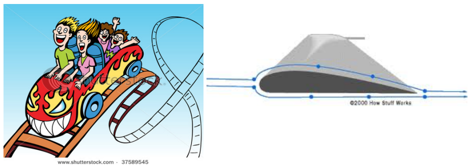
Energy: Part 3 – moving on to Fluids!



Can we apply notion of energy conservation to lift a plane?
(even if the seats are loose)

Lecture 11:
- KE
- Water distribution

Reminders:
Next week new materials…

Register to vote:
http://www.sos.state.co.us/pubs/elections/vote/VoterHome.html

Online discussion forum for questions?

Summary

- Energy and work so far:
  - GPE
  - Work done by friction (⇒heat)
  - Work done by applied forces (by me)

$$\boxed{W_{ext} - |W_{friction}| = \Delta PE + \Delta KE}$$

  - Power
- This time
  - Kinetic energy
  - Water distribution (energy of running water)

New form of energy to think about. Motional or kinetic energy.

$$KE = \frac{1}{2}\text{ mass x (velocity)}^2$$

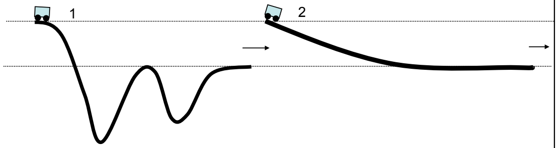
$$KE = ½ mv^2$$

Notice that this has the right units to be an energy:

$KE = kg×m^2/s^2$

$Energy = N×m$
$= kg×(m/s^2)×m$
$= kg×m^2/s^2$

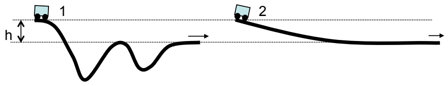
Conservation of energy questions



Two identical cars with really good bearings and solid tires roll on a very smooth, hard track (without friction). They start from rest and coast downhill, ending at the same height. How do their speeds compare at the end?

a. 1 is going much faster than 2
b. 1 is going a little faster than 2
c. 1 is going the same speed as 2
d. 1 is going a little slower than 2
e. 1 is going much slower than 2

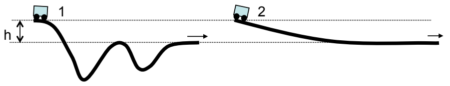
Conservation of energy questions



How do their speeds compare at the end?
c. 1 is going the same speed as 2

$\cancel{W_{ext}} - \cancel{|W_{friction}|} = \Delta PE + \Delta KE$

Conservation of energy questions



How do their speeds compare at the end?

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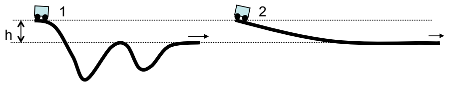
$0 = \Delta PE + \Delta KE$

Conservation of energy questions



How do their speeds compare at the end?

c. 1 is going the same speed as 2

$\Delta PE + \Delta KE = 0$

PE + KE = Constant:

No ext. work, No friction: Mechanical energy (KE+PE) of cart is conserved

$\Delta KE = -\Delta PE$:

KE gained = GPE lost

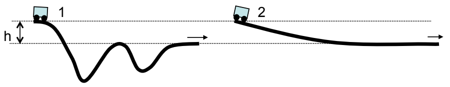
$1/2\ mv^2 = mgh$

$v^2 = 2gh$

$v = \text{sqrt}(2gh)$

- v and h are directly related at all points along track
- Mass cancels out. v determined by height lost only

Conservation of energy questions



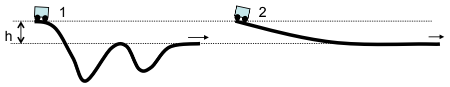
$v = \text{sqrt}(2gh)$

h = 20 cm

What is the speed of the cart at the end of the tracks?

a. 0 m/s
b. 0.2 m/s
c. 2 m/s
d. 20 m/s
e. 200 m/s

Conservation of energy questions



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e. 200 m/s

No friction, No external force

KE gained = GPE lost

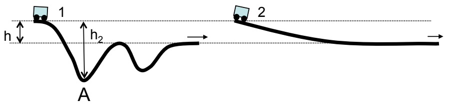
$1/2\ mv^2 = mgh$

$v = \text{sqrt}(2gh)$

$= \text{sqrt}(2*10*0.20)$

$= 2$ m/s

Conservation of energy questions

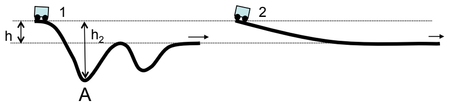


h_2 = 45cm

What is the speed of the cart at A?

a. Faster than final speed
b. Slower than final speed
c. Same as final speed

Conservation of energy questions



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KE gained = GPE lost

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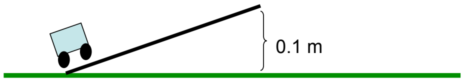
$v = \text{sqrt}(2gh)$

More height lost ⇒ bigger speed

Push a 0.5 kg cart up 0.10 m.
How much gravitational potential energy (GPE) does it gain?

Answer: GPE = mg x change in height h,
= 0.5 kg x 9.8 m/s^2 x 0.1 m = 0.49 J (= 0.5 J)



If we let it roll back down, how fast will it be going when it is back to where it started from?
a. 9.8 m/s , b. 1.4 m/s, c. 2 m/s, d. 1 m/s, e. 0.2 m/s

Push a 0.5 kg cart up 0.10 m.
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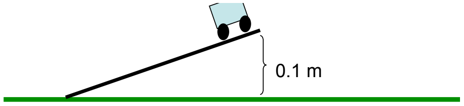

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$\cancel{W_{ext}} - |\cancel{W_{friction}}| = \Delta PE + \Delta KE$

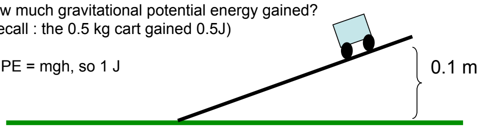
KE gained = GPE lost
$1/2\ m\ v^2 = mgh$
$v^2 = 2gh$
v = sqrt((2)(9.8)(0.1)) = 1.4m/s

Or
$½\ mv^2 = 0.5$ J
$v^2 = 0.5$ J x 2 / 0.5 kg
v = sqrt (2) = 1.4m/s

Conservation of energy questions

Push a 1 kg cart up 0.10 m.
How much gravitational potential energy gained?
(Recall : the 0.5 kg cart gained 0.5J)

GPE = mgh, so 1 J



Push the 1 kg cart up 0.10 m. How fast will it be going after it rolls back down?

a. same as the 0.5 kg cart, b. twice as fast, c. ½ as fast,
d. sqrt(2) x faster, e. sqrt(1/2) slower.

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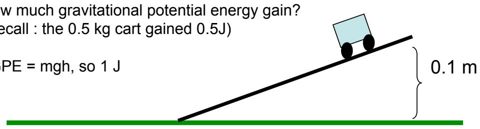

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a. same as the 0.5 kg cart, b. twice as fast, c. ½ as fast,
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KE gained = GPE lost
$1/2\ m\ v^2 = mg(h)$
v = sqrt(2gh)

Mass cancels out ⇒ v does not depend on m

Lets check!

Conservation of energy questions

Push a 1 kg cart up 0.10 m.
How much gravitational potential energy gain?
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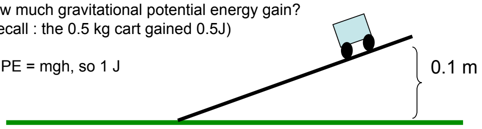

Push the 1 kg cart up 0.10 m. How fast will it be going after it rolls back down?

a. same as the 0.5 kg cart,

This experiment should ring bells……we have looked at it before from point of view of forces and acceleration

- Net force part of force of gravity = Fraction × mg
- $F_{net} = ma \Rightarrow$ Fraction × mg = ma
  a = fraction × g.

When net force due to gravity, acceleration and hence velocity independent of mass

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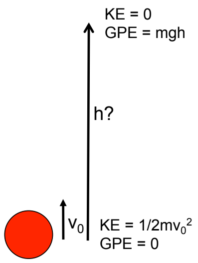
How high do balls go if you toss them?

(Conservation of energy - The easy method)



$\cancel{W_{ext}} - |\cancel{W_{friction}}| = \Delta PE + \Delta KE$

KE lost = GPE gained

Consider the changes between start and top of flight:

KE lost = GPE gained
$½\ mv_0^2 = mgh$

$\boxed{h = v_0^2/2g}$

Example of "expert" approach:

Realize same physics principle applies to any situation where you're just trading between gravitational and kinetic energy
(No friction/external forces)

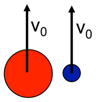
e.g. Cars on ramps, Balls in air -

Look different but SAME physics, SAME maths

How high do balls go if you toss them?

Consider 2 balls of different sizes, different masses tossed up with same initial velocity.
Compare the maximum heights that they reach:

a) Heavier ball goes higher
b) Same height
c) Lighter ball goes higher



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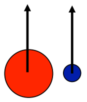

Energy conservation tells us they will go to same height.

$h = v_0^2/2g$

Connection to stuff we already know:
You could have figured this out by considering the force of gravity and acceleration, velocity, and position as function of time (but C of E method produces ***much*** easier math).

Dropping cannonballs onto a board.

What is going to happen to the board?

What forms of energy are involved?

a. Gravitational potential energy
b. Gravitational potential energy and Kinetic energy
c. Kinetic energy and Thermal energy
d. Chemical energy, Gravitational Potential energy, and Thermal energy
e. Gravitational Potential energy, Kinetic energy, and Thermal energy

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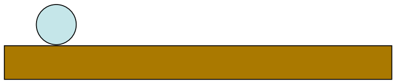
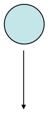


Dropping cannonballs onto board.

What is going to happen to the board?



Board heats up where the cannonball hits.
GPE ⇒ Kinetic ⇒ Thermal
mgh ⇒ ½ mv² ⇒ Constant x temp change.

Transfer of Energy

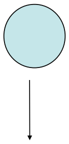
Gravitational Potential ➔ Kinetic ➔ Thermal

Work of gravity on ball = $F_{gravity}$ x distance of fall

Work of ball on board = F_{ball} x distance of crunch

Dropping cannonballs onto board.

Now drop the ball from twice the height.
What will the measured temperature rise be?



a. same, b. somewhat hotter, c. not as hot, d. 10 times hotter.

Answer is b.
mgh is twice as large, so the ball can deliver twice as much thermal energy.
(This is hard to hard to measure, though).

Conservation of energy summary

- Often your best route for answering mechanics problems (especially when time is not involved)

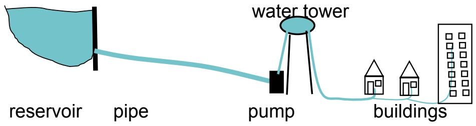
$$W_{ext} - |W_{friction}| = \Delta PE + \Delta KE$$

- Conservation of energy has been checked in thousands of experiments.
- This principle *always works*.

- There is no such thing as energy for free, or a perpetual motion machine!

When tackling a new situation/problem to get a solution or to make a prediction about behavior, usually the **first** thing a physicist does is figure out the different forms of energy, how much there is of each, and how it is being converted between various forms.

Water distribution



Where does the water flow?
What determines the water pressure in different homes/heights?
How fast does water flow out of a faucet?
How do you pump water out of wells?

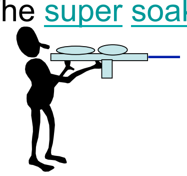
ALL ABOUT CONSERVATION OF ENERGY!

GPE = mgh KE = ½ mv^2 PPE = PV

Pumps do work (Force x distance)

(all of the physics of water distribution system)

The super soaker (e.g. squirt guns)

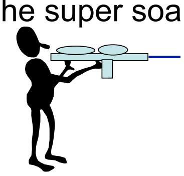


Pump up the pressure inside just a little bit and squirt. If we pump it up more, the water coming out will be:

a. going slower than before, b. going the same speed,
c. going faster.

(all of the physics of water distribution system)

The super soaker (e.g. squirt guns)



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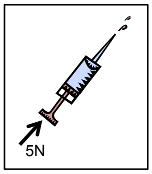
c. going faster.

- Think conservation of energy.
  - Pumping does work, energy in arm → stored (potential) energy in SS
  - When press trigger, PPE → KE of water
- Test: Shoot water up.
  Just like tossing a ball up, faster water goes higher.

Pressure potential energy (PPE)



What the heck is pressure anyway?

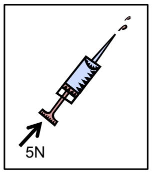
Pressure = $\frac{\text{Force}}{\text{Area}}$

Units: 1Pascal (Pa) = 1N/m^2

The plunger of a syringe has an area of 1cm^2.
I push the plunger with a force of 5N.
What's the pressure exerted by the plunger on the fluid inside?

a) 5N/m
b) 5 N/m^2
c) 500N/m^2
d) 50,000N/m
e) 50,000N/m^2

Pressure potential energy (PPE)



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e) 50,000N/m^2

Pressure = F/A = 5N/((0.01)(0.01))m^2 = 50000 N/m^2
= 50000 Pa

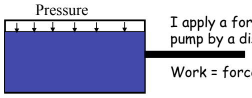
Pressure potential energy (PPE)

- New form of potential energy for fluids
- PE is the energy of an object (or fluid) due to its CONDITION (situation, surroundings etc)
- Water of mass m, at height h has associated GPE = mgh because of its (vertical) position
  - work (mgh) was done to get the water from ground to that height
  - Physical details of how the work was done or how water is being supported is not important
- Water of volume V at pressure P has associated PPE = PV
  - Work (PV) was done to pressurize the water
  - Physical details of how the work was done or how the pressure is being maintained are not important
- Check that PV has units of energy (J)

$$PV = \frac{N}{m^2} \times m^3 = Nm = J$$ ✓

Forms of energy in Super Soaker

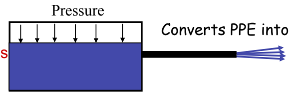
1. Pumping does work transforming chemical energy in my arm into PPE.



I apply a force, compress pump by a distance.

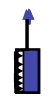
Work = force X distance.

2. When I pull the trigger, pressure does work on the water.



Converts PPE into KE = $\frac{1}{2} mv^2$

3. When I fire upward, this KE turns into GPE

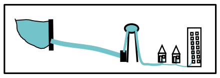


Energy in a water distribution system

- The same three forms of energy exist in a water distribution system
- If we add up energy in these forms, the sum must be constant.
- It just sloshes back and forth between forms!

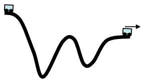
PPE + KE + GPE = E_{total} (constant)

$PV + \frac{1}{2} mv^2 + mgh = E_{total}$



Since E_{total} is constant:

- If one form of E changes, the other quantities must change correspondingly.
  - If pressure changes (water comes out of nozzle), v changes.
  - If height changes (go up in building), pressure or v changes, etc.

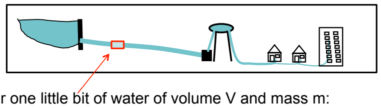


- Like the cart coasting up and down hills with no friction. Velocity and height are always connected
- ***If you know velocity and height at one place, can calculate it at all others.***

Bernoulli's Equation

$PV + \frac{1}{2} mv^2 + mgh = E_{total}$

But what mass of water are we talking about, what height?



Consider one little bit of water of volume V and mass m:

Replace m = ρV where ρ is the fluid density (ρ = mass/volume = 1000kg/m³ for water)

$PV + \frac{1}{2} \rho Vv^2 + \rho Vgh = E_{total}$

We can divide through by V to get the standard form for Bernoulli's equation:

$$\boxed{P + \frac{1}{2} \rho v^2 + \rho gh = E_{total}/V \qquad (E_{total\ per\ unit\ volume})}$$

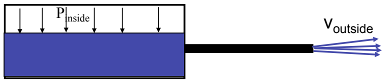
Just good old conservation of energy with the terms relabeled

Since $E_{total\ per\ vol}$ is constant:

Know P, v and h at one point ⇒ can calculate these quantities at another

Apply Bernoulli to Squirt Gun

How is velocity of water out related to pressure inside gun?



$P + \frac{1}{2} \rho v^2 + \cancel{\rho gh} = E_{total\ per\ vol}$ Height constant so ignore GPE

⇒ $\mathbf{P + \frac{1}{2} \rho v^2 = E_{total\ per\ vol}}$ **(constant)**

Inside gun: P = Atmos Pres. + P_{pump} v= 0 ⇒ AP + P_{pump} = $E_{total\ per\ vol}$

Outside gun: P =Atmos Pres., $v_{outside}$ is big ⇒ AP + ½ ρ $v_{outside}^2$ = $E_{total\ per\ vol}$

$$\cancel{AP} + P_{pump} = \cancel{AP} + \frac{1}{2} \rho\, v_{outside}^2$$

$$P_{pump} = \frac{1}{2} \rho\, v_{outside}^2$$

$$v_{outside} = \mathrm{sqrt}(2\, P_{pump} / \rho)$$