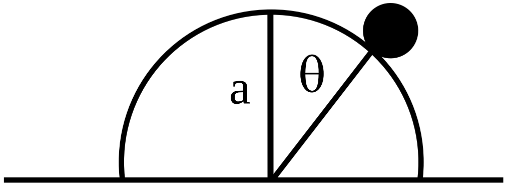
Physics 3210

Week 3 clicker questions

A particle of mass m slides on the outside of a cylinder of radius a. What condition must be satisfied by the force of constraint at the point (angle θ_0) where the particle leaves the cylinder?



A. $Q_r = mg$
B. $Q_r = mg \sin \theta_0$
C. $Q_r = 0$
D. $Q_r = -mg$
E. $Q_r = -mg \sin \theta_0$

What can you conclude from the following relation?

$$\delta\mathcal{L} = \sum_i \frac{\partial\mathcal{L}}{\partial x_i}\delta x_i = 0$$

A. $\sum_i \frac{\partial\mathcal{L}}{\partial x_i} = 0$

B. $\frac{\partial\mathcal{L}}{\partial x_i} = 0$ for all i

C. $\delta x_i = 0$ for all i

D. $\nabla\mathcal{L}\cdot\delta\vec{r} = 0$

What can you conclude from the following relation?

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \text{constant} \quad \text{for all i}$$

A. Nothing.

B. Angular momentum is conserved (for a closed system).

C. Momentum is conserved (for a closed system).

D. Energy is conserved (for a closed system).

E. Lagrangian mechanics is fun.

Which of these choices is a valid way to rewrite the following relation?

$$\delta\mathcal{L} = \sum_i \left[\frac{\partial \mathcal{L}}{\partial x_i}\delta x_i + \frac{\partial \mathcal{L}}{\partial \dot{x}_i}\delta \dot{x}_i \right] = 0$$

A. $\delta\mathcal{L} = \sum_i [p_i \delta x_i + \dot{p}_i \delta \dot{x}_i]$

B. $\delta\mathcal{L} = \sum_i [p_i \delta x_i - \dot{p}_i \delta \dot{x}_i]$

C. $\delta\mathcal{L} = \sum_i [\dot{p}_i \delta x_i - p_i \delta \dot{x}_i]$

D. $\delta\mathcal{L} = \sum_i [\dot{p}_i \delta x_i + p_i \delta \dot{x}_i]$

How many of the following statements are true?

1. Momentum is conserved because space is homogeneous.
2. Angular momentum is conserved because space is homogeneous.
3. Angular momentum is conserved because space is isotropic.
4. Momentum is conserved because space is isotropic.
5. Energy is conserved because space is homogeneous and isotropic.

A. Exactly one
B. Exactly two
C. Exactly three
D. Exactly four
E. All five

Which of these choices is the correct expression for the time derivative of the Lagrangian of a closed system?

A. $$\frac{d\mathcal{L}}{dt} = \sum_j \frac{\partial \mathcal{L}}{\partial q_j} \dot{q}_j$$

B. $$\frac{d\mathcal{L}}{dt} = \sum_j \left[\frac{\partial \mathcal{L}}{\partial q_j} \ddot{q}_j + \frac{\partial \mathcal{L}}{\partial \dot{q}_j} \dot{q}_j \right]$$

C. $$\frac{d\mathcal{L}}{dt} = \sum_j \frac{\partial \mathcal{L}}{\partial \dot{q}_j} \ddot{q}_j$$

D. $$\frac{d\mathcal{L}}{dt} = \sum_j \left[\frac{\partial \mathcal{L}}{\partial q_j} \dot{q}_j + \frac{\partial \mathcal{L}}{\partial \dot{q}_j} \ddot{q}_j \right]$$

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Week 3 - Wednesday clicker questions

How can we rewrite the following expression?

$$\int_{t_1}^{t_2} p_j \frac{d}{dt}\delta q_j \, dt = ?$$

A. $-\int_{t_1}^{t_2} \dot{q}_j \delta p_j \, dt$

B. $-\int_{t_1}^{t_2} \dot{p}_j \delta q_j \, dt$

C. $-\int_{t_1}^{t_2} q_j \delta \dot{p}_j \, dt$

D. $-\int_{t_1}^{t_2} \dot{q}_j \delta \dot{p}_j \, dt$

If we know that

$$\int_{t_1}^{t_2}\sum_j\left[-\left(\dot{p}_j+\frac{\partial H}{\partial q_j}\right)\delta q_j+\left(\dot{q}_j-\frac{\partial H}{\partial p_j}\right)\delta p_j\right]dt=0$$

what can we say about the terms in parentheses?

A. Either $\left(\dot{p}_j+\frac{\partial H}{\partial q_j}\right)$ or $\left(\dot{q}_j-\frac{\partial H}{\partial p_j}\right)$ vanishes, but not both.

B. Both $\left(\dot{p}_j+\frac{\partial H}{\partial q_j}\right)$ and $\left(\dot{q}_j-\frac{\partial H}{\partial p_j}\right)$ vanish independently.

C. The sum $\left(\dot{p}_j+\frac{\partial H}{\partial q_j}\right)+\left(\dot{q}_j-\frac{\partial H}{\partial p_j}\right)$ vanishes.

D. The ratio $\left(\dot{p}_j+\frac{\partial H}{\partial q_j}\right)\Big/\left(\dot{q}_j-\frac{\partial H}{\partial p_j}\right)=\frac{\delta p_j}{\delta q_j}$.

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Week 3 - Friday clicker questions

There has been a request to have the homework due Thursday (instead of Wednesday) to avoid conflicts with other classes. Which would you prefer?

A. Change to Thursday.
B. Stick with Wednesday.
C. I don’t care.

There has been a request to change the course grading so that the lowest of the 3 midterm exam grades is dropped. Which would you prefer?

A. Count all 3 exams.
B. Drop the lowest of the 3 exams.
C. I don’t care.

Hamilton's canonical equations are: $$\dot{q}_j = \frac{\partial H}{\partial p_j}$$

$$\dot{p}_j = -\frac{\partial H}{\partial q_j}$$

If the Hamiltonian is independent of p_j, which of the following is true?

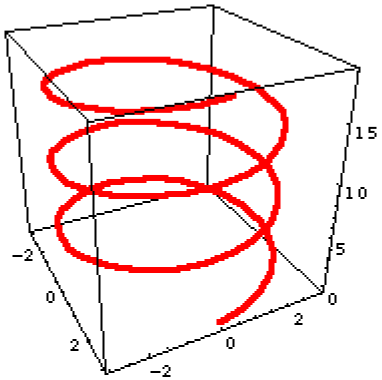
A. q_j is constant

B. q_j is zero

C. p_j is constant

D. p_j is zero

A particle slides on a helical wire defined by $z=k\theta$, r=const. If we use θ as the generalized coordinate, what is the generalized momentum p?



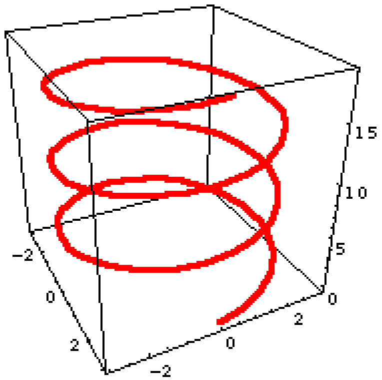
A. $p=\frac{1}{2}m\left(r^2+k^2\right)\dot{\theta}^2$

B. $p=m\dot{\theta}$

C. $p=m(r+k)\dot{\theta}$

D. $p=m\left(r^2+k^2\right)\dot{\theta}$

A particle slides on a helical wire defined by z=kθ, r=const. If we use θ as the generalized coordinate, the generalized momentum $p = m\left(r^2 + k^2\right)\dot{\theta}$. What is the Hamiltonian of the system?



A. $H = \dfrac{p^2}{2m} + mgk\theta$

B. $H = \dfrac{p^2}{2m} - mgk\theta$

C. $H = \dfrac{p^2}{2m(r+k)} - mgk\theta$

D. $H = \dfrac{p^2}{2m\left(r^2 + k^2\right)} + mgk\theta$

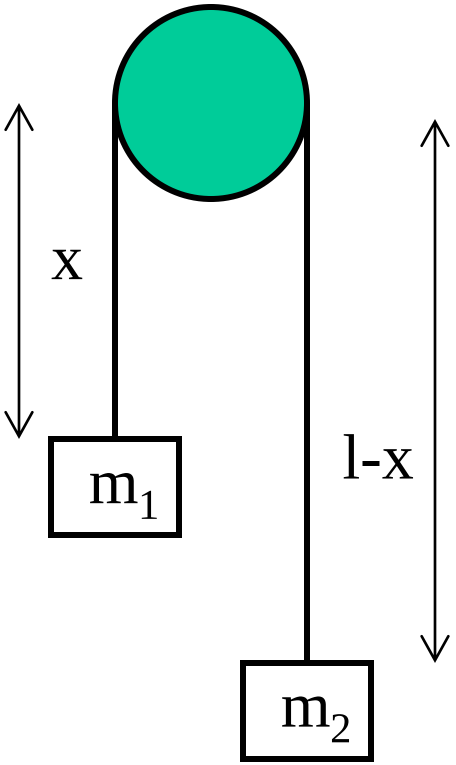
An Atwood machine has total free-hanging length of string l connected to two masses m_1 (on the left) and m_2 (on the right). Use x as a generalized coordinate. What is the generalized momentum p?

A. $p = \frac{1}{2}(m_1 + m_2)\dot{x}^2$

B. $p = (m_1 + m_2)\dot{x}$

C. $p = (m_1 - m_2)\dot{x}$

D. $p = m_1\dot{x}$



An Atwood machine has a Hamiltonian:

$$H = \frac{p^2}{2(m_1 + m_2)} - (m_1 - m_2)gx - m_2 gl$$

What are Hamilton's equations of motion?

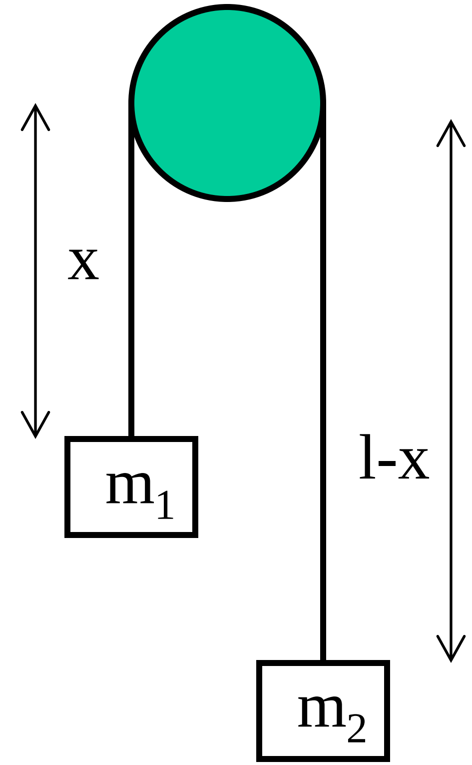
A. $$\dot{x} = \frac{p}{(m_1 + m_2)}$$
$$\dot{p} = -(m_1 - m_2)g$$

B. $$\dot{x} = \frac{p}{(m_1 + m_2)}$$
$$\dot{p} = -(m_1 + m_2)g$$

C. $$\dot{x} = \frac{p}{(m_1 - m_2)}$$
$$\dot{p} = (m_1 + m_2)g$$

D. $$\dot{x} = \frac{p}{(m_1 - m_2)}$$
$$\dot{p} = -(m_1 - m_2)g$$

E. $$\dot{x} = \frac{p}{(m_1 + m_2)}$$
$$\dot{p} = (m_1 - m_2)g$$



An Atwood machine has the following Hamiltonian equations of motion:

$$\dot{x} = \frac{p}{(m_1 + m_2)}$$

$$\dot{p} = (m_1 - m_2)g$$

How does the system move?

A. x moves with constant gravitational acceleration with effective mass $(m_1 + m_2)$
B. x moves with constant gravitational acceleration with effective mass $(m_1 - m_2)$
C. x moves with constant gravitational acceleration with effective mass $(m_1 - m_2)/(m_1 + m_2)$
D. x moves with constant velocity
E. x doesn't change