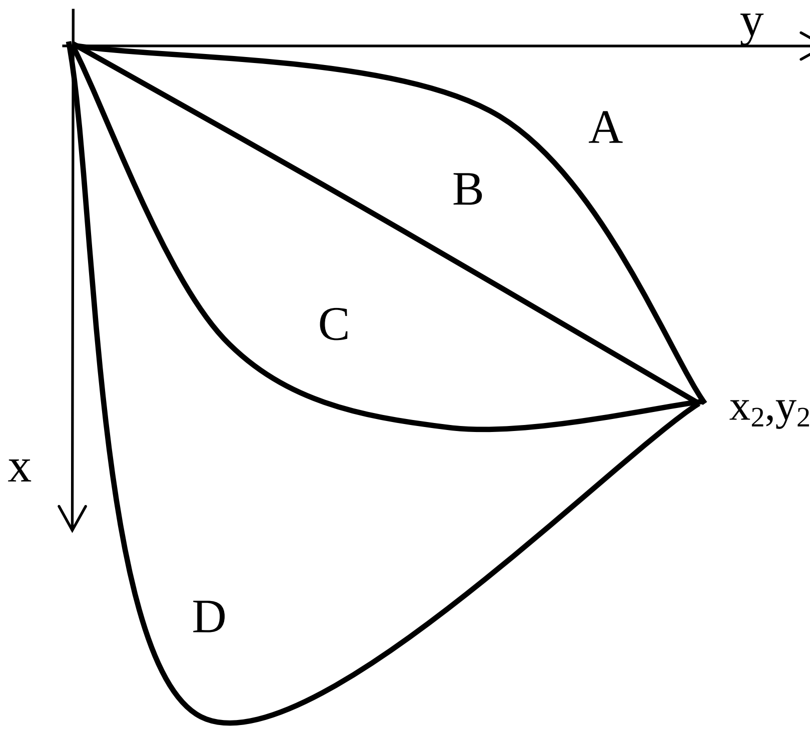
CT14-1. Which of these problems could be solved using the calculus of variations?

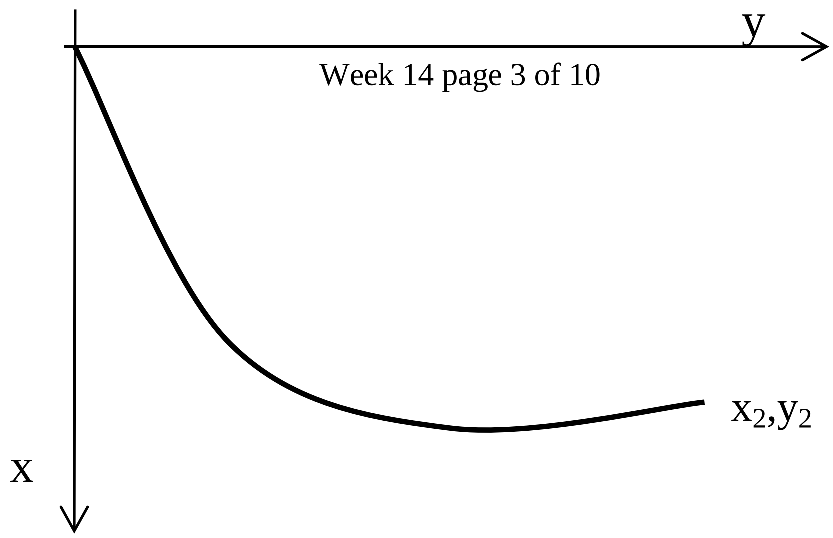
1. Find the period of small oscillations for a particle sliding (without friction) on the inside of a sphere.
2. Find the surface with fixed area that encloses the maximum volume.
3. Find the path between two points that minimizes the time for a particle to slide (without friction) between the points.
4. Find the path of a projectile (with no air resistance) that leads to the maximum range.

A) None B) Only one C) Exactly two
D) Exactly three E) All four

CT14-2. Which of these paths could be a solution to the brachistochrone problem?



CT14-3. If the particle starts from rest at x=y=0 and slides under the gravitational force (without friction), what is the relationship between x, y, and the speed of the particle?



A) $\frac{1}{2}mv^2 - mgy = \text{const.}$

B) $\frac{1}{2}mv^2 + mgy = \text{const.}$

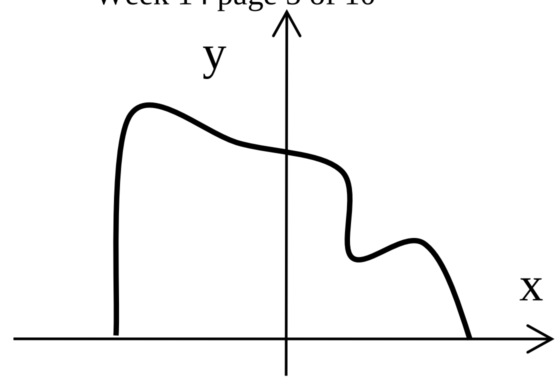
C) $\frac{1}{2}mv^2 - mgx = \text{const.}$

D) $\frac{1}{2}mv^2 + mgx = \text{const.}$

CT14-4. Which of these problems could be solved using the calculus of variations with constraints?

1. Find the shortest path between two points on the surface of a cylinder.
2. Find the surface with fixed area that encloses the maximum volume.
3. Find the path between two points that minimizes the time for a particle to slide (without friction) between the points.
4. Find the path between two points that minimizes the area of the surface of revolution (formed by rotating the path around an axis).

A) None
B) Only one
C) Exactly two
D) Exactly three
E) All four



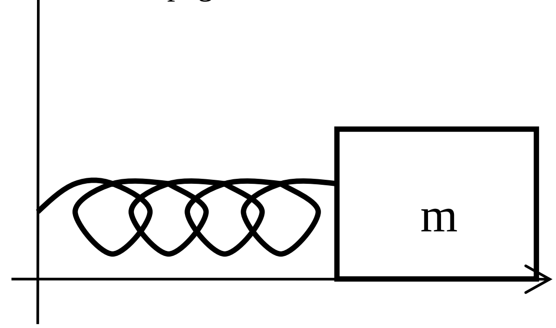
CT14-5. To solve Dido's problem, what is the function that must be maximized?

A) $J[y]=\int_{-a}^{a}\left(y+\lambda\sqrt{1+y'^2}\right)dx$

B) $J[y]=\int_{-a}^{a}\left(y+\lambda y'^2\right)dx$

C) $J[y]=\int_{-a}^{a}\left(y^2+\lambda\left[1+y'^2\right]\right)dx$

D) $J[y]=\int_{-a}^{a}\left(\sqrt{1+y'^2}+\lambda y\right)dx$



CT14-6. What is the Lagrangian of a particle (mass m) attached to a spring (spring constant k)?

A) $\mathcal{L}(x,\dot{x},t)=\frac{1}{2}k\dot{x}^2-\frac{1}{2}mx^2$

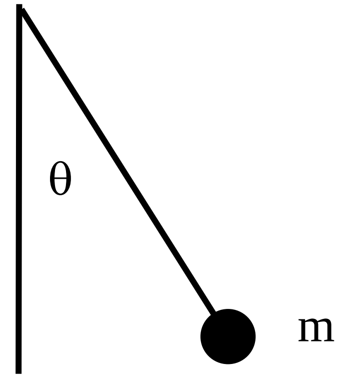
B) $\mathcal{L}(x,\dot{x},t)=m\dot{x}^2-kx^2$

C) $\mathcal{L}(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$

D) $\mathcal{L}(x,\dot{x},t)=m\dot{x}^2+kx^2$

E) $\mathcal{L}(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$

CT14-7. What is the Lagrangian of a pendulum (mass m, length l)? Assume the potential energy is zero when θ is zero.



A) $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\ell^2\dot{\theta}^2-mg\ell(1-\cos\theta)$

B) $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\ell^2\dot{\theta}^2+mg\ell(1-\cos\theta)$

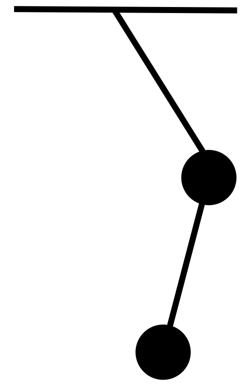
C) $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\dot{\theta}^2+mg\ell(1-\cos\theta)$

D) $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\dot{\theta}^2-mg\ell(1-\cos\theta)$

CT14-8. Which of these constraints is holonomic?

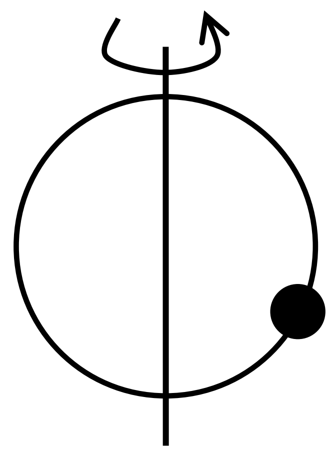
1. A particle is constrained to slide on the inside of a sphere.
2. A cylinder rolls down an inclined plane.
3. Two moving particles are connected by a rod of length l.
4. A moving car is constrained to obey the speed limit.

A) None
B) Only one
C) Exactly two
D) Exactly three
E) All four



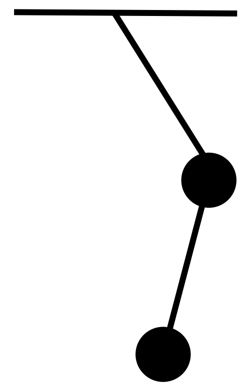
CT14-9. For which of these systems could you use Lagange's equations of motion?

1. A double pendulum: a pendulum (mass m, length l) has a second pendulum (mass m, length l) connected to its bob.
2. A projectile moves in two dimensions with gravity and air resistance.
3. A bead slides without friction on a circular, rotating wire.



A) 1 only B) 2 only C) 3 only
D) 1 and 2 E) 1 and 3

CT14-10. What would be a good choice of generalized coordinates for the double pendulum? (Assume the pendula are constrained to move in the x-y plane.)



A) the Cartesian coordinates of the bob positions: x_1, y_1 (first bob) and x_2, y_2 (second bob)
B) the Cartesian coordinates of the first bob: x_1, y_1 and the Cartesian coordinates of the second bob, treating the first bob as the origin: x'_2, y'_2
C) the angles made between each pendulum rod and the vertical: θ_1, (first bob) θ_2 (second bob)
D) the angles made between a line drawn from each pendulum bob to the pivot and the vertical: α_1, (first bob) α_2 (second bob)

