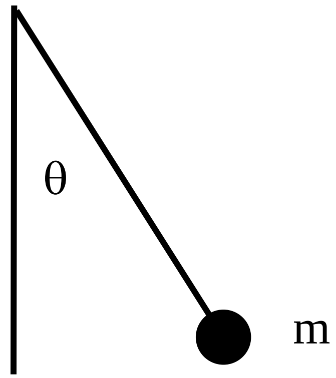
Physics 3210

Week 2 clicker questions

What is the Lagrangian of a pendulum (mass m, length l)? Assume the potential energy is zero when θ is zero.



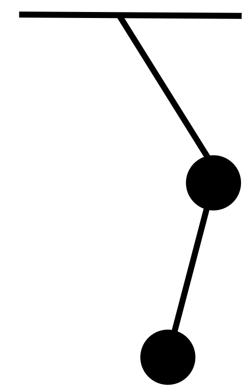
A. $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\ell^2\dot{\theta}^2-mg\ell(1-\cos\theta)$

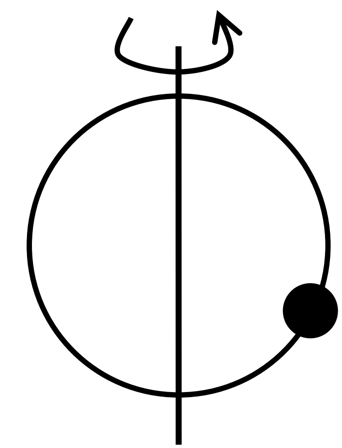
B. $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\ell^2\dot{\theta}^2+mg\ell(1-\cos\theta)$

C. $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\dot{\theta}^2+mg\ell(1-\cos\theta)$

D. $\mathcal{L}(\theta,\dot{\theta},t)=\frac{1}{2}m\dot{\theta}^2-mg\ell(1-\cos\theta)$

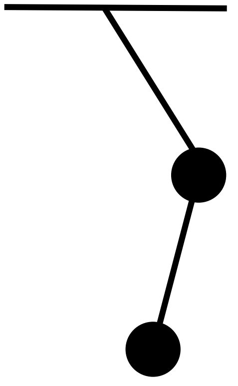
For which of these systems could you use Lagange's equations of motion?
1. A double pendulum: a pendulum (mass m, length l) has a second pendulum (mass m, length l) connected to its bob.
2. A projectile moves in two dimensions with gravity and air resistance.
3. A bead slides without friction on a circular, rotating wire.





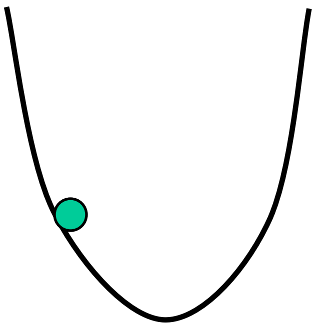
A. 1 only
B. 2 only
C. 3 only
D. 1 and 2
E. 1 and 3

What would be a good choice of generalized coordinates for the double pendulum? (Assume the pendula are constrained to move in the x-y plane.)



A. the Cartesian coordinates of the bob positions: x_1, y_1 (first bob) and x_2, y_2 (second bob)
B. the Cartesian coordinates of the first bob: x_1, y_1 and the Cartesian coordinates of the second bob, treating the first bob as the origin: x'_2, y'_2
C. the angles made between each pendulum rod and the vertical: θ_1, (first bob) θ_2 (second bob)
D. the angles made between a line drawn from each pendulum bob to the pivot and the vertical: α_1, (first bob) α_2 (second bob)

What is the kinetic energy of a particle sliding on the parabola $y=x^2$?



A. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2$

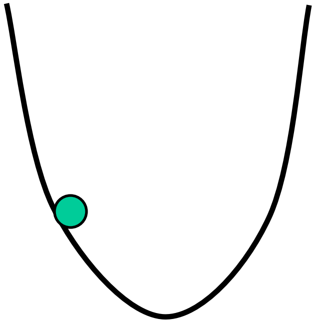
B. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+x)$

C. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2(1+4x)$

D. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2\left(1+2x^2\right)$

E. $T(x,\dot{x},t)=\frac{1}{2}m\dot{x}^2\left(1+4x^2\right)$

What is the generalized force of a particle sliding on the parabola $y=x^2$?



A. $-mgx$

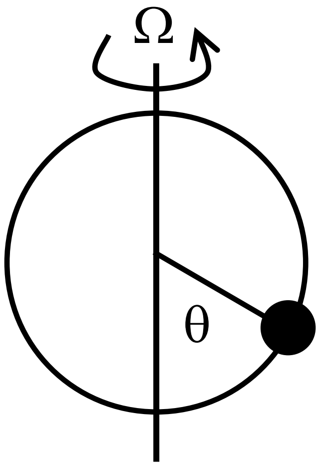
B. $-2mgx$

C. $4mx\dot{x}^2 - 2mgx$

D. $2mx\dot{x}^2 - mgx$

E. $4mx^2\dot{x} - 2mgx$

A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. What is the kinetic energy of the bead?



A. $T(\theta,\dot{\theta},t)=\frac{1}{2}mR^2\dot{\theta}^2+\frac{1}{2}mR^2\Omega^2$

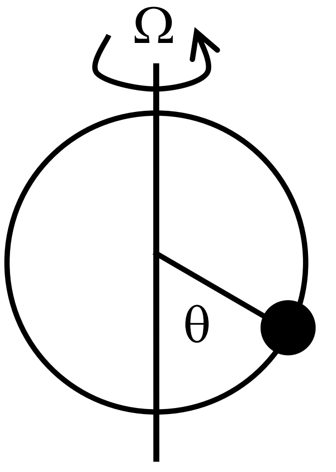
B. $T(\theta,\dot{\theta},t)=\frac{1}{2}mR^2\dot{\theta}^2+\frac{1}{2}mR^2\cos^2\theta\Omega^2$

C. $T(\theta,\dot{\theta},t)=\frac{1}{2}mR^2\dot{\theta}^2+\frac{1}{2}mR^2\sin^2\theta\Omega^2$

D. $T(\theta,\dot{\theta},t)=\frac{1}{2}mR^2\dot{\theta}^2-\frac{1}{2}mR^2\Omega^2$

E. $T(\theta,\dot{\theta},t)=\frac{1}{2}mR^2\left(\dot{\theta}+\Omega\cos\theta\right)^2$

A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. When the bead is at angle θ, how high is the bead above the lowest point of the wire?



A. $h = R\cos\theta$

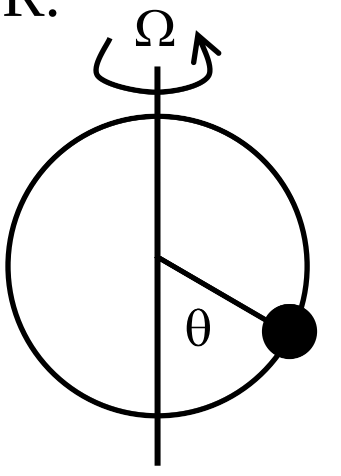
B. $h = R\sin\theta$

C. $h = R(\sin\theta - \cos\theta)$

D. $h = R(1 - \sin\theta)$

E. $h = R(1 - \cos\theta)$

A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. The equation of motion of the bead is



$$\ddot{\theta}+\frac{g}{R}\sin\theta-\Omega^2\sin\theta\cos\theta=0$$

What are the equilibrium value(s) of θ?

A. $\theta=0$

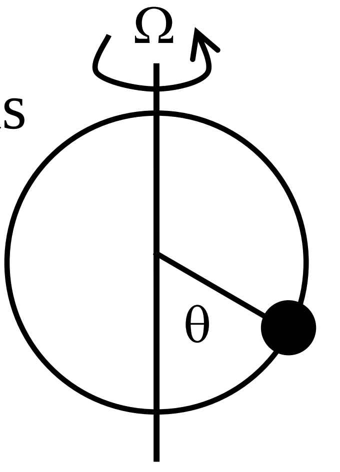
B. $\theta=\cos^{-1}\left(\frac{g}{R\Omega^2}\right)$

C. $\theta=\sin^{-1}\left(\frac{g}{R\Omega^2}\right)$

D. $\theta=0$ and $\theta=\cos^{-1}\left(\frac{g}{R\Omega^2}\right)$

E. $\theta=0$ and $\theta=\sin^{-1}\left(\frac{g}{R\Omega^2}\right)$

A bead of mass m slides on a circular wire of radius R. The wire rotates about a vertical axis with angular velocity Ω. The equation of motion for small motions about the equilibrium $\theta_0 = \cos^{-1}\left(\frac{g}{R\Omega^2}\right)$ is



$$\delta\ddot{\theta} + \Omega^2 \sin^2\theta_0 \, \delta\theta = 0$$

What is the oscillation frequency of the bead?

A. $\omega = \Omega$

B. $\omega = \Omega \sin\theta$

C. $\omega = \Omega \sin\theta_0$

D. $\omega = \Omega^2$

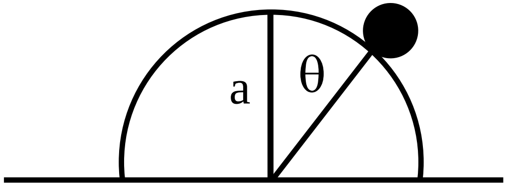
E. $\omega = \Omega^2 \sin^2\theta_0$

Which of these constraints is holonomic?

1. A particle is constrained to slide on the inside of a sphere.
2. A disk rolls without slipping down an inclined plane (in one dimension).
3. A disk rolls without slipping on a table (in two dimensions).
4. A moving car is constrained to obey the speed limit.

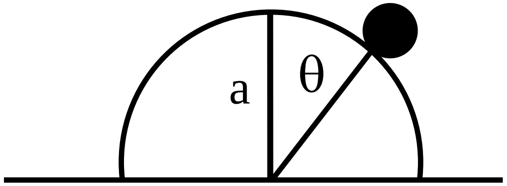
A. None
B. Only one
C. Exactly two
D. Exactly three
E. All four

A particle of mass m slides on the outside of a cylinder of radius a. A good choice of generalized coordinates is (r, θ). What is the constraint equation?



A. $f(r, \theta)=r$
B. $f(r, \theta)=a$
C. $f(r, \theta)=r+a$
D. $f(r, \theta)=r-a$

A particle of mass m slides on the outside of a cylinder of radius a. What is the force of constraint (in the radial direction) when $\theta=0$?



A. $Q_r(\theta=0)= mg$

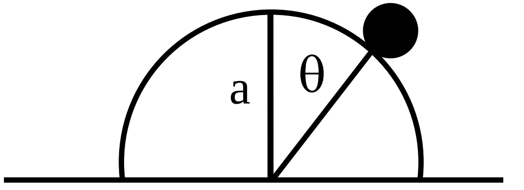
B. $Q_r(\theta=0)= mg/2$

C. $Q_r(\theta=0)= 0$

D. $Q_r(\theta=0)= -mg/2$

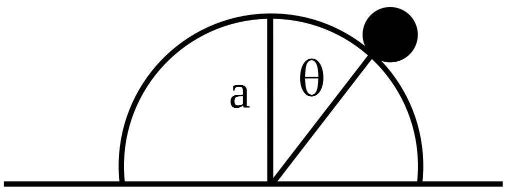
E. $Q_r(\theta=0)= -mg$

A particle of mass m slides on the outside of a cylinder of radius a. What happens to the (magnitude of the) force of constraint as θ increases?



A. The constraint force increases.
B. The constraint force decreases.
C. The constraint force is constant.

A particle of mass m slides on the outside of a cylinder of radius a. What are the kinetic and potential energy of the particle?



A. $T = \frac{1}{2} m r^2 \dot{\theta}^2$, $U = mgr\cos\theta$

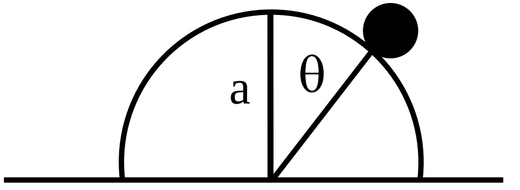
B. $T = \frac{1}{2} m r^2 \dot{\theta}^2$, $U = mgr\sin\theta$

C. $T = \frac{1}{2} m \left(\dot{r}^2 + \dot{\theta}^2 \right)$, $U = mgr\sin\theta$

D. $T = \frac{1}{2} m \left(\dot{r}^2 + r^2\dot{\theta}^2 \right)$, $U = mgr\sin\theta$

E. $T = \frac{1}{2} m \left(\dot{r}^2 + r^2\dot{\theta}^2 \right)$, $U = mgr\cos\theta$

A particle of mass m slides on the outside of a cylinder of radius a. What condition must be satisfied by the force of constraint at the point (angle θ_0) where the particle leaves the cylinder?



A. $Q_r = mg$
B. $Q_r = mg \sin \theta_0$
C. $Q_r = 0$
D. $Q_r = -mg$
E. $Q_r = -mg \sin \theta_0$