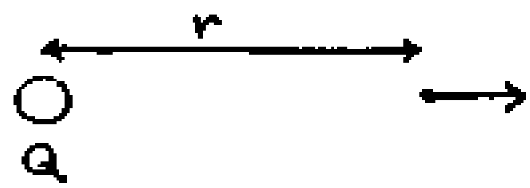
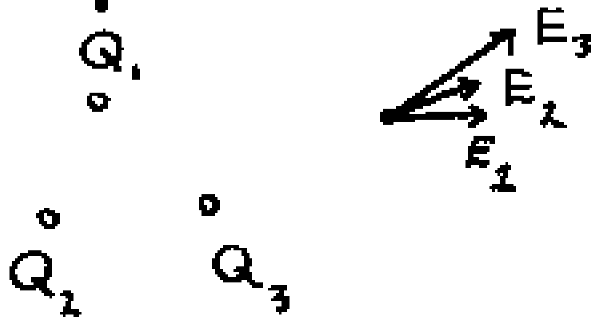
Ch. 26 More about E-fields

E-field due to 1 point charge:



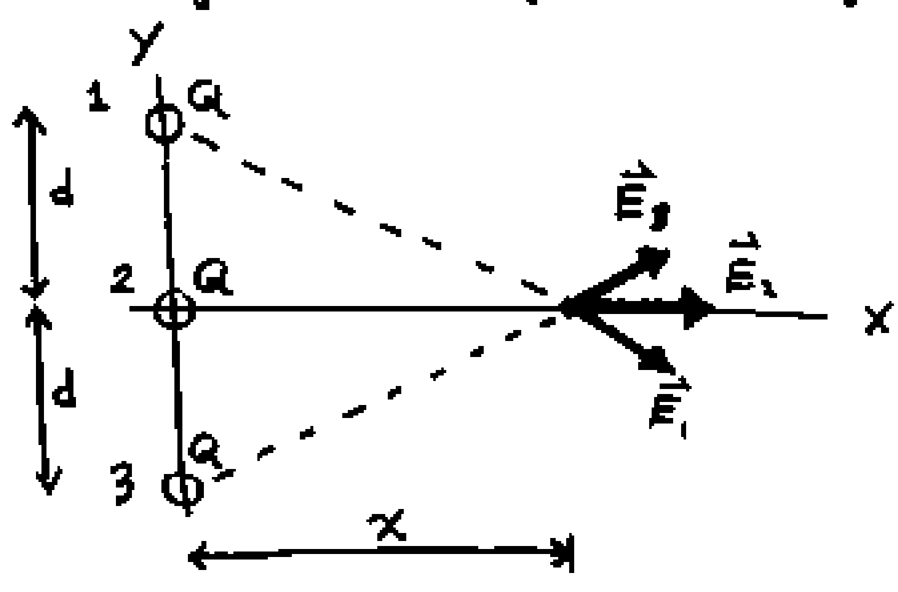
$$\vec{E} = \frac{kQ}{r^2}\,\hat{r} \qquad \left(k = \frac{1}{4\pi\epsilon_0}\right)$$

E-field due to several discrete point charges:



$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots$$

Example: 3 equal charges on y-axis:



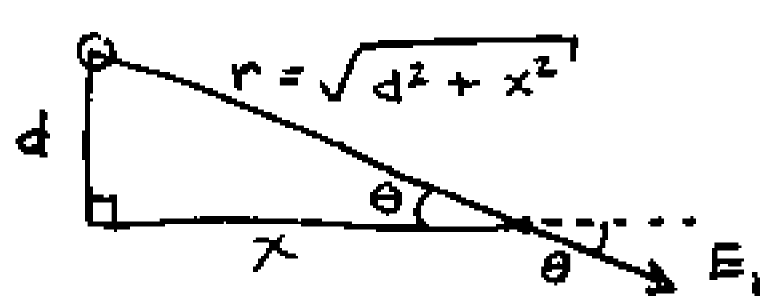
By symmetry,

$$E_{net,y} = 0$$

$$E_{net,x} = E_{1,x} + E_{2,x} + E_{3,x}$$

$$= 2 \cdot E_{1x} + E_{2,x}$$

$$E_{2,x} = E_2 = kQ/x^2 \quad (\text{since } r = x)$$



$$E_1 = \frac{kQ}{r^2} = \frac{kQ}{d^2 + x^2}$$

$$E_{1,x} = E_1 \cos\theta$$

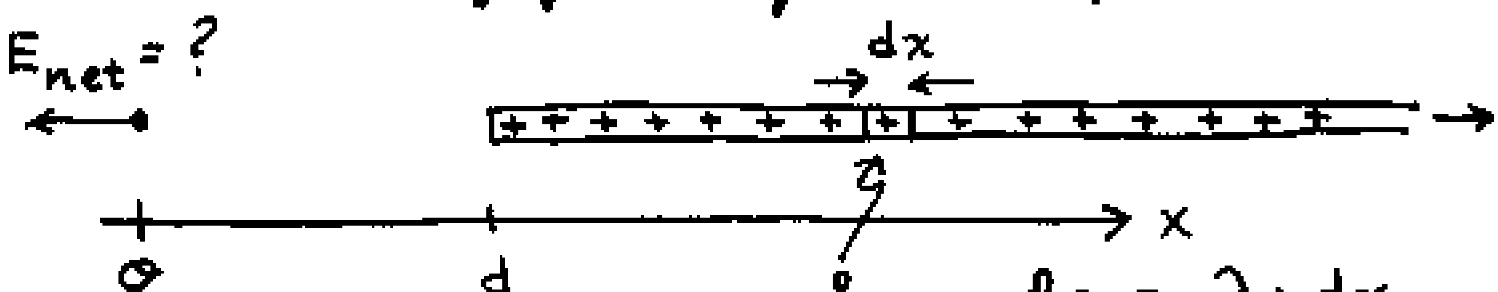
$$E_{1,x} = E_1 \cos\theta = \frac{kQ}{(d^2+x^2)} \cdot \frac{x}{\sqrt{d^2+x^2}} = \frac{kQx}{(d^2+x^2)^{3/2}}$$

$$E_{net} = 2E_{1x} + E_{2x} = \frac{2kQx}{(d^2+x^2)^{3/2}} + \frac{kQ}{x^2}$$

$\vec{E}$ - field due to continuous distribution of charge

$$\vec{E}_{net} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\,\hat{r}$$

Example: semi-infinite line of charge w/ charge per length $= \lambda$, $[\lambda] = C/m$



charge $dq = \lambda \cdot dx$

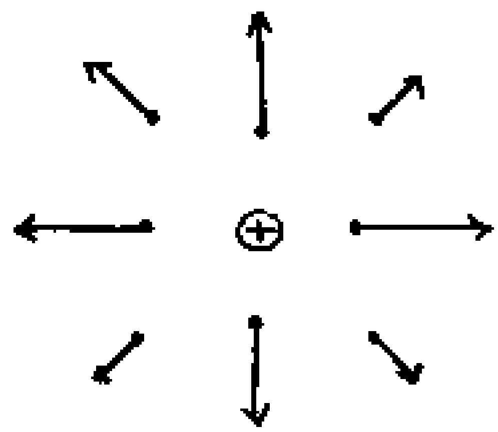
dx = "little bit of x", dq = "little bit of charge"

$$dE = \frac{k\,dq}{r^2} = \frac{k\,\lambda \cdot dx}{x^2}$$ = "little bit of $|\vec{E}|$ due to dq"

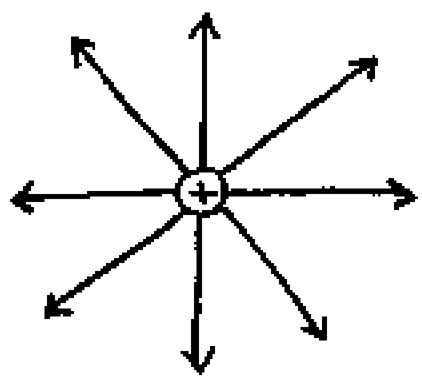
$$E_{net} = \int dE = \int_d^\infty \frac{k\lambda\,dx}{x^2} = k\lambda\left(\frac{-1}{x}\right)\Big|_d^\infty = \frac{k\lambda}{d}$$

(check units! $[\lambda]$ = charge/length)

The E-field



can be represented by drawing ...

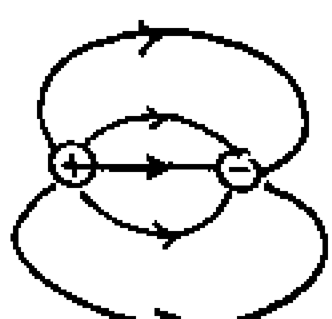


← E-field <u>lines</u>.

Rules for drawing field lines:

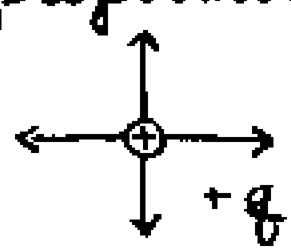
(1) Direction of lines = dir. of $\vec{E}$

(2) Lines begin ~~at~~ on (+) charges (or at ∞) and end on (−) charges (or at ∞)

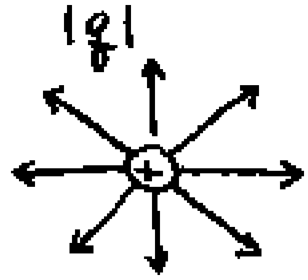


(3) Magnitude of $\vec{E}$ proportional to <u>density</u> of lines (lines closer together $\Rightarrow$ bigger E)

(4) # lines beginning or ending on q is proportional to $|q|$

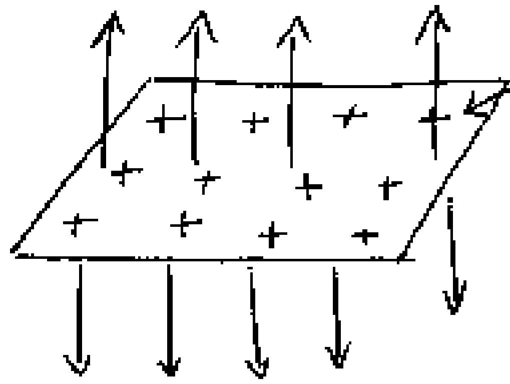


$+q$



$+2q$

E-field due to <u>infinite</u> plane of charge:



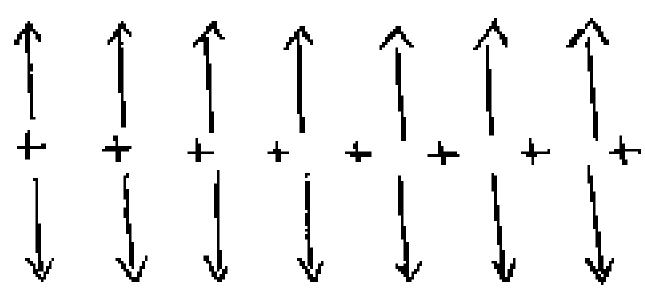
<u>uniform</u> charge per area $= \sigma$ ("sigma")

(text uses η instead)

$\vec{E} \perp$ plane, $E =$ <u>const</u> $= \dfrac{\sigma}{\varepsilon_0}$

(will prove later)

Sideview:



$E = \sigma/\varepsilon_0$

<u>independent</u> of distance from plane

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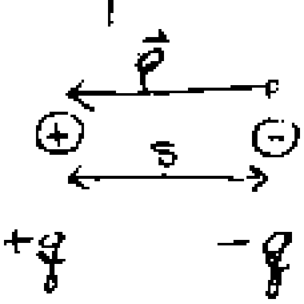
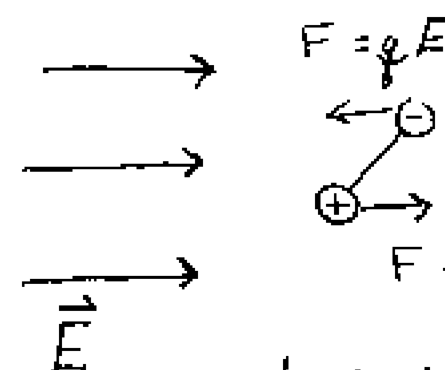
Dipole moment $\vec{p}$ in uniform, const E-field feels $\vec{F}_{net} = 0$, but does feel <u>torque</u> $\vec{\tau}$ which tends to <u>align</u> $\vec{p}$ + $\vec{E}$

dipole moment $\vec{p}$



$|\vec{p}| = q \cdot s$

In const uniform $\vec{E}$-field



torque $\vec{\tau} = \vec{p} \times \vec{E}$