## Some Applications of Newton's Laws.

In this chapter, we get some practice applying Newton's laws to various physical problems. We do not introduce any new laws of physics.

## Solving $\mathrm{F}_{\text {net }}=\mathbf{m a}$ problems with multiple bodies

Problem: A mass $m_{1}$ is pulled up a frictionless incline by a string over a pulley and a hanging mass $\mathrm{m}_{2}$.


We know $\mathrm{m}_{1}, \mathrm{~m}_{2}$, and the angle $\theta$.
We seek :
$\mathrm{T}=$ tension in the cord,
$\mathrm{a}=$ acceleration of the mass,
$\mathrm{N}=$ normal force on $\mathrm{m}_{1}$

## Step 1

Draw a free-body diagram for each moving object.


Label the force arrows with the magnitudes of the forces.

Notice that T is the same for both objects (by NIII).

Use $\mathrm{m}_{1}, \mathrm{~m}_{2}$, in the diagrams, not $m$.

## Step 2

For each object, choose $\mathrm{x} \cdot \mathrm{y}$ axes so that the acceleration vector $\mathbf{a}$ is in the (+) direction.


Notice that we can choose different axes for the different objects. And we can tilt the axes if necessary.

Step 3 For each object, write the equations $\sum \mathrm{F}_{\mathrm{x}}=\mathrm{ma}_{\mathrm{x}}, \quad \sum \mathrm{F}_{\mathrm{y}}=\mathrm{m} \mathrm{a}_{\mathrm{y}}$

$\mathrm{m}_{1}: \begin{array}{ll}\mathrm{x} \text {-eqn } & (1) \\ \mathrm{y} \text {-eqn } & +\mathrm{T}-\mathrm{m}_{1} \mathrm{~g} \sin \theta=\mathrm{m}_{1} \mathrm{a} \\ +\mathrm{N}-\mathrm{m}_{1} \mathrm{~g} \cos \theta=0\end{array}$
$\mathrm{m}_{2}$ :
(3) $+\mathrm{m}_{2} \mathrm{~g}-\mathrm{T}=\mathrm{m}_{2} \mathrm{a}$

Notice that $|\vec{a}|=a$ is the same for both $m_{1}$ and $m_{2}$ since they are connected by a string that doesn't stretch.

Now we have a messy algebra problem with 3 equations in 3 unknowns:
(1) $+\mathrm{T}-\mathrm{m}_{1} \mathrm{~g} \sin \theta=\mathrm{m}_{1} \mathrm{a}$
(2) $+\mathrm{N}-\mathrm{m}_{1} \mathrm{~g} \cos \theta=0 \quad$ The unknowns are $\mathrm{T}, \mathrm{N}$, and a .
(3) $+\mathrm{m}_{2} \mathrm{~g}-\mathrm{T}=\mathrm{m}_{2} \mathrm{a}$

We can solve for N right away. $\operatorname{Eqn}(2) \Rightarrow \mathrm{N}=\mathrm{m}_{1} \mathrm{~g} \cos \theta$
Now we have 2 equations [(1) \& (3)] in 2 unknowns ( $\mathrm{a} \& \mathrm{~T}$ ).
Solve (1) for T and plug into (3) to get an equation without an T :
(1) $\Rightarrow \mathrm{T}=\mathrm{m}_{1} \mathrm{~g} \sin \theta+\mathrm{m}_{1} \mathrm{a}=\mathrm{m}_{1}(\mathrm{~g} \sin \theta+\mathrm{a})$
(3) $\Rightarrow \mathrm{m}_{2} \mathrm{~g}-\left[\mathrm{m}_{1}(\mathrm{~g} \sin \theta+\mathrm{a})\right]=\mathrm{m}_{2} \mathrm{a}$

Now solve this last equation for a:

$$
\mathrm{m}_{2} \mathrm{~g}-\mathrm{m}_{1} \mathrm{~g} \sin \theta=+\mathrm{m}_{1} \mathrm{a}+\mathrm{m}_{2} \mathrm{a}=\mathrm{a}\left(\mathrm{~m}_{1}+\mathrm{m}_{2}\right)
$$

$$
a=\frac{m_{2} g-m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)}
$$

Finally, if you have any strength left, we can solve for tension T by plugging our expression for acceleration a back into either (1) or (3) and solving for T. From (3), we have
$\mathrm{T}=\mathrm{m}_{2} \mathrm{~g}-\mathrm{m}_{2} \mathrm{a}=\mathrm{m}_{2}(\mathrm{~g}-\mathrm{a})$.

Plugging in our big expression for $a$, we get

$$
\mathrm{T}=\mathrm{m}_{2}(\mathrm{~g}-\mathrm{a})=\mathrm{m}_{2}\left(\mathrm{~g}-\frac{\mathrm{m}_{2} \mathrm{~g}-\mathrm{m}_{1} \mathrm{~g} \sin \theta}{\left(\mathrm{~m}_{1}+\mathrm{m}_{2}\right)}\right)
$$

We can simplify:

$$
\begin{aligned}
& \mathrm{T}=\mathrm{m}_{2}\left(\frac{\mathrm{~g}\left(\mathrm{~m}_{1}+\mathrm{m}_{2}\right)}{\left(\mathrm{m}_{1}+\mathrm{m}_{2}\right)}-\frac{\mathrm{m}_{2} g-\mathrm{m}_{1} g \sin \theta}{\left(\mathrm{~m}_{1}+\mathrm{m}_{2}\right)}\right)=\mathrm{m}_{2}\left(\frac{m_{1} g+m_{2} g-m_{2} g+m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)}\right) \\
& \mathrm{T}=\mathrm{m}_{2}\left(\frac{m_{1} g+m_{1} g \sin \theta}{\left(m_{1}+m_{2}\right)}\right)=\mathrm{T}=\frac{m_{2} m_{1} g}{\left(m_{1}+m_{2}\right)}(1+\sin \theta)
\end{aligned}
$$

Do these expressions make sense? Let's check some limits.
If $\mathrm{m}_{1}=0$, then $\mathrm{m}_{2}$ should be freely falling with $\mathrm{a}=\mathrm{g}$ and the tension T should be zero. Check that this is so.

If $\mathrm{m}_{2}=0$, then $\mathrm{m}_{1}$ should slide down the incline with acceleration $\mathrm{a}=-\mathrm{g} \sin \theta$ (since it would be accelerating in the negative direction). Check that this is so.

## Forces and circular motion

NII: $\overrightarrow{\mathrm{F}}_{\text {net }}=\mathrm{m} \overrightarrow{\mathrm{a}} \Rightarrow$ To make something accelerate, we need a force in the same direction as the acceleration. $\Rightarrow$ Centripetal acceleration is always caused by centripetal force, a force toward center. "Centripetal" means "toward the center". "Centrifugal" means something totally different. Centrifugal forces don't exist! More on that below.

Example: Rock twirled on a string. (Assume no gravity)


Given: $\mathrm{m}=0.1 \mathrm{~kg}, \mathrm{~T}($ period $)=1 \mathrm{~s}$, radius $\mathrm{r}=1 \mathrm{~m}$
What is tension $\mathrm{F}_{\mathrm{T}}$ in the string? (Here, we use symbol $\mathrm{F}_{\mathrm{T}}$, since T already taken by period.)
$\mathrm{F}_{\mathrm{T}}$ is the only force acting. (No such thing as "centrifugal force"!)
$\mathrm{F}_{\mathrm{T}}=\mathrm{ma}=\mathrm{m} \frac{\mathrm{v}^{2}}{\mathrm{r}}, \quad \mathrm{v}=\frac{2 \pi \mathrm{r}}{\mathrm{T}}, \quad \Rightarrow$
$\mathrm{F}_{\mathrm{T}}=\mathrm{m} \frac{(2 \pi \mathrm{r} / \mathrm{T})^{2}}{\mathrm{r}}=4 \pi^{2} \mathrm{~m} \frac{\mathrm{r}}{\mathrm{T}^{2}}=4 \pi^{2}(0.1) \frac{1}{1^{2}}=3.9 \mathrm{~N} \approx 1$ pound

## What about the outward 'centrifugal force"?

A person on a merry-go-round (or twirled on a rope by a giant) "feels" an outward force. This is an illusion! There is no outward force on the person. Our intuition is failing us. Our intuition about forces was developed over a lifetime of experiences in inertial (non-accelerating) reference frames. If we are suddenly placed in an accelerating reference frame, our brains (wrongly) interpret our sense impressions as if we were still in a non-accelerating frame.

The result is that the direction of the perceived force is exactly opposite the direction of the true force. Example: A person in car accelerating forward. The chair pushes the driver forward. The force on the driver is in the forward direction. But the driver "feels" herself pressed back into the seat. It seems there is some force pushing the driver backward. WRONG!

"Centrifugal force" (not to be confused with "centripetal force") is also called a "pseudo-force" or "fictitious force" . Newton's Laws are only valid in a non-accelerating reference frame (an inertial frame). If we try to analyze motion in a non-inertial frame (for instance, in a rotating
frame) then Newton's Laws don't hold. However, we can pretend that Newton's Laws hold in an accelerating frame if we pretend that "pseudo-forces" exist. That is, we can get the right answer if we makes two mistakes. In my opinion, this is a Devil's bargain. Computational convenience has come at the price of endless confusion of millions of physics students (and many professional engineers!). My advice: If you have choice, NEVER do calculations in non-inertial frames. Avoid using fictitious forces.

Consider the rock on the string again (still no gravity). If the string breaks, then there is no longer any force on the rock and it will move in a straight line with constant velocity [according to NI: if $\mathrm{F}_{\text {net }}=0$, then $\mathbf{v}=$ constant]. The reason the rock does not move in a straight line is because the string keeps pulling it inward, turning it away from its straight-line path. There is NO outward force on the rock.


Friction ...
is very useful! We need friction to walk. Friction is not well understood. The amount of friction between two surfaces depends on difficult-tocharacterize details of the surfaces, including microscopic roughness, cleanliness, and chemical composition.

-- friction involves tearing, wear between microscopically rough surfaces.

If two metal surfaces are atomically smooth and clean (almost impossible to achieve), they will bond on contact = "cold weld".

Empirical observations about friction:

- the magnitude of the force of friction $f$ between 2 surfaces is proportional to the normal force $N$, not the area of contact, $(f \propto N)$.


Pull a block of mass $m$ along a surface. Regardless of orientation, you get the same normal force ( $\mathrm{N}=\mathrm{mg}$ ), and you get the same frictional force $f$.

Why not $\mathrm{f} \propto$ area of contact? Because more area $\Rightarrow$ less weight per area.

- static friction is different than sliding friction (also called kinetic friction). The maximum magnitude of static friction force usually larger than the magnitude of kinetic friction force.

Kinetic Friction (also called sliding friction)
$\mathrm{f}=\mu_{\mathrm{K}} \mathrm{N} \quad$ (Not a law, just an empirical observation - usually, but not always, true)
$\mu_{\mathrm{K}}=$ coefficient of kinetic friction $=$ dimensionless number $\mu_{\mathrm{K}}>0$
( $\mu_{\mathrm{K}}$ can be greater than 1 but, usually, $\mu_{\mathrm{K}}<1$.)
Example: A block of mass $m$ is being pushed along a rough horizontal table. One maintains a constant velocity v with a horizontal external force of magnitude $\mathrm{F}_{\mathrm{ext}}$. What is $\mu_{\mathrm{K}}$ ?


Free-body diagram: direction of frictional force $f$ is always opposite to the motion:
velocity $=$ constant $\Rightarrow \mathrm{a}=0 \Rightarrow \mathrm{~F}_{\text {net }}=0$

$\Rightarrow$ In y-direction: $N=m g \quad$ In x-direction: $F_{\text {ext }}=\mu_{\mathrm{K}} \mathrm{N} \quad$ So, $\mathrm{F}_{\mathrm{ext}}=\mu_{\mathrm{K}} \mathrm{mg}$, or...

$$
\mu_{\mathrm{K}}=\frac{\mathrm{F}_{\mathrm{ext}}}{\mathrm{mg}}
$$

## Static Friction

$\mathrm{f}_{\text {static }}<\mathrm{f}_{\text {static, } \max }=\mu_{\mathrm{S}} \mathrm{N} \quad$ (the maximum magnitude of the static friction force is $\mu_{\mathrm{S}} \mathrm{N}$ )
$\mu_{\mathrm{S}}=$ coefficient of static friction $=$ dimensionless number $\quad \mu_{\mathrm{S}}>0$
Usually, $\mu_{\mathrm{S}}>\mu_{\mathrm{K}}$ (maximum static friction is greater than kinetic friction)
Consider a book sitting on a table. You pull on a book with a small force $\mathrm{F}_{\text {ext }}$ to the right, but it doesn't move. There must be a frictional force to the left (otherwise the book would move).


If you increase the external force and the book still does not move, the frictional force must have gotten bigger to match.

If you make the external force big enough, the book will suddenly start to move. Just before the book moved, the static friction was at its maximum value. So the magnitude of the static friction
force can be anything between zero and a maximum value, given by $\mathrm{f}_{\max }=\mu_{\mathrm{S}} \mathrm{N}$. The book will remain stationary until $\mathrm{F}_{\text {ext }}>\mathrm{f}_{\max }=\mu_{\mathrm{S}} \mathrm{N}$. Then the book will start to slide.

Usually, $\mu_{\mathrm{S}}>\mu_{\mathrm{K}} \Rightarrow$ large force is needed to start an object sliding, but then a smaller force is needed to keep it sliding. Anyone who has pushed a fridge across the kitchen floor knows this.

There is no good theory of friction $\Rightarrow \mu^{\prime}$ s cannot be computed; instead, they are determined experimentally.

Example: Friction on an inclined plane


A mass $m$ on an incline at angle $\theta$, with kinetic friction coefficient $\mu_{\mathrm{K}}$. What size external force $\mathrm{F}_{\text {ext }}$ is required to maintain an acceleration of magnitude $a$ up the incline?

Step 1: Free-body diagram.

Step 2: Choose coordinate system.
(Make direction of acceleration $=+$ direction)

Step 3: $\sum \mathrm{F}_{\mathrm{x}}=\mathrm{ma} \mathrm{a}_{\mathrm{x}}, \quad \sum \mathrm{F}_{\mathrm{y}}=\mathrm{ma} \mathrm{a}_{\mathrm{y}}$


Notice:
x -component of weight $=-\mathrm{mg} \sin \theta$
y -component of weight $=-\mathrm{mg} \cos \theta$
$\mathrm{Y}: \quad \mathrm{a}_{\mathrm{y}}=0 \Rightarrow+\mathrm{N}-\mathrm{mg} \cos \theta=0, \quad \mathrm{~N}=\mathrm{mg} \cos \theta$
$\mathrm{X}: \quad \mathrm{a}_{\mathrm{x}}=\mathrm{a} \Rightarrow+\mathrm{F}_{\mathrm{ext}}-\mu_{\mathrm{K}} \mathrm{N}-\mathrm{mg} \sin \theta=\mathrm{ma}$


Combine X and Y equations:
$+\mathrm{F}_{\mathrm{ext}}-\mu_{\mathrm{K}} \mathrm{mg} \cos \theta-\mathrm{mg} \sin \theta=\mathrm{ma} \Rightarrow \mathrm{F}_{\mathrm{ext}}=\mathrm{ma}+\mu_{\mathrm{K}} \mathrm{mg} \cos \theta+\mathrm{mg} \sin \theta$
(Whew! )

Example: A mass $m$ on a flat table, with sliding friction coefficient $\mu_{\mathrm{K}}$, is pulled along the table by a force $\mathrm{F}_{\text {ext }}$ at angle $\theta$. What is the (magnitude of the) acceleration $a$ ?


Steps 1, 2:


Step 3:
Y-motion: $\sum \mathrm{F}_{\mathrm{y}}=\mathrm{ma}_{\mathrm{y}}, \mathrm{a}_{\mathrm{y}}=0 \Rightarrow \sum \mathrm{~F}_{\mathrm{y}}=0 \Rightarrow \sum$ (forces up) $=\sum$ (forces down) $\Rightarrow$
$\mathrm{N}+\mathrm{F}_{\text {ext }} \sin \theta=\mathrm{mg}, \quad \mathrm{N}=\mathrm{mg}-\mathrm{F}_{\mathrm{ext}} \sin \theta \quad$ (Do you understand the $\sin \theta$ here?)
X-motion: $\sum \mathrm{F}_{\mathrm{x}}=\mathrm{ma}_{\mathrm{x}}, \mathrm{a}_{\mathrm{x}}=\mathrm{a} \Rightarrow \mathrm{F}_{\mathrm{ext}} \cos \theta-\mu_{\mathrm{K}} \mathrm{N}=\mathrm{ma}$

Now combine X and Y results:
$\mathrm{F}_{\text {ext }} \cos \theta-\mu_{\mathrm{K}}\left(\mathrm{mg}-\mathrm{F}_{\text {ext }} \sin \theta\right)=\mathrm{ma} \Rightarrow \mathrm{a}=\underbrace{\frac{\mathrm{F}_{\text {ext }}}{\mathrm{m}}\left(\cos \theta+\mu_{\mathrm{K}} \sin \theta\right)-\mu_{\mathrm{K}} \mathrm{g}}_{\text {notice that all terms have units of }[\mathrm{a}]}$

Example: Rotation with friction. A car rounds a curve on a flat road (not banked). The radius of the circular curve is $r=100 \mathrm{~m}$, and the speed of the car is $\mathrm{v}=30 \mathrm{~m} / \mathrm{s}(\cong 68 \mathrm{mph})$. How large a static friction coefficient ( $\mu_{\mathrm{s}}$, not $\mu_{\mathrm{K}}$ !) is needed for the car to not skid off the road?


Top view


## View from rear of car

$\mathrm{F}_{\text {net }}=\mathrm{F}_{\text {fric }}=\mathrm{ma} \Rightarrow \quad \mu_{\mathrm{S}} \mathrm{N}=\mathrm{mv}^{2} / \mathrm{r} \quad\left(\right.$ car about to skid $\left.\Rightarrow \mathrm{F}_{\text {fric }}=\mu_{\mathrm{S}} \mathrm{N}\right)$
$\mathrm{N}=\mathrm{mg} \quad \Rightarrow \quad \mu_{\mathrm{S}} \mathrm{mg}=\mathrm{mv}^{2} / \mathrm{r} \quad$ (m's cancel) $\Rightarrow$

$$
\mu_{\mathrm{s}}=\frac{\mathrm{v}^{2}}{\mathrm{gr}}=\frac{(30 \mathrm{~m} / \mathrm{s})^{2}}{\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(100 \mathrm{~m})}=0.92 \text { (no units) }
$$

So, need $\mu_{\mathrm{S}} \geq 0.92$ or else car will skid. For rubber on dry asphalt $\mu_{\mathrm{S}} \cong 1.0$, for rubber on wet asphalt $\mu_{\mathrm{S}} \cong 0.7$. So car will skid if road is wet.

