What follows is a rough outline of physics 1120 material, it is by no means complete but is meant to highlight basic ideas and topics. It is certainly not meant to be used as a "crib sheet" for your exam. As usual the equations by themselves are meaningless so please read the text of this document to aid in understanding their meaning. Where vector arrows are omitted for vector quantities it denotes the amplitude only. The reader must determine the direction.

1. ELECTROSTATICS

A. Electric Fields and Forces

The electric field is a vector field which obeys the laws of **superposition**, i.e. each contributing charge or charge distribution can be considered separately and then the **vector sum** of all the contributions can be taken to give the total field. Electric field lines are a tool for visualizing a field. The strength of the field is proportional to the density of field lines. Lines originate at positive charges and terminate on negative charges.

$$\text{Electric Field of a point charge: } \vec{E} = \frac{kq}{r^2}\hat{r}$$

$$\text{Force on a charged particle due to an electric field: } \boxed{\vec{F}_e = q\vec{E}}$$

$$\text{Force between two charged particles (combine last two eqns.): } \boxed{\vec{F}_e = \pm k\frac{|q_1||q_2|}{r^2}\hat{r}}$$

In the last equation the plus or minus sign is chosen so that like charges repel and opposite charges attract. A couple of examples of electric fields for specific charge distributions follow:

$$\text{Electric field for a single infinite sheet of charge: } E = \frac{\sigma}{2\epsilon_o}$$

$$\text{Electric field of two infinite planes of opposite charge (between the plates): } E = \frac{\sigma}{\epsilon_o}$$

This last result can be seen from the superposition of two infinite plate fields, where each plate has an opposite charge (but equal magnitudes). Of course the field outside of two oppositely charged parallel plates is zero (why?). All of the expressions for electric fields that you will see can be derived from Gauss's Law (see below). **Electric flux** is defined to be

$$\Phi_e = \int \vec{E} \cdot d\vec{A}$$

where since we are integrating over an area the double integral is implied. Flux is interpreted as the amount of field lines penetrating a surface. If the surface is closed (ie a sphere) then the outward normal is traditionally taken as the positive. For a non-closed surface the choice for the direction of the area vector lies with you. For a uniform electric field and a flat surface the above equation reduces to

$$\Phi_e = \vec{E} \cdot \vec{A} = EA\cos(\theta) \text{ (uniform field and flat area only)}$$

Here θ is the angle between the area vector and the electric field.

To find the electric field for a charge distribution with high symmetry a useful tool is **Gauss's law**:

$$\boxed{\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_o}}$$

Here q_{enc} is of course the amount of charge enclosed by our (closed) imaginary Gaussian surface. The most common Gaussian surfaces are spheres, cylinders, and boxes. Without high symmetry Gauss's law is essentially useless. It is only useful in cases where we can choose a surface that is perpendicular to the E

field everywhere, and everywhere along that surface the field has the same magnitude. Then we can back the electric field out of the integral, leaving a simple integral over dA which just gives the area of our gaussian surface (know your area formulas). It is then a simple matter to solve for the electric field.

Conductors in electrostatic equilibrium have a few properties:

- The electric field inside a conductor is zero. (make sure to know why)

- On the surface of a conductor the electric field points perpendicular to the surface (or parallel to the normal). This is due to the fact that the entire conductor is an equipotential (see below).

B. Electric potential and Electric potential energy

Since the electric field is a vector quantity, it is mathematically a pain in the ass to deal with at times. A very useful concept which is both convenient and offers a bridge into the world of electronics is **electric potential field**. As with gravitational potential (with which you are hopefully more familiar) the zero point of the potential can be chosen anywhere we want as long as we are consistent after that choice. Since this means that the absolute value of the potential has no meaning, the only thing that is meaningful are changes in potential. In terms of the electric field, change in electric potential is defined to be

$$\boxed{\Delta V = -\int \vec{E} \cdot d\vec{l}}$$

This is a path integral, the dot product serves to pick out components of our path that are along the electric field (through the cosine dependence). Thus if we move perpendicularly to the field we move along an equipotential line. Instead of drawing field lines, we can draw equipotential lines (or surfaces if you are a talented artist) to represent a field. By the preceding argument, field lines are always perpendicular to equipotential surfaces. In the gravity analogy, the gravitational force acts downward, and equipotential lines/surfaces are then planes of constant height. If we are in a uniform field, like in a parallel plate capacitor, and choose a path along the direction of the field, the above integral reduces to

$$\Delta V = -E\Delta l$$

For non-infinite charge distributions we normally take the zero point for the potential to be at infinity. With this convention we can write

$$\text{Electric potential due to a point charge: } V = \frac{kq}{r}$$

Notice that potential can be either positive or negative depending on the sign of the charge. Since we integrate the field, the electric potential for a point charges falls off like $1/r$ in contrast to the electric field that falls off like $1/r^2$.

Just as an object with mass in a gravitational field can have gravitational potential energy, a object with charge can have **electric potential energy (EPE)**.

$$EPE = qV \text{ for a point charge}$$

This is the potential energy associated with a charged particle (q) being placed in a electric potential field created by other charge(s). Moving a charged particle in an electric field can change its potential. The work required and the corresponding change in potential are related by the following:

$$\Delta V = -\frac{\text{Work}_{elec}}{q} = \frac{\text{Work}_{ext}}{q} = \frac{\Delta EPE}{q}$$

The subscripts "elec" and "ext" represent the works done by the field itself and an external agent respectively. The minus sign is important, convince yourself why. Notice that if we move along an equipotential, ΔEPE is zero and so no work is done by either the field or the external agent. Recall that Work $= \vec{F} \cdot \vec{d}$, the **dot product** of the force and the displacement. Another useful result is the **work-kinetic energy theorem**:

$$\boxed{\text{Work}_{net} = \Delta KE}$$

where net means the total work done on an object, by both conservative and non-conservative forces. The energy density of an electric field is given by

$$u_E = \frac{\epsilon_o E^2}{2}$$

To find the energy stored in an electric field you have to integrate this energy density over the volume in question.

C. Capacitors

Capacitors are objects that store energy in the form of electric fields. They are used in electronic filters, high current discharge devices, and for burning people. Capacitance is defined to be the (positive) quantity

$$\boxed{C = \frac{q}{\Delta V}}$$

The simplest type of capacitor is a parallel plate capacitor that consists of two plates of area A and a distance d apart.

$$C = \frac{\epsilon_o A}{d} \text{ parallel plate capacitor}$$

Notice that the capacitance is only a function of the geometry of the object, in that manner it is a property of the system and not the parameters q, ΔV etc. For a parallel plate capacitor filled with a dielectric material, we must multiply the above equation by the unit-less dielectric constant of said material to find the capacitance. When working with dielectrics it is usually a good rule of thumb to replace ϵ_o everywhere with ϵ_r, the dielectric constant of the dielectric. Common problems include relating the quantities described thus far for varying situations (plates hooked up across a battery or not etc.). The definition of potential difference in terms of the $\vec{E}$ field, the electric field between two very large (infinite) plates, and the previous relation for capacitance of a parallel plate capacitor can be used to solve such problems.

The **energy stored in a capacitor** is given by

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{q^2}{C}$$

where I used U for the energy instead of E to avoid confusion with the electric field.

2. MAGNETOSTATICS

A. Magnetic fields

Sources of magnetic fields (which is also a vector field which obeys superposition) are currents, or as you know magnets themselves. Field directions are given by the right hand rule. When placed in a magnetic field, a magnet (ie a compass) will rotate as to align its north pole with the direction of the field. A few magnetic fields are given by

$$\text{Magnetic field due to a moving point charge: } \vec{B} = \frac{\mu_o}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

where $\hat{r}$ points from the charge to the point of interest (where we calculate the field).

$$\text{Magnetic field due to a long current carrying wire: (outside) } B = \frac{\mu_o I}{2\pi d}$$

$$\text{Magnetic field due to a long current carrying wire: (inside) } B = \frac{\mu_o I}{2\pi R^2}r$$

where R is the radius of the wire and r is your distance from the center.

$$\text{Magnetic field at the center of a coil with N turns and radius R: } B = \frac{\mu_o N I}{2R}$$

Magnetic field of a long solenoid (inside), (its 0 outside): $B = \mu_o n I$

where n is the number of turns per unit length. The **magnetic dipole moment** of a current carrying loop of area A is defined as

$$\vec{\mu} = AI$$

The direction is the same as the magnetic field at the center of the loop, due to the current carried by the loop (use RHR). Similar to Gauss's law in electrostatics, **Ampere's Law** is a useful tool for determining the magnetic field in magnetostatics. It is

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_o I_{enc}}$$

where I_{enc} is the current enclosed by the imaginary (closed) amperian loop. As with Gauss's law, this law is most useful with highly symmetric situations where B can be taken outside the integral and then easily solved for. All of the magnetic field equations above (except the point charge field) can be derived using ampere's law.

B.   Magnetic Forces

The magnetic force on a charged particle in a magnetic field is given by

$$\boxed{F_{mag} = q\vec{v} \times \vec{B}}$$

Notice that due to the cross product, the magnetic force is always perpendicular to both the charge's velocity and the magnetic field. Since the velocity of a particle is always parallel to its displacement (at least for small displacements) **the work done by the magnetic force on a charged particle is zero**, ie it cannot speed the particle up or slow it down, it can only change its direction (see work-kinetic energy theorem above with net work equalling zero).

From the force on a single charge we can derive the following:

Force on a current carrying wire in a magnetic field: $\vec{F} = I\vec{L} \times \vec{B}$

The $\vec{L}$ vector points in the same direction as the current in the wire and has magnitude equal to the length of the wire. Using the magnetic field expression for a wire and the above equation gives

Force between two parallel wires a distance d apart: $F = \dfrac{\mu_o L I_1 I_2}{2\pi d}$

Notice that I didn't give the direction. For currents in the same direction the wires attract. Currents in opposite directions repel. Due to the forces on segments of current in a magnetic field, a current loop feels a torque when placed in a magnetic field, it is given by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where $\vec{\mu}$ is the magnetic dipole moment defined above. The energy density of a magnetic field is

$$u_B = \frac{B^2}{2\mu_o}$$

To find the energy stored in a magnetic field you must integrate this energy density over the volume in question.

C.   Induced EMF: Faraday's Law

A rather surprising aspect of nature is that it fights a change in magnetic flux. Similar to electric flux, **magnetic flux** is defined to be

$$\Phi_m = \int_{\text{area of loop}} \vec{B} \cdot d\vec{A}$$

When the magnetic flux through a surface **changes**, an electromotive force (EMF), also known as a potential difference, is created that attempts to create a current to maintain the previous flux. Mathematically this is **Faraday's Law**

$$\epsilon = \left| \frac{d\Phi_m}{dt} \right|$$

Which says that a changing magnetic flux produces an emf. Don't confuse the emf symbol ϵ with the universal constant ϵ_o. The direction of the resulting current [1] is such that the change in flux is minimized. Faraday's law is used in all kinds of applications, electric motors, transformers, generators, metal detectors, etc.

D. Inductors

Inductors are devices that store energy in a magnetic field (similar to the capacitor that stores energy in the electric field). A solenoid (discussed above and in your text) acts as an inductor. Inductance is defined to be

$$L = \frac{\Phi_m}{I}$$

It turns out that

The inductance of a solenoid: $L_{\text{solenoid}} = \frac{\mu_o N^2 A}{l}$

Notice that the inductance of a solenoid is only a function of its geometry (N-number of turns, A-cross sectional area, l-length). An inductor is essentially a series of many conducting loops, so using Faraday's law we know that if we try to change the current through an inductor it will fight that change with an induced potential difference. This is the potential difference across an inductor in exactly this situation:

$$\Delta V_L = -L \frac{dI}{dt}$$

Note that if we change the current through an inductor very rapidly, for example by closing or opening a switch, the voltage across the inductor can be huge. Inductors have many applications listed above for Faraday's law as well as many circuit applications. The energy stored in an inductor is given by

$$U_L = \frac{1}{2} L I^2$$

3. CIRCUITS

A. Components and Analysis

We've already discussed two circuit components, inductors and capacitors. Other basic components include batteries and resistors. **A battery's entire purpose is to maintain a constant potential difference between its terminals**. Resistors are exactly that, parts of the circuit that resist current flow by dissipating energy. Light bulbs act as resistors, the power that they dissipate is in the form of light and heat. What causes current to flow in a wire or circuit component is an electric field that pushes the charges along. Batteries (as well as changing magnetic flux) produce a potential difference which is related to an electric field that exerts forces on the charges. The current density J is related to this electric field strength by

$$J = \sigma E$$

where σ is called the conductivity of the material. The inverse of σ is $\rho = 1/\sigma$ and is called the resistivity. Technically speaking the relation involving J is ohm's law. You probably call the following **ohm's law**:

$$\boxed{\Delta V = IR}$$

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[1]A current from the potential difference will ensue only if there is a conductor at the location in question. However, the emf is generated regardless of whether there is anything there.

which <u>only holds for ohmic materials obeying $J = \sigma E$</u>. Broken wires do not obey ohm's law, neither do capacitors. In the above relation causality is difficult to see. It is the potential difference across something that creates the current through it. The resistance of an object (R) is related to its resistivity. For a wire the resistance is given by

$$R = \frac{\rho L}{A}$$

where A is the cross sectional area and L is the length. So if we double a wire's length we double its resistance, if we double its radius the resistance goes down by a factor of 4. Commonly in circuits the wires are taken to be ideal, that is they have no resistance so you need not worry about it. This assumption is valid if the resistance of your circuit components is much larger than the wires used which is almost always true.

Circuit analysis uses the following rules:

- $$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$

- $$\sum I_{in} = \sum I_{out}$$

The first says that if you go around a complete loop in a circuit and add up the voltage drops/rises across each component as you cross it you will get zero. You must end where you began. The signs in the equation can be tricky, look at examples in you text etc. The second is the junction rule and simply conveys that all the current into a junction leaves that junction. This says that current is conserved, it is not "used up" by any circuit component. Of course this is not true for potential difference which is in a sense "used up" as you progress along the circuit. These two rules let you solve (via a system of linear equations) for the currents in any loop that you choose, which using ohm's law ($V = IR$) will give you the potential drop across any circuit component of interest.

I mentioned that resistors dissipate power. The amount of **power dissipated by a resistor or light bulb** with resistance R is

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

where ΔV is the potential drop across the resistor/bulb and I is the current. The numerous expressions are attained from the first using ohm's law. The expression that is most convenient to use is determined by what you know.

There are a few more rules for circuits that are of use, review series and parallel if needed:

- The current through any components in series is the same.

- The potential difference across any components in parallel is the same.

B. Component addition

We can combine a set of many resistors and replace it by an equivalent resistor. The way they combine differs depending on whether they are in parallel or series.

$$R_T = R_1 + R_2 \quad \text{(2 resistors in series)}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{(2 resistors in parallel)}$$

Inductors add just like resistors do. Capacitors are just the opposite:

$$C_T = C_1 + C_2 \quad \text{(2 capacitors in parallel)}$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(2 capacitors in series)}$$

It is also worthy of note that the charge on any two capacitors in series is the same. Also, an uncharged capacitor behaves just like an ideal wire. A fully charged one behaves like an open circuit, where no current can flow. Many complicated circuits can be simplified via these equations. When determining equivalent resistance or capacitance, I suggest you start with a small part of the circuit, draw a new picture at each step, and be careful to only combine components that are strictly in series or parallel.

C. Common circuits

The RC circuit

is a simple circuit that contains a switch, a resistor and a charged capacitor all in series. Zero time is usually defined to be the moment that you close the switch. The RC circuit has some interesting features, the charge on the capacitor changes exponentially with time:

$$Q = Q_o e^{-t/\tau} \quad \text{where} \quad \tau = RC$$

τ is called the time constant of the circuit and controls how fast the capacitor will charge or discharge. The current in the circuit behaves similarly:

$$I = I_o e^{-t/\tau}$$

Notice after a long time (relative the the time constant) the current is essentially zero, this agrees with the fact that a fully charged capacitor acts like an open switch.

The RL circuit

The current in an LR circuit behaves just like the current in the RC circuit but with a different time constant (how can it be the same, there isn't any C anymore)...

$$I = I_o e^{-t/\tau} \quad \text{where} \quad \tau = \frac{L}{R}$$

The situation for the LR circuit corresponding to this equation is quite different than the RC circuit, see pg . 1073 in your text.

4. MAXWELL'S EQUATIONS AND SOLUTIONS

All of classical electricity and magnetism can be described using 5 equations. The first is the Lorentz force which combines the forces on a charged particle from electric and magnetic fields:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

The other 4 equations that are needed are the Maxwell's equations, which combine all the information discussed thusfar...

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o} \qquad \text{Gauss's Law} \tag{2}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{No magnetic monopoles} \tag{3}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt} \qquad \text{Faraday's law} \tag{4}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc} + \epsilon_o \mu_o \frac{d\Phi_E}{dt} \qquad \text{Ampere-Maxwell law} \tag{5}$$

These are Maxwell's equations in integral form, there also exist differential forms. The last equation (5) conveys that there are two sources of magnetic fields, changing electric flux and currents. As you have hopefully seen, solutions to Maxwell's equations are electromagnetic waves that travel at the speed of light!

This is how Maxwell concluded that light was an electromagnetic wave. It turns out that the velocity of propagation for light in a vacuum (c) is

$$v_{em} = c = \frac{1}{\sqrt{\epsilon_o \mu_o}} = 3 \times 10^8 m/s$$

The simplest type of electromagnetic wave is a plane wave. It is important to understand the picture (in your text and tutorials) that represents plane waves. The direction of travel can be determined from the equations for the electric and magnetic traveling waves as we discussed in tutorial, it is also given by $\vec{E} \times \vec{B}$. The **intensity** of an electromagnetic wave is given by

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}} = \frac{E_{max}^2}{2c\mu_o} = \frac{c\epsilon_o E_{max}^2}{2}$$

It turns out that our eyes sense intensities (also our ears but with sound waves), not amplitudes so for sensing purposes the intensity is the interesting quantity. There is also the relation that

$$c = \frac{E}{B}$$

which is true at all times since the two waves mirror each other. The maximum amplitude of a sine or cosine wave is related to the rms (average) value by a constant:

$$A_{rms} = \frac{A_{max}}{\sqrt{2}}$$

The rms value is a little more than 2/3 of the maximum value.

Polarization

Electromagnetic waves can be polarized in certain directions. That direction is given by the plane in which the electric field is oscillating. Polarizing filters are used to control the polarization of light by filtering out all polarizations that are not wanted. Of course you lose some intensity of the wave when you filter off components. The following relation is **Malus's law** and mathematically tells you exactly how much intensity you lose.

$$I_{transmitted} = I_{original} \cos^2 (\theta)$$

The angle in question here is the angle between the original polarization direction (just before the filter) and the filter's polarization axis (discussed in you text). When you have more than one filter in a row, the intensity losses add multiplicatively. You must be sure to use the correct angles in these cases. A common error is to use the angle between the nth polarizing filter and the original polarization axis, when you should use the relative angle between the nth filter and the (n-1)th filter. For example
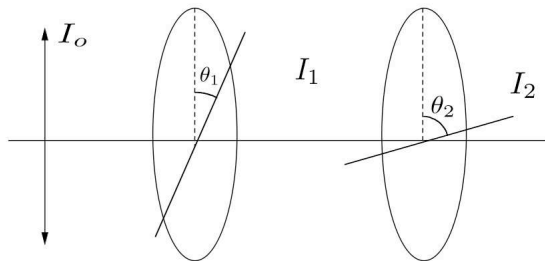
$$I_2 = I_1 \cos^2 (\theta_2 - \theta_1) = I_0 \cos^2 (\theta_1) \cos^2 (\theta_2 - \theta_1)$$

Notice that the answer is not

$$I_2 \neq I_o \cos^2 (\theta_1) \cos^2 (\theta_2)$$