Energy: Part 4

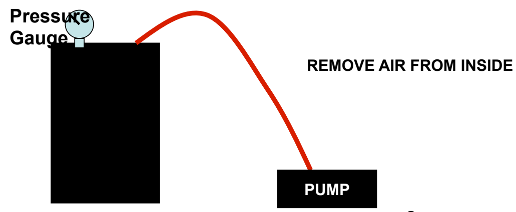
Bernoulli and Water distribution systems

Lecture 12:
- Water distribution
- Air pressure

Reminders:
HW5 due Monday
A word about HW experiments.
Reading for Tuesday: 16.1 / 16.2 Thurs
Midterm 2: 2 weeks today

What will happen if we remove the air from inside the drum?

a. Nothing
b. The drum will explode
c. The drum will implode
d. Something else



2

Reading quiz

1. Bernoulli's equation is all about
   a. Conservation of momentum
   b. Conservation of heat
   c. Conservation of water
   d. Conservation of potential energy
   e. None of the above
2. Bernoulli's equation describes
   a. How the temperature of water changes as it flows through pipes.
   b. The different amounts of water distributed to houses and industry in a typical city.
   c. The relationship between pressure, velocity, and height of water in a pipe.
   d. The relationship between the thickness of water pipes and the pressure of the water they contain
3. When water leaves a hose through a nozzle, the pressure
   a. Increases
   b. Decreases
   c. Stays the same.

Reading quiz

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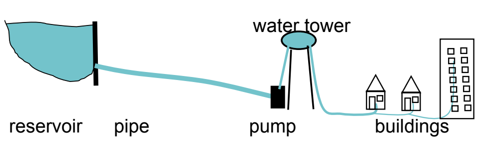
Water distribution



Where does the water flow?
What determines the water pressure in different homes/heights?
How fast does water flow out of a faucet?
How do you pump water out of wells?

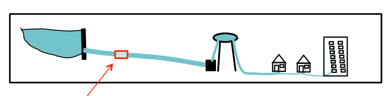
ALL ABOUT CONSERVATION OF ENERGY!

GPE = mgh KE = ½ mv^2 PPE = PV

Pumps do work (Force x distance)

Bernoulli's Equation

$$PV + \frac{1}{2} mv^2 + mgh = E_{total}$$



Consider one little bit of water of volume V and mass m:

Replace m = ρV where ρ is the fluid density (ρ = mass/volume = 1000kg/m^3 for water)

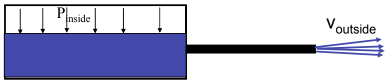
$$PV + \frac{1}{2} \rho V v^2 + \rho V g h = E_{total}$$

$$\boxed{P + \frac{1}{2} \rho v^2 + \rho g h = E_{total}/V \quad (E_{total\ per\ unit\ volume})}$$

$E_{total\ per\ vol}$ is constant:
Know P, v and h at one point ⇒ can calculate these quantities at another

Apply Bernoulli to Squirt Gun

How is velocity of water out related to pressure inside gun?



$P + \frac{1}{2}\rho v^2 + \cancel{\rho g h} = E_{total\ per\ vol}$ Height constant so ignore GPE

$\Rightarrow$ **$P + \frac{1}{2}\rho v^2 = E_{total\ per\ vol}$ (constant)**

Inside gun: P = Atmos Pres. + P_{pump} $v = 0$ $\Rightarrow$ $AP + P_{pump} = E_{total\ per\ vol}$

Outside gun: P = Atmos Pres., $v_{outside}$ is big $\Rightarrow$ $AP + \frac{1}{2}\rho v_{outside}^2 = E_{total\ per\ vol}$

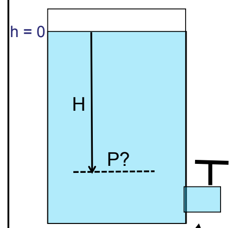
$$\cancel{AP} + P_{pump} = \cancel{AP} + \frac{1}{2}\rho v_{outside}^2$$

$$P_{pump} = \frac{1}{2}\rho v_{outside}^2$$

$$v_{outside} = \mathrm{sqrt}(2 P_{pump} / \rho)$$

More on pressure

Here's a bucket of water with a faucet attached.
What is the pressure at a depth H?



Faucet shut off, so water is not moving.

Bernoulli's Equation:

$P + \frac{1}{2}\rho v^2 + \rho g h = E_{tpv}$

Compare water at surface and at depth H

$v = 0$ everywhere $\Rightarrow P + \rho g h = E_{tpv}$

At surface: $P = AP$, $h = 0$

$\Rightarrow E_{tpv} = AP$

At depth H: $P = AP + P_w$, height = $-H$

$\Rightarrow E_{tpv} = AP + P_w + \rho g(-H)$

E_{tpv} constant $\Rightarrow AP = AP + P_w - \rho g H$

$0 = P_w - \rho g H$

$\boxed{P_{of\ water} = \rho g H}$ $\boxed{P_{at\ H} = AP + \rho g H}$

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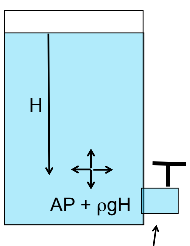

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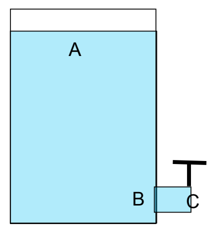
E_{tpv} constant $\Rightarrow AP = AP + P_w - \rho g H$

$\boxed{P_w = \rho g H}$ $\boxed{P_H = AP + \rho g H}$

This pressure is exerted equally in all directions

With the faucet off, the water is stopped at point C.
Rank the pressures at the three locations shown.



a) $P_A < P_B < P_C$
b) $P_A < P_B = P_C$
c) $P_A = P_B = P_C$
d) $P_A = P_B > P_C$

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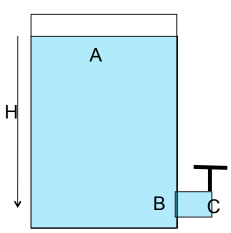

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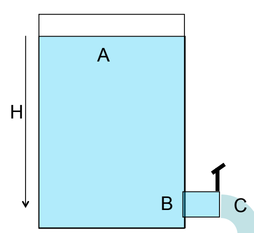
A: AP
B: AP + $\rho g H$
C: same pressure as B because they are at the same depth.

Now open the faucet. What is the pressure at point C, just outside the faucet?



a) Atmospheric pressure
b) The same as at B
c) Less than atmospheric pressure

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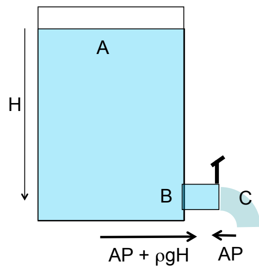

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At faucet exit, water is exposed to air. The little bit of water at faucet exit experiences a pressure imbalance and hence a net force which causes it to accelerate and flow out

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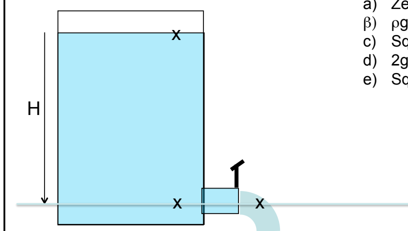
Consider a little bit of water leaving the faucet. What is its velocity at the faucet exit?

(Hint: Question about pressure and velocity changes in fluid
⇒ Apply Bernoulli's Eqn)



a) Zero
β) ρgH
c) Sqrt(ρgH)
d) 2gH
e) Sqrt (2gH)

$P + \frac{1}{2}\rho v^2 + \rho gh = E_{tpv}$

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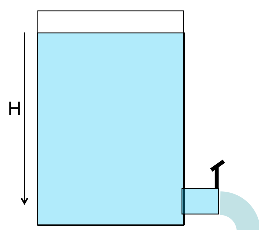


e) Sqrt (2gH)

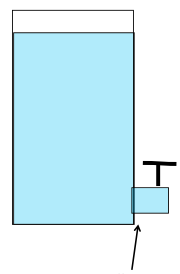
$P + \frac{1}{2}\rho v^2 + \rho gh = E_{tpv}$

Consider the water at 2 points
- just inside faucet
- just outside faucet

h doesn't change ⇒ $P + \frac{1}{2}\rho v^2 = E_{tpv}$

Inside: $AP + \rho gH + \frac{1}{2}\rho v_{in}^2 = E_{tpv}$
Outside: $AP + \frac{1}{2}\rho v_{out}^2 = E_{tpv}$
$AP + \rho gH + 0 = AP + \frac{1}{2}\rho v_{out}^2$
$\rho gH = + \frac{1}{2}\rho v_{out}^2$
$v_{out}^2 = 2gH$
$v_{out} = \text{sqrt}(2gH)$

More on Atmospheric pressure

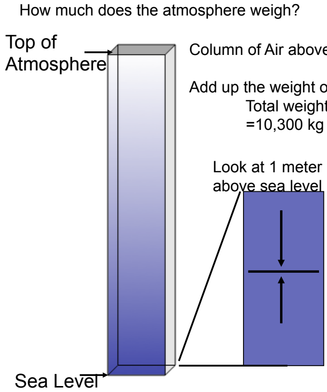


Faucet shut off, so water is not moving.

Pressure at surface of water
= Atmospheric pressure (AP)
≈ 100,000 Pa

- Pressure due to air molecules hitting surface and exerting a force
- Always present at surface of earth
- Usually only interested in CHANGES in water pressure so AP cancels out
- If so can set zero of water pressure at AP (like setting zero of height somewhere convenient)

How much does the atmosphere weigh?



Column of Air above 1 m^2 of the ground.

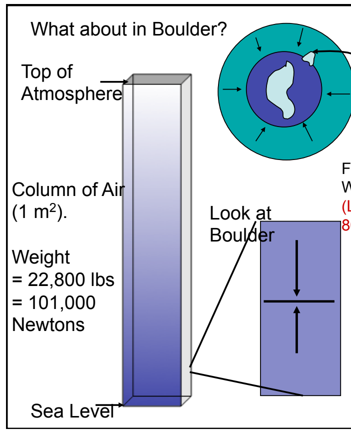
Add up the weight of every air molecule in column:
Total weight = 22,800 lbs or
=10,300 kg * 9.8 m/s² = 101,000 Newtons

Force Down = Weight of Atmosphere above

Force Up ?
Force up MUST equal Force down otherwise column above would **fall.**
Air pushes up with force.

Pressure = Force/Area
= 101,000 N / 1m²
= 22,800 lbs / 1550 in²
= 14.7 lbs/in²

What about in Boulder?



Force Down = Weight of Atmosphere ABOVE
(Less than at sea level… about 80% above us, 20% below us)

So Force Up is Less:
Air pushes up with less force than at sea level

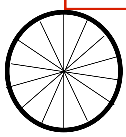
Pressure = Force/Area

Air Pressure in Boulder LESS than at Sea Level

Air and pressure

Air : Lots of small molecules zooming around.
Hit objects and each other … exert a force

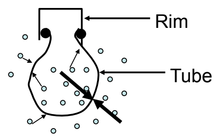
Force exerted = pressure of air x area
OR Pressure = Force/Area



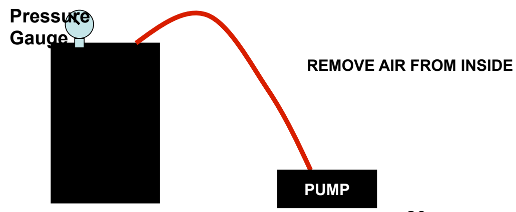
A *totally flat* bike tire (tubeless tire), the air pressure on the inside of the tire is:
a. less than the air pressure on the outside
b. greater than the air pressure on the outside
c. equal to the air pressure on the outside.

Answer is c.

Pressure on inside,
same as pressure on outside
Pressure at sea level =
14.7 lbs of force per square inch
Force of air on inside balances outside.



What will happen if we remove the air from inside the drum?
a. Nothing
b. The drum will explode
c. The drum will implode
d. Something else



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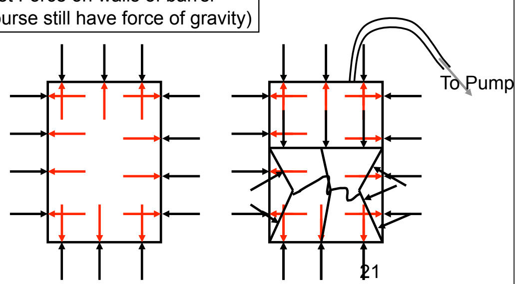
The mighty force of Air Pressure!

Initially:
Air both inside and out
Pressure inside = Pressure outside
No Net Force on walls of barrel
(of course still have force of gravity)

As Air removed:



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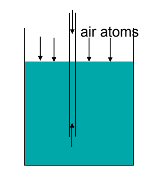
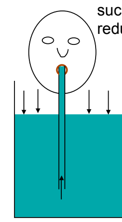
Straws

air pressure, 100,000Pa



- The pressures up and down balance….
⇒ So there is no flow through straw.

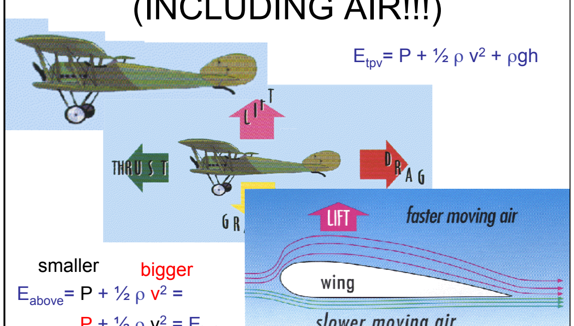
suck out air,
reduced pressure down



- The air pressure on water surface pushes the water up the straw.
- The maximum pressure difference inside/outside straw = 100,000 Pa = 1atm.
- The highest straw is 10 m (can calc.)

Bernoulli applies to all fluids (INCLUDING AIR!!!)

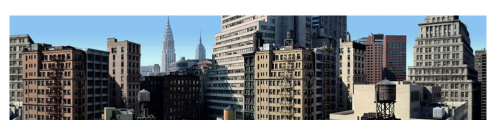
$E_{tpv} = P + \frac{1}{2} \rho v^2 + \rho gh$



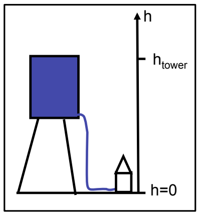
smaller bigger
$E_{above} = P + \frac{1}{2} \rho v^2 =$
$P + \frac{1}{2} \rho v^2 = E_{below}$
bigger smaller

Water towers are everywhere!

But what exactly are they for?



Bernoulli's equation and water distribution systems



ρ_{water} = 1000 kg/m^3

Note: I choose h=0 at bottom!

E_{tpv}= P + ½ ρ v² + ρgh

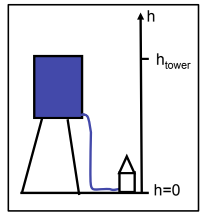
Consider water at top of tower:
P = 0, v=0 and height = h_{tower}
E_{tpv}= 0 + 0 + $\rho g h_{tower}$

Consider water at house:
P = P_{house}, v ≈ 0, and h = 0
E_{tpv} = P_{house} + 0 +0

Cons. of energy:
⇒ E_{tpv} = the same everywhere in system
⇒ P_{house} = $\rho g h_{tower}$
= (1000)(9.8)h_{tower}

Strictly speaking, there is also one atmosphere of pressure at the top and at the bottom, but only the difference in pressure matters.

Bernoulli's equation and water distribution systems



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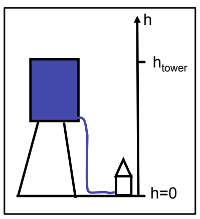
= (1000)(9.8)h_{tower}

Atmospheric pressure 100,000 Pa (= 14.6psi)
About how high would a water tower have to be to provide ~1 atmosphere of *water* pressure to the house?

a. 2 m, b. 5 m c. 10 m d. 20 m, e. 50 m.

Bernoulli's equation and water distribution systems



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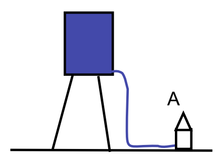
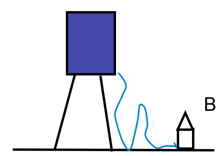
a. 2 m, b. 5 m c. 10 m d. 20 m, e. 50 m.

P_{house} = (1000)(9.8)h_{tower}
⇒ h_{tower} = 100,000/(1000 × 9.8)
≈ 10m

What if the pipe from the water tower takes a weird path (say, over hills) to get to a house at the same height?



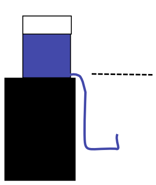


a) House A has higher pressure
b) House B has higher pressure
c) The pressure is the same at both houses.

Answer: c. P = ρgh, so only the height of the house matters.

Hint: ignore friction (we'll come back to this)

Here I have a tank of water with a hose connected to the bottom. When I take my finger off the hose, water (under pressure) will squirt into the air. Will the water go higher or lower than the opening in the tank (dashed line)?



a. Higher
b. Right exactly to the dashed line
c. Lower
d. Impossible to predict
e. None of the above.

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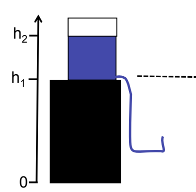

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Think conservation of total energy per volume

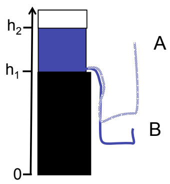
At top of tank: E_{tpv} = $\rho g h_2$

Water squirts out of hose and reaches highest point of flight:

P =0, v =0, height = h_{top}
E_{tpv} = $\rho g h_{top}$

E_{tpv} the same everywhere
⇒ h_2 = h_{top}

Here I have a tank of water with a hose connected to the bottom. When I take my finger off the hose, water (under pressure) will squirt into the air. I can hold the hose high (at A) or low (at B). From which location will the water squirt higher (relative to the ground)?



a. A, the higher location
b. B, the lower location
c. Water reaches the same height from both locations
d. Impossible to predict
e. None of the above.

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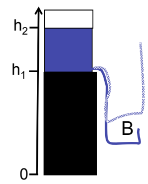
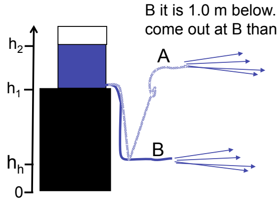


a. A, the higher location
b. B, the lower location
c. Water reaches the same max. height from both locations
d. Impossible to predict
e. None of the above.

From previous question: $E_{tpv} = \rho g h_2$
At top of flight (after leaving hose) $E_{tpv} = \rho g h_{top}$ (all GPE)
$\Rightarrow h_2 = h_{top}$
The height of the hose end doesn't come into it
At A, water leaves hose slower, but starts higher
At B, water leaves hose faster, but starts lower
$\Rightarrow$Reach same max height
(Note: Friction may reduce h_{top} slightly below h_2)

At A, the hose is 0.25 m below the water's surface. At B it is 1.0 m below. How much faster will the water come out at B than at A?



a. ¼ as fast at B as at A.
b. ½ as fast at B as at A
c. same speed
d. 2 times faster at B than at A
e. 4 times faster at B than at A

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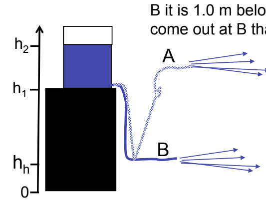

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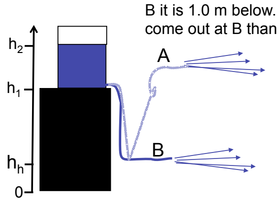
d. 2 times faster at B than at A
e. 4 times faster at B than at A

$E_{tpv} = P + ½ \rho v^2 + \rho g h$
Consider water at surface of tank: $E_{tpv} = \rho g h_2$
Consider water leaving hose at height h_h : $E_{tpv} = \rho g h_h + ½ \rho v^2$
$\Rightarrow \rho g h_2 = \rho g h_h + 1/2 \rho v^2$
$\Rightarrow v^2 = 2g(h_2 – h_h)$
$\Rightarrow v \propto$ sqrt(drop from surface to hose)
If drop increases by factor of 4, v increases by factor of sqrt(4) = 2

At A, the hose is 0.25 m below the water's surface. At B it is 1.0 m below. How much faster will the water come out at B than at A?



a. ¼ as fast at B as at A.
b. ½ as fast at B as at A
c. same speed
d. 2 times faster at B than at A
e. 4 times faster at B than at A

Quick route:
- Think: This is all about conservation of energy
- PPE not involved
$\Rightarrow$ GPE lost per vol = KE gained per vol
$\rho g h_{drop} = ½ \rho v^2$
$\Rightarrow v = \text{sqrt}(2 g h_{drop})$ as before