Energy: Part 4

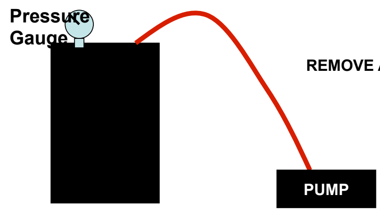
Bernoulli and Water distribution systems

Lecture 12:
- Water distribution
- Air pressure

Reminders:
HW5 due Monday
A word about HW experiments.
Reading for Tuesday: 16.1 / 16.2 Thurs
Midterm 2: 2 weeks today

What will happen if we remove the air from inside the drum?

a. Nothing
b. The drum will explode
c. The drum will implode
d. Something else



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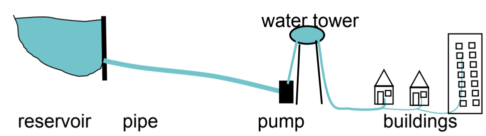
Reading quiz

1. Bernoulli's equation is all about
   a. Conservation of momentum
   b. Conservation of heat
   c. Conservation of water
   d. Conservation of potential energy
   e. None of the above
2. Bernoulli's equation describes

3. When water leaves a hose through a nozzle….

Water distribution



Where does the water flow?
What determines the water pressure in different homes/heights?
How fast does water flow out of a faucet?
How do you pump water out of wells?

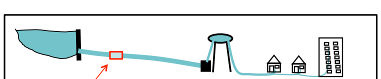
ALL ABOUT CONSERVATION OF ENERGY!

GPE = mgh KE = $\frac{1}{2}mv^2$ PPE = PV

Pumps do work (Force x distance)

Bernoulli's Equation

$PV + \frac{1}{2}mv^2 + mgh = E_{total}$



Consider one little bit of water of volume V and mass m:

Replace $m = \rho V$ where ρ is the fluid density (ρ = mass/volume = 1000kg/m³ for water)

$PV + \frac{1}{2}\rho V v^2 + \rho V g h = E_{total}$

$P + \frac{1}{2}\rho v^2 + \rho g h = E_{total}/V$ ($E_{total\ per\ unit\ volume}$)

$E_{total\ per\ vol}$ is constant:

Know P, v and h at one point ⇒ can calculate these quantities at another

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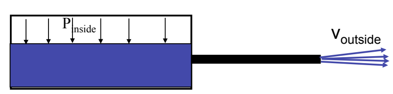
Apply Bernoulli to Squirt Gun

How is velocity of water out related to pressure inside gun?



$P + \frac{1}{2}\rho v^2 + \rho g h = E_{total\ per\ vol}$ Height constant so ignore GPE

⇒ **$P + \frac{1}{2}\rho v^2 = E_{total\ per\ vol}$** **(constant)**

Inside gun: P = Atmos Pres. + P_{pump} v= 0 ⇒ $AP + P_{pump} = E_{total\ per\ vol}$

Outside gun: P = Atmos Pres., $v_{outside}$ is big ⇒ $AP + \frac{1}{2}\rho v_{outside}^2 = E_{total\ per\ vol}$

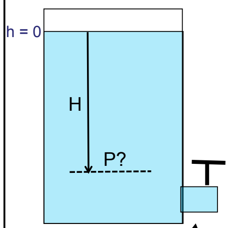
$AP + P_{pump} = AP + \frac{1}{2}\rho v_{outside}^2$

$P_{pump} = \frac{1}{2}\rho v_{outside}^2$

$v_{outside} = \mathrm{sqrt}(2 P_{pump} / \rho)$

More on pressure

Here's a bucket of water with a faucet attached.
What is the pressure at a depth H?



Faucet shut off, so water is not moving.

Bernoulli's Equation:

$P + \frac{1}{2}\rho v^2 + \rho g h = E_{tpv}$

Compare water at surface and at depth H

$v = 0$ everywhere $\Rightarrow P + \rho g h = E_{tpv}$

At surface: $P = AP$, $h = 0$

$\Rightarrow E_{tpv} = AP$

At depth H: $P = AP + P_w$, height = -H

$\Rightarrow E_{tpv} = AP + P_w + \rho g(-H)$

E_{tpv} constant $\Rightarrow AP = AP + P_w - \rho g H$

$0 = P_w - \rho g H$

$P_{\text{of water}} = \rho g H$ $\quad$ $P_{\text{at } H} = AP + \rho g H$

More on pressure

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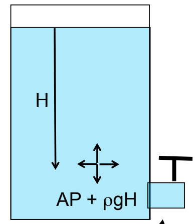

Faucet shut off, so water is not moving.

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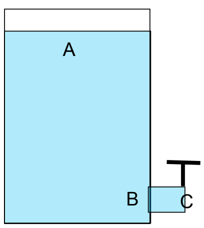
  $\Rightarrow E_{tpv} = AP + P_w + \rho g(-H)$

E_{tpv} constant $\Rightarrow AP = AP + P_w - \rho g H$

$P_w = \rho g H$ $\quad$ $P_H = AP + \rho g H$

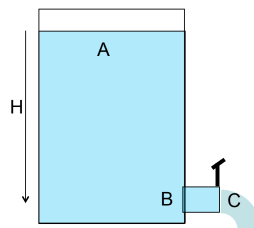
This pressure is exerted equally in all directions

With the faucet off, the water is stopped at point C.
Rank the pressures at the three locations shown.



a) $P_A < P_B < P_C$
b) $P_A < P_B = P_C$
c) $P_A = P_B = P_C$
d) $P_A = P_B > P_C$

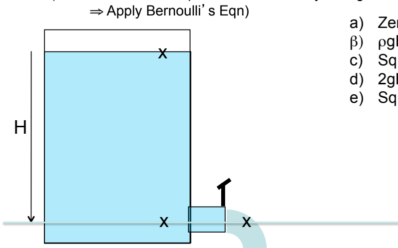
Now open the faucet. What is the pressure at point C, just outside the faucet?



a) Atmospheric pressure
b) The same as at B
c) Less than atmospheric pressure

Consider a little bit of water leaving the faucet. What is its velocity at the faucet exit?

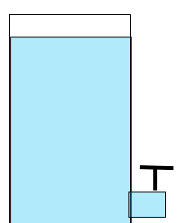
(Hint: Question about pressure and velocity changes in fluid
$\Rightarrow$ Apply Bernoulli's Eqn)



a) Zero
β) ρgH
c) Sqrt(ρgH)
d) 2gH
e) Sqrt (2gH)

$P + \frac{1}{2}\rho v^2 + \rho g h = E_{tpv}$

More on Atmospheric pressure

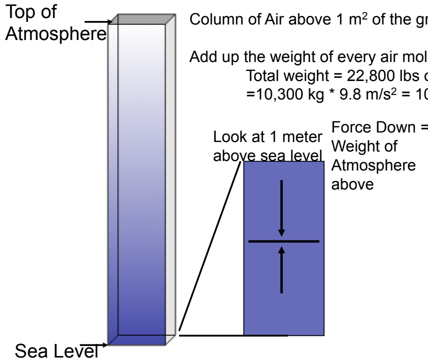


Faucet shut off, so water is not moving.

Pressure at surface of water
= Atmospheric pressure (AP)
≈ 100,000 Pa

- Pressure due to air molecules hitting surface and exerting a force
- Always present at surface of earth
- Usually only interested in CHANGES in water pressure so AP cancels out
- If so can set zero of water pressure at AP (like setting zero of height somewhere convenient)

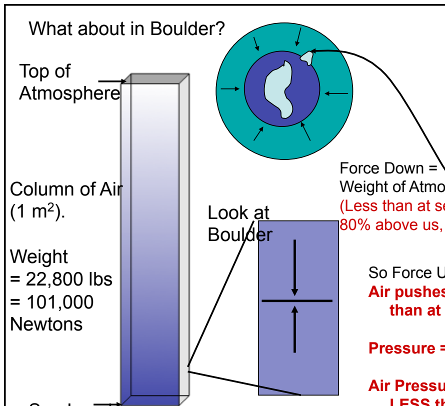
How much does the atmosphere weigh?



Column of Air above 1 m^2 of the ground.

Add up the weight of every air molecule in column:
Total weight = 22,800 lbs or
=10,300 kg * 9.8 m/s^2 = 101,000 Newtons

What about in Boulder?



Force Down =
Weight of Atmosphere ABOVE
(Less than at sea level… about 80% above us, 20% below us)

So Force Up is Less:
Air pushes up with less for than at sea level

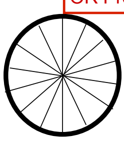
Pressure = Force/Area

Air Pressure in Boulder LESS than at Sea Level

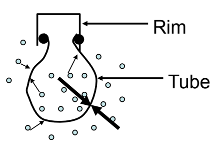
Air and pressure

Air : Lots of small molecules zooming around.
Hit objects and each other … exert a force

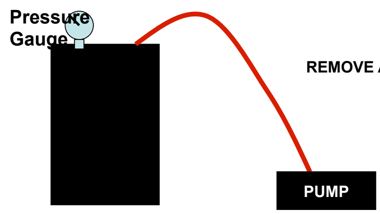
Force exerted = pressure of air x area
OR Pressure = Force/Area



A ***totally flat*** bike tire (tubeless tire), the air pressure on the inside of the tire is:
a. less than the air pressure on the outside
b. greater than the air pressure on the outside
c. equal to the air pressure on the outside.



What will happen if we remove the air from inside the drum?
a. Nothing
b. The drum will explode
c. The drum will implode
d. Something else



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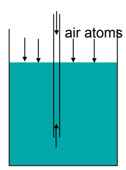
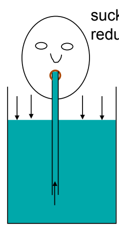
Straws

air pressure, 100,000Pa



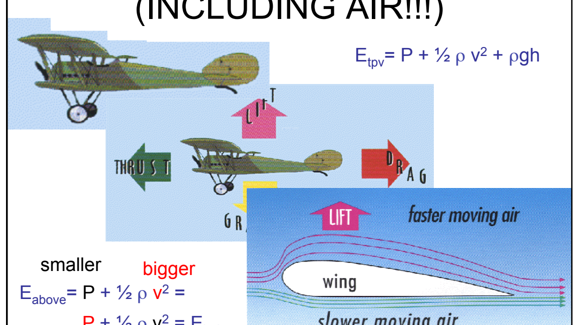
- The pressures up and down balance….
⇒ So there is no flow through straw.

suck out air,
reduced pressure down



- The air pressure on water surface pushes the water up the straw.
- The maximum pressure difference inside/outside straw = 100,000 Pa = 1atm.
- The highest straw is 10 m (can calc.)

Bernoulli applies to all fluids (INCLUDING AIR!!!)



$E_{tpv} = P + \frac{1}{2} \rho v^2 + \rho gh$

smaller bigger

$E_{above} = P + \frac{1}{2} \rho v^2 =$

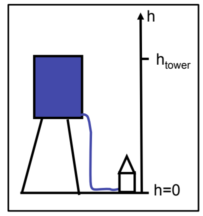
$P + \frac{1}{2} \rho v^2 = E_{below}$

bigger smaller

Water towers are everywhere!

But what exactly are they for?

Bernoulli's equation and water distribution systems



ρ_{water} = 1000 kg/m^3

E_{tpv}= P + ½ ρ v² + ρgh

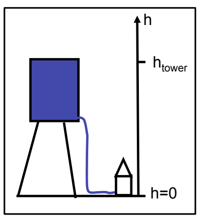
Consider water at top of tower:
P = 0, v=0 and height = h_{tower}
E_{tpv}= 0 + 0 + $\rho g h_{tower}$

Consider water at house:
P = P_{house}, v ≈ 0, and h = 0
E_{tpv} = P_{house} + 0 +0

Cons. of energy:
⇒ E_{tpv} = the same everywhere in system
⇒ P_{house} = $\rho g h_{tower}$
= $(1000)(9.8)h_{tower}$

Strictly speaking, there is also one atmosphere of pressure at the top and at the bottom, but only the difference in pressure matters.

Bernoulli's equation and water distribution systems



ρ_{water} = 1000 kg/m^3

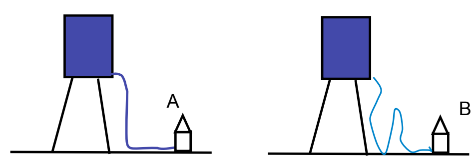
E_{tpv}= P + ½ ρ v² + ρgh

Cons. of energy:
⇒ E_{tpv} = the same everywhere in system
⇒ P_{house} = $\rho g h_{tower}$
= $(1000)(9.8)h_{tower}$

Atmospheric pressure 100,000 Pa (= 14.6psi)
About how high would a water tower have to be to provide ~1 atmosphere of *water* pressure to the house?

a. 2 m, b. 5 m c. 10 m d. 20 m, e. 50 m.

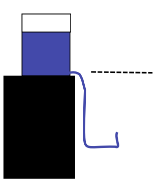
What if the pipe from the water tower takes a weird path (say, over hills) to get to a house at the same height?



a) House A has higher pressure
b) House B has higher pressure
c) The pressure is the same at both houses.

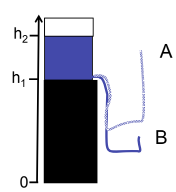
Hint: ignore friction (we'll come back to this)

Here I have a tank of water with a hose connected to the bottom. When I take my finger off the hose, water (under pressure) will squirt into the air. Will the water go higher or lower than the opening in the tank (dashed line)?



a. Higher
b. Right exactly to the dashed line
c. Lower
d. Impossible to predict
e. None of the above.

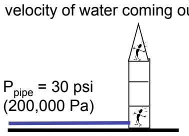
Here I have a tank of water with a hose connected to the bottom. When I take my finger off the hose, water (under pressure) will squirt into the air. I can hold the hose high (at A) or low (at B). From which location will the water squirt higher (relative to the ground)?



a. A, the higher location
b. B, the lower location
c. Water reaches the same height from both locations
d. Impossible to predict
e. None of the above.

Where does the shower work best?

In the house below, water pressure on the1st floor is 30 psi. What is the velocity of water coming out of the shower on the 1st floor?



a. 2 m/s,
b. 5 m/s,
c. 20 m/s,
d. 100 m/s,
e. 414 m/s

$P + \frac{1}{2}\rho v^2 + \rho gh = E_{tpv}$

Does h change on first floor?

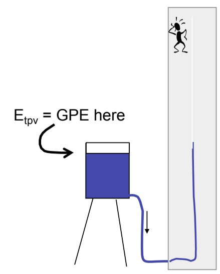
$E_{tpv\ INSIDE} = E_{tpv\ OUTSIDE}$

ignore v inside pipe.

P (inside shower) $= \frac{1}{2}\rho v^2$

$\rho = 1000\ kg/m^3$

Water distribution in skyscrapers

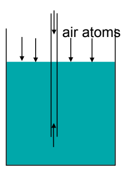


The skyscraper water problem:

- Less pressure on the higher floors,
- Water won't make it to the top floor….

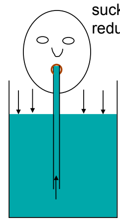
How can you solve this problem?

Straws



- The pressures up and down balance….

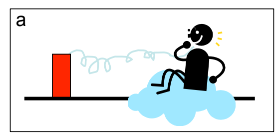
⇒ So there is no flow through straw.



- The air pressure on water surface pushes the water up the straw.
- The maximum pressure difference inside/outside straw = 100,000 Pa = 1atm.
- The highest straw is 10 m (Beqn)

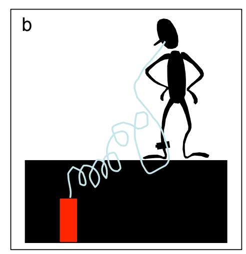
You suck on 30 foot straw.

a. sitting on floor
b. standing on desk



For you to suck up the red juice

a. will have to suck the same
b. will have to suck less hard from floor
c. suck less hard from desk



Straw height and Bernoulli's Eqn.

(Go through on own)

$P + \rho gh = E_{tpv}$ (KE term ~ 0 v is close to 0)

How to apply this to figure out what pressure is needed to suck fluid up straw?

Bottom of straw: P = 1atm, h = 0 (define)

E_{tpv} = 1atm (100,000Pa)

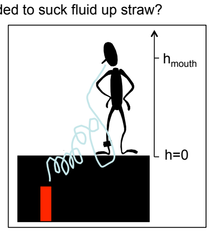
Top of straw: $P = 1atm - \Delta P_{suck}$, $h = h_{mouth}$

$E_{tpv} = (1atm - \Delta P_{suck}) + \rho g(h_{mouth})$

⇒ $1atm = (1atm - \Delta P_{suck}) + \rho g(h_{mouth})$

⇒ $\Delta P_{suck} = \rho gh_{mouth}$

⇒ The higher you stand the harder it is to suck fluid up the straw



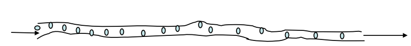
Notice how P = 1 atm canceled from both sides. Always does so it's easiest to always just use P as difference from 1 atmosphere.
i.e. set zero of pressure at 1atm

Bernoulli's Equation in Real Life

Total Energy per volume $= P + \frac{1}{2}\rho v^2 + \rho gh$

- This is a good approximation but it cannot be perfectly correct
- What type of energy does it ignore?

⇒ Think about a narrow pipe.



- Does not consider energy going into thermal energy- from friction with walls etc.
- For example, for high speed flow in a narrow pipe, more water molecules bounce off walls, creating significant friction and energy loss as heat
- But for most water distribution systems, friction can be ignored and BE works very well.