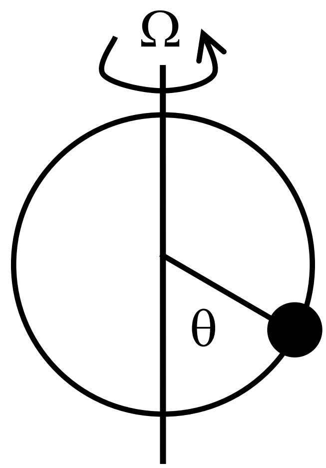
CT15-1. A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. What is the kinetic energy of the bead?



A) $T = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\Omega^2$

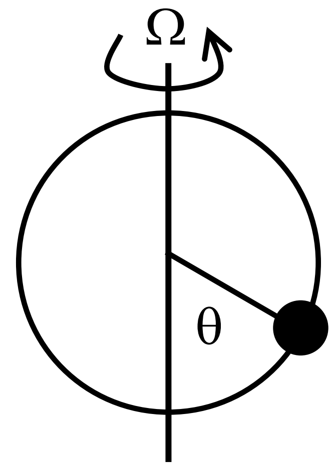
B) $T = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\cos^2\theta\,\Omega^2$

C) $T = \frac{1}{2}mR^2\dot{\theta}^2 + \frac{1}{2}mR^2\sin^2\theta\,\Omega^2$

D) $T = \frac{1}{2}mR^2\dot{\theta}^2 - \frac{1}{2}mR^2\Omega^2$

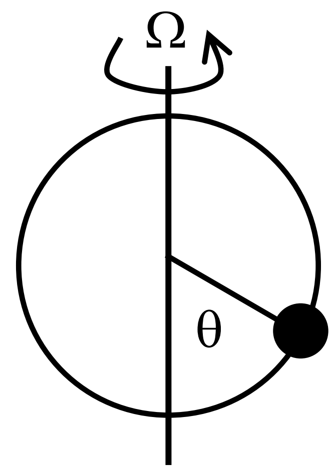
E) $T = \frac{1}{2}mR^2\left(\dot{\theta} + \Omega\sin\theta\right)^2$

CT15-2. A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. When the bead is at angle θ, how high is the bead above the lowest point of the wire?



A) $h = R\cos\theta$
B) $h = R\sin\theta$
C) $h = R(\sin\theta - \cos\theta)$
D) $h = R(1 - \sin\theta)$
E) $h = R(1 - \cos\theta)$

CT15-3. A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. When the bead is at angle θ, what is the θ component of the centripetal force on the bead?



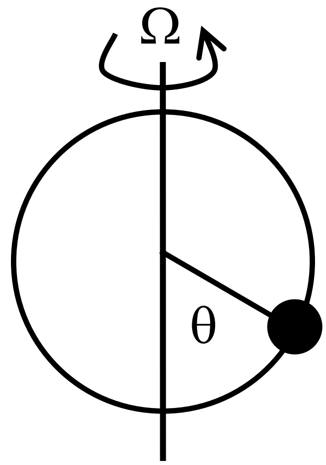
A) $F_{c\theta} = mR^2\Omega^2 \cos\theta$

B) $F_{c\theta} = mR^2 \sin\theta$

C) $F_{c\theta} = mR^2\Omega^2 \sin\theta$

D) $F_{c\theta} = mR^2\Omega^2 \sin\theta \cos\theta$

E) $F_{c\theta} = mR^2 \sin\theta \cos\theta$



CT15-4. A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. The equation of motion of the bead is

$$\ddot{\theta} + \frac{g}{R}\sin\theta - \Omega^2 \sin\theta\cos\theta = 0$$

What are the equilibrium value(s) of θ?

A) $\theta = 0$

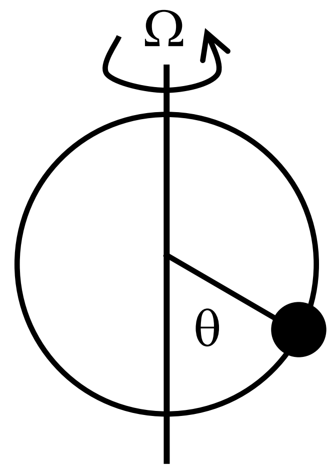
B) $\theta = \cos^{-1}\left(\frac{g}{R\Omega^2}\right)$

C) $\theta = \sin^{-1}\left(\frac{g}{R\Omega^2}\right)$

D) $\theta = 0$ and $\theta = \cos^{-1}\left(\frac{g}{R\Omega^2}\right)$

E) $\theta = 0$ and $\theta = \sin^{-1}\left(\frac{g}{R\Omega^2}\right)$

CT15-5. A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. The equation of motion for small motions about the equilibrium $\theta_0 = \cos^{-1}\left(\frac{g}{R\Omega^2}\right)$ is



$$\delta\ddot{\theta} + \Omega^2 \sin^2\theta_0 \, \delta\theta = 0$$

What is the oscillation frequency of the bead?

A) $\omega = \Omega$

B) $\omega = \Omega^2$

C) $\omega = \Omega \sin\theta_0$

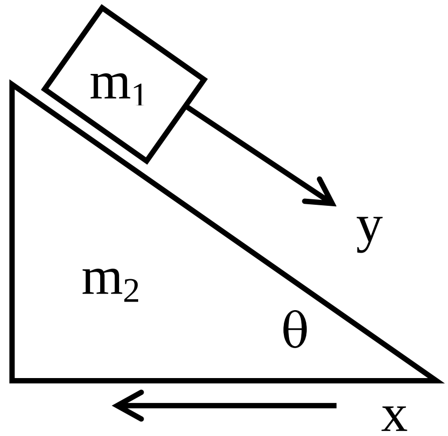
D) $\omega = \Omega^2 \sin^2\theta_0$

E) $\omega = \Omega \sin\theta$

CT15-6. A block of mass m_1 slides on an inclined plane of mass m_2. The Lagrangian of the system is

$$\mathcal{L}(x,\dot{x},y,\dot{y}) = \frac{1}{2}m_1\left(\dot{y}\cos\theta - \dot{x}\right)^2 + \frac{1}{2}m_1\dot{y}^2\sin^2\theta + \frac{1}{2}m_2\dot{x}^2 + m_1 g y\sin\theta$$

What is the Lagrangian equation of motion for the variable x?



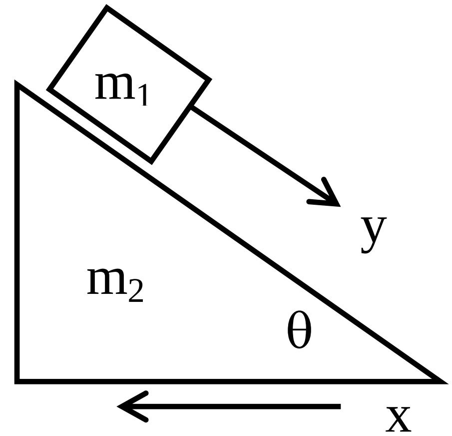
A) $m_1\ddot{x} = 0$

B) $(m_1 + m_2)\ddot{x} = 0$

C) $m_2\ddot{x} - m_1\cos\theta\,\ddot{y} = 0$

D) $\ddot{x} - \cos\theta\,\ddot{y} = 0$

E) $(m_1 + m_2)\ddot{x} - m_1\cos\theta\,\ddot{y} = 0$



CT15-7. A block of mass m_1 slides on an inclined plane of mass m_2. The equations of motion are

$$\ddot{x} = \frac{m_1}{m_1 \sin^2\theta + m_2} g \sin\theta \cos\theta$$

$$\ddot{y} = \frac{m_1 + m_2}{m_1 \sin^2\theta + m_2} g \sin\theta$$

What is a good approximation to these equations in the limit of a very heavy plane?

A) $\ddot{x} = 0 \qquad \ddot{y} = g \sin\theta$

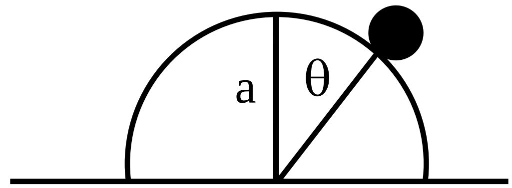
B) $\ddot{x} = \frac{m_1}{m_2} g \sin\theta \cos\theta \qquad \ddot{y} = \frac{m_1 + m_2}{m_2} g \sin\theta$

C) $\ddot{x} = g \sin\theta \cos\theta \qquad \ddot{y} = g \sin\theta$

D) $\ddot{x} = g \frac{\cos\theta}{\sin\theta} \qquad \ddot{y} = g \frac{1}{\sin\theta}$

E) $\ddot{x} = g \frac{\cos\theta}{\sin\theta} \qquad \ddot{y} = g$

CT15-8. A particle of mass m slides on the outside of a cylinder of radius a. A good choice of generalized coordinates is (r, θ) What are the kinetic and potential energy of the particle?



A) $T = \frac{1}{2} m r^2 \dot{\theta}^2$, $U = mgr\cos\theta$

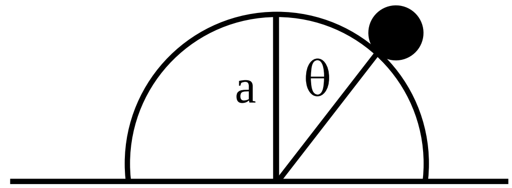
B) $T = \frac{1}{2} m r^2 \dot{\theta}^2$, $U = mgr\sin\theta$

C) $T = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$, $U = mgr\sin\theta$

D) $T = \frac{1}{2} m\left(\dot{r}^2 + \dot{\theta}^2\right)$, $U = mgr\cos\theta$

E) $T = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$, $U = mgr\cos\theta$

CT15-9. A particle of mass m slides on the outside of a cylinder of radius a. What condition must be satisfied by the force of constraint at the point (angle θ_0) where the particle leaves the cylinder?



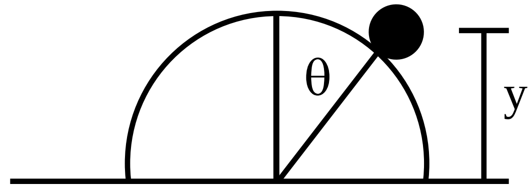
A) $Q_r = mg$

B) $Q_r = -mg$

C) $Q_r = mg\sin\theta$

D) $Q_r = mg\cos\theta$

E) $Q_r = 0$



CT15-10. A particle of mass m slides on the outside of a cylinder of radius a. The particle leaves the cylinder at angle θ_0 and speed v, where

$$\cos\theta_0 = \frac{2}{3}$$
$$v = \sqrt{\frac{2ag}{3}}$$

Once the particle leaves the cylinder, what is its height y(t) above the ground?

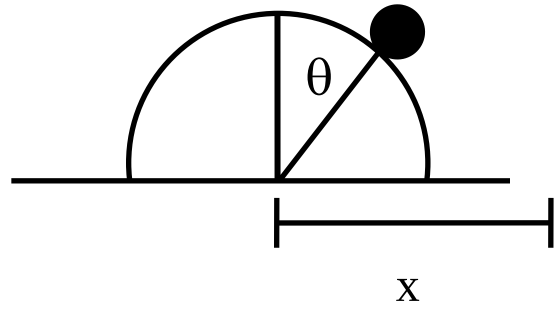
A) $y = a - v\sin\theta_0\, t - \frac{1}{2}gt^2$

B) $y = a + v\sin\theta_0\, t - \frac{1}{2}gt^2$

C) $y = \frac{2a}{3} - v\sin\theta_0\, t - \frac{1}{2}gt^2$

D) $y = \frac{2a}{3} + v\sin\theta_0\, t - \frac{1}{2}gt^2$

E) $y = \frac{2a}{3} - v\sin\theta_0\, t - gt^2$



CT15-11. A particle of mass m slides on the outside of a cylinder of radius a. The particle leaves the cylinder at angle θ_0 and speed v, where

$$\cos\theta_0 = \frac{2}{3}$$

$$v = \sqrt{\frac{2ag}{3}}$$

The particle then flies through the air for time

$$t_f = v\sin\theta_0\left(\sqrt{1+\frac{4ag}{3v^2\sin^2\theta_0}}-1\right)$$

How far from the center of the cylinder does the particle land?

A) $x = vt_f\cos\theta_0 + a\sin\theta_0$

B) $x = (vt_f + a)\sin\theta_0$

C) $x = vt_f\sin\theta_0 + a\cos\theta_0$

D) $x = (vt_f + a)\cos\theta_0$

E) $x = vt_f - a\sin\theta_0$

