx$

E) $A = 2\pi \int_{x_1}^{x_2} x \sqrt{1+y'^2}\, dx$

CT13-7. What is the correct expression for $\frac{\partial f}{\partial \alpha}$ ? Assume that

$$f = f\left[y(x); y'(x); x\right]$$

A) $\frac{\partial f}{\partial \alpha} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \alpha} - \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial \alpha}$

B) $\frac{\partial f}{\partial \alpha} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \alpha}$

C) $\frac{\partial f}{\partial \alpha} = \eta\frac{\partial f}{\partial y} + \eta'\frac{\partial f}{\partial y'}$

D) $\frac{\partial f}{\partial \alpha} = \frac{\partial f}{\partial y'}\frac{\partial y'}{\partial \alpha}$

E) $\frac{\partial f}{\partial \alpha} = \eta'\frac{\partial f}{\partial y} + \eta\frac{\partial f}{\partial y'}$

CT13-8. When determining the shortest distance between two points in the plane, what is the differential equation determined by Euler's equation? In other words, given $f = \sqrt{1+y'^2}$ , what ODE for y results from plugging f into Euler's equation?

A) $y' + \frac{d}{dx}\left[\frac{y'}{\sqrt{1+y'^2}}\right] = 0$

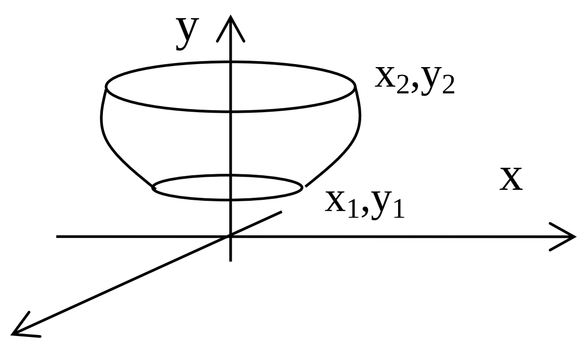
B) $\frac{d}{dx}\left[\frac{y'}{\sqrt{1+y'^2}}\right] = 0$

C) $\frac{d}{dx}\left[\frac{y''}{\sqrt{1+y'^2}}\right] = 0$

D) $y'' + \frac{d}{dx}\left[\frac{1}{\sqrt{1+y'^2}}\right] = 0$

E) $\frac{d}{dx}\left[\sqrt{1+y'^2}\right] = 0$

CT13-9. Suppose a surface is described by a curve from $(x_1,y_1)$ to $(x_2,y_2)$ revolved about the y axis. When determining the surface of minimum area, what is the differential equation determined by Euler's equation? In other words, given $f = x\sqrt{1+y'^2}$, what ODE for y results from plugging f into Euler's equation?




A) $\dfrac{d}{dx}\left[\dfrac{xy'}{\sqrt{1+y'^2}}\right]=0$

B) $\dfrac{d}{dx}\left[\dfrac{y'}{\sqrt{1+x^2}}\right]=0$

C) $\dfrac{d}{dx}\left[\dfrac{2x^2y'}{\sqrt{1+y'^2}}\right]=0$

D) $\dfrac{d}{dx}\left[\dfrac{xy'}{\sqrt{1+x^2}}\right]=0$

E) $\dfrac{d}{dx}\left[\dfrac{y'}{\sqrt{1+y'^2}}\right]=0$

CT13-10. When determining the shortest distance between two points in the plane, what is the differential equation determined by the second form of Euler's equation? In other words, given $f = \sqrt{1+y'^2}$ , what ODE for y results from plugging f into the second form of Euler's equation?

A) $\sqrt{1+y'^2} - \frac{y'}{\sqrt{1+y'^2}} = c$

B) $\sqrt{1+y'^2} - \frac{y'^2}{\sqrt{1+y'^2}} = 0$

C) $1 - \frac{y'^2}{1+y'^2} = 0$

D) $y'^2\left(\sqrt{1+y'^2} - \frac{1}{\sqrt{1+y'^2}}\right) = c$

E) $\sqrt{1+y'^2} - \frac{y'^2}{\sqrt{1+y'^2}} = c$