Physics 3210

Week 14 clicker questions

When expanding the potential energy about a minimum (at the origin), we have

$$V(q_j) = V(0) + \sum_j \left.\frac{\partial V}{\partial q_j}\right|_0 q_j + \sum_{j,k} \left.\frac{\partial^2 V}{\partial q_j \partial q_k}\right|_0 q_j q_k + \cdots$$

What can we say about the coefficients of the second term?

A. $\left.\frac{\partial V}{\partial q_j}\right|_0 > 0$ for some j

B. $\left.\frac{\partial V}{\partial q_j}\right|_0 = 0$ for all j

C. $\left.\frac{\partial V}{\partial q_j}\right|_0 = 0$ for some j

D. $\left.\frac{\partial V}{\partial q_j}\right|_0 > 0$ for all j

E. $\left.\frac{\partial V}{\partial q_j}\right|_0 < 0$ for all j

We can write the Lagrangian near a minimum of the potential as

$$\mathcal{L} = T - V = \frac{1}{2}\sum_{j,k}\left(a_{jk}\dot{q}_j\dot{q}_k - k_{jk}q_jq_k\right)$$

What can we say about the matrices $A=[a_{jk}]$ and $K=[k_{jk}]$?

A. Both A and K are symmetric.

B. Both A and K are antisymmetric.

C. A is symmetric and K is antisymmetric.

D. A is antisymmetric and K is symmetric.

To solve the system of n linear, coupled, homogenous, 2nd order ODEs described by

$$A\ddot{\mathbf{q}} + K\mathbf{q} = 0$$

what form of solution is a good guess?

A. Growing exponential.
B. Decaying exponential.
C. Linear function.
D. Linear function times an exponential.
E. Oscillating function.

For a non-trivial solution of the linear system

$$\left(\mathrm{K} - \omega^2 \mathrm{A}\right)\mathbf{q}_0 = \mathrm{M}\mathbf{q}_0 = \mathbf{0}$$

to exist, what properties must the matrix M satisfy?

A. M must be invertible.

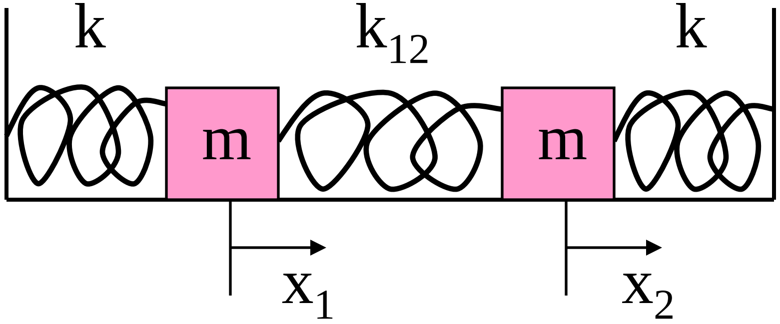
B. M must be non-invertible.

C. M must be symmetric.

D. M must be antisymmetric.

E. M must be orthogonal.

For the problem of two coupled harmonic oscillators (as sketched), we have



$$T = \frac{1}{2} m\dot{x}_1^2 + \frac{1}{2} m\dot{x}_2^2 = \frac{1}{2}\sum_{j,k} a_{jk}\dot{x}_j\dot{x}_k$$

What is the correct matrix $A=[a_{jk}]$?

A. $A = \begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix}$

B. $A = \begin{bmatrix} 0 & -m \\ m & 0 \end{bmatrix}$

C. $A = \begin{bmatrix} m & m \\ m & m \end{bmatrix}$

D. $A = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$

E. $A = \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}$

Physics 3210

Wednesday clicker questions

Which of the following are rank-zero tensors (aka scalars)?

I. The dot product of two position vectors, **u** and **v**.
II. The kinetic energy of a particle.
III. The x component of a particle's velocity.

A. I only.
B. I and II.
C. I, II, and III.
D. II and III.
E. I and III.

Which of the following are rank-one tensors (aka vectors)?

I. The momentum of a particle.

II. The angular velocity of a particle.

III. The center of mass position of a system of particles.

A. I only.

B. I and II.

C. I, II, and III.

D. II and III.

E. I and III.

Which of the following is the correct part of the Levi-Civita antisymmetric tensor ε_{ij1}?

A. $\varepsilon_{ij1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B. $\varepsilon_{ij1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\varepsilon_{ij1} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

D. $\varepsilon_{ij1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

E. $\varepsilon_{ij1} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

The vector **v** has components v_x, v_y, v_z. How is this vector written in a new coordinate system in which the coordinate axes are inverted (reflected through the origin)?

A. $\mathbf{v}' = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$

B. $\mathbf{v}' = \begin{bmatrix} v_x \\ v_y \\ -v_z \end{bmatrix}$

C. $\mathbf{v}' = \begin{bmatrix} -v_x \\ -v_y \\ -v_z \end{bmatrix}$

D. $\mathbf{v}' = \begin{bmatrix} -v_x \\ -v_y \\ v_z \end{bmatrix}$

E. $\mathbf{v}' = \begin{bmatrix} -v_x \\ v_y \\ v_z \end{bmatrix}$

Which of the following are pseudovectors (rather than true vectors)?

I. The momentum of a particle.
II. The angular velocity of a particle.
III. The torque on a particle.

A. I only.
B. I and II.
C. I, II, and III.
D. II and III.
E. I and III.

Physics 3210

Friday clicker questions

Why is it useful to describe coupled oscillations using normal modes?

A. The normal modes describe the transfer of energy between different oscillators.
B. Each normal modes describes an oscillation at a single frequency.
C. Only one normal mode can be excited at a time.
D. It allows for greater mathematical complexity.

The motion of a system of coupled oscillators can be described using normal coordinates:

$$\mathbf{q}(t) = \sum_j \mathbf{q}_{j0} \alpha_j e^{i\omega_j t} = \sum_j \mathbf{q}_{j0} \eta_j(t)$$

What equation is satisfied by the normal coordinate?

A. $\ddot{\eta}_j + \omega_j \eta_j = 0$

B. $\ddot{\eta}_j + \omega_j^2 \eta_j = 0$

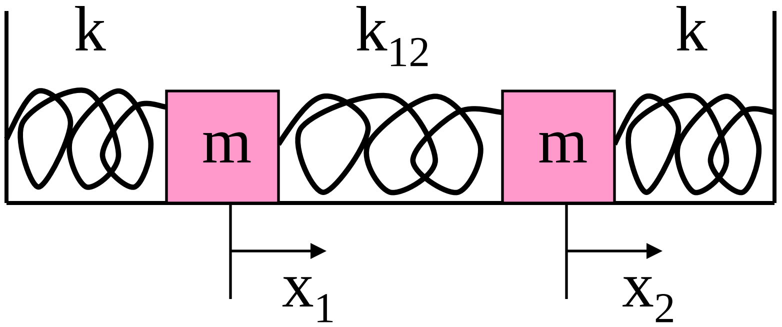
C. $(K - \omega_j A)\eta_j = 0$

D. $(K - \omega_j^2 A)\eta_j = 0$

A system of two coupled harmonic oscillators has general solution

$$x_1(t) = \alpha_1 e^{i\omega_1 t} + \alpha_2 e^{i\omega_2 t}$$

$$x_2(t) = -\alpha_1 e^{i\omega_1 t} + \alpha_2 e^{i\omega_2 t}$$



What is the second normal coordinate in terms of x_1 and x_2?

A. $\eta_1(t) = x_1(t) + x_2(t)$

B. $\eta_1(t) = \dfrac{x_1(t) + x_2(t)}{2}$

C. $\eta_1(t) = x_1(t) - x_2(t)$

D. $\eta_1(t) = -x_1(t) + x_2(t)$