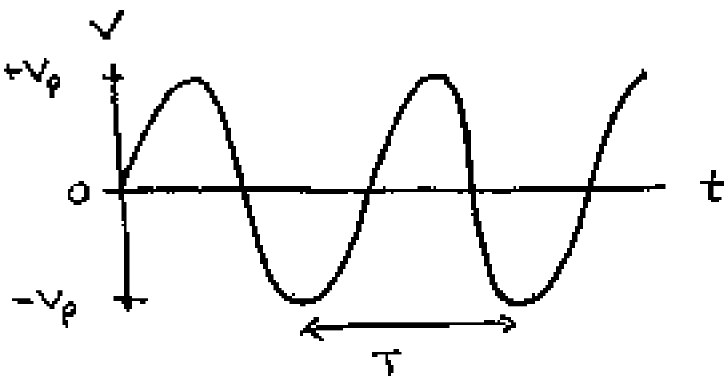
Ch. 33 AC Circuits

The voltage in your wall sockets at home is AC.

AC = alternating-current = sinusoidal in time



$$V(t) = V_p \sin(\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

period $T = \frac{1}{60}\,s$, $f = \frac{1}{T} = 60\,Hz$

$V_{rms} = V_{\text{root-mean-square}} = \sqrt{\overline{V^2}}$ ← bar means time-avg

$$V_{rms} = V_p \sqrt{\overline{\sin^2(\omega t)}} = V_p \cdot \sqrt{\tfrac{1}{2}} = \frac{V_p}{\sqrt{2}}$$

the average of $\sin^2 = \frac{1}{2}$

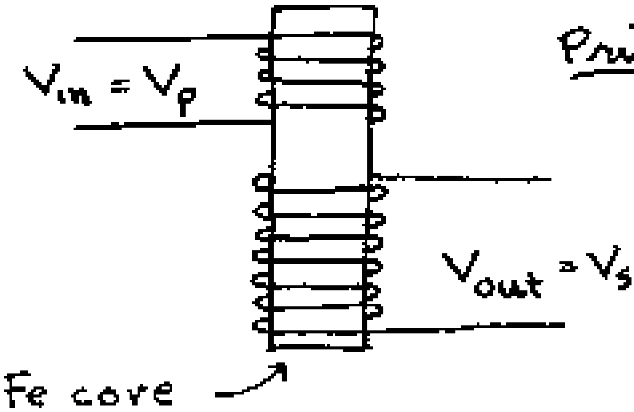
(sin varies from +1 to −1; $\sin^2$ varies from $0 \to 1$)

For wall socket voltage or "line voltage"

$V_{rms} = 120\,V$ $\quad V_p = \sqrt{2}\, V_{rms} \simeq 170\,V$

Transformer – device for transforming AC voltage from 1 value (say 120VAC) to another value (like say 10VAC or 2000VAC)

- made of 2 coils of wire, usually wrapped around iron core; <u>no</u> moving parts.



<u>Primary</u> coil = input
N_p = # turns

<u>Secondary</u> coil = output
N_s turns

Will show $V_{out} = V_s = \frac{N_s}{N_p} \cdot V_p$ or $\boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$

- only works for <u>AC</u> voltage

V_{in} DC $\Rightarrow V_{out} = 0$

Ratio of turns = ratio of voltage

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} > 1 \Rightarrow \text{"step-up transformer"}$$

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} < 1 \Rightarrow \text{"step-down transformer"}$$

(Step-down transformer gives less V, but more I)

Transformers work because of Faraday's Law:

$$(1) \quad V_s = N_s \frac{d\Phi}{dt}$$

$$(2) \quad V_p = N_p \frac{d\Phi}{dt}$$

$\Phi = B \cdot A$ = flux thru each turn is <u>same</u> in primary + secondary because iron core "guides flux" from S to P.

$$(1) \div (2) \Rightarrow \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$V_p(AC) \Rightarrow I_p(AC) \Rightarrow B_p(AC) \Rightarrow B_s(AC) \Rightarrow \mathcal{E} = V_s$$

If transformer is well-designed, only 1-5% of power is lost to heating of coils + eddy currents in Fe core

$$\Rightarrow P_{out} \simeq P_{in} \Rightarrow V_s I_s = V_p I_p$$

$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

Step-down transformer ⇒ less voltage but more I
⇒ same $P = V \cdot I$

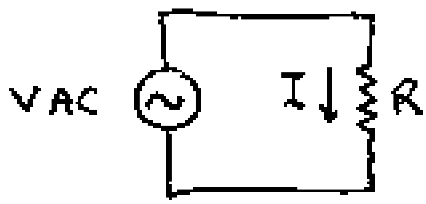
Light bulbs and appliances w/ motors (vacuum cleaners, blenders) use AC voltage to operate. But devices w/ electronic circuits (TV's, computers, phones, etc.) need DC (constant) voltage to function. The "power supply" in computers, TV's converts AC voltage from the wall into DC volts (usually ≈ 10-15V) that the circuitry needs.

The term "AC voltage" means $V_{rms} = \frac{V_p}{\sqrt{2}}$

Likewise, "AC current" means $I_{rms} = I_p/\sqrt{2}$.

rms average values of AC voltage, current are always used because all the old DC formulas involving V, I, R, P (power) work OK w/ <u>rms</u> <u>values</u>.

Example: AC voltage across a resistor

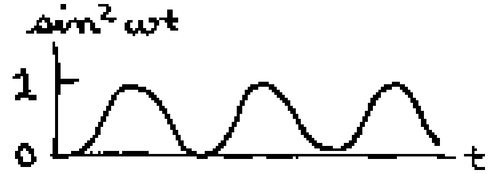


$V = V_p \sin \omega t$

$I = \frac{V}{R} = \frac{V_p}{R} \sin \omega t = I_p \sin \omega t$

Power $P = IV = I_p V_p \sin^2 \omega t$

$\sin^2 \omega t$ varies from $0 \rightarrow 1$



<u>instantaneous</u> power is rapidly alternating

between $P_{min} = 0$ and $P_{max} = I_p V_p$,

but the <u>average</u> power $P_{avg} = I_p V_p \underbrace{\overline{\sin^2 \omega t}}_{1/2}$ ← bar means avg

$$P_{avg} = \frac{I_p}{\sqrt{2}} \cdot \frac{V_p}{\sqrt{2}} = I_{rms} V_{rms}$$

$\underbrace{P = IV}_{DC}$ vs. $\underbrace{P_{avg} = I_{rms} V_{rms}}_{AC}$ same formula!

$P = IV = I^2 R = \frac{V^2}{R}$ all these formula are true for AC (as well as DC) if you use I_{rms}, V_{rms}, and P_{avg}.

AC voltage is used to distribute power to homes (rather than DC voltage) because it's easy to transform AC voltages using transformers.

Electrical power is transmitted from power plant to city w/ big aluminum power cables. Much energy is wasted because cables heat up (I^2R losses). To reduce waste, power is transmitted at <u>very</u> <u>high</u> <u>voltage</u> (100 kV → 1 MV)

$P_{\text{from plant to city}} = IV =$ <u>constant</u> (set by needs of city, capability of plant)

Bigger V ⟺ smaller I ⇒ less I^2R loss in cable.

Example: say, P_0 = power output of plant $= 10^8$ W (100 MW)

$$R_{cable} = 10\,\Omega$$

$P_0 = IV$, $\quad I = \frac{P_0}{V}$

$$P_{lost} = I^2 R_{cable} = \frac{P_0^2}{V^2} \cdot R_{cable}$$

Fraction of power from plant wasted =

$$\frac{P_{lost}}{P_0} = \frac{P_0}{V^2} \cdot R_{cable}$$

$V = 50,000\ V$: $\frac{P_{lost}}{P_0} = \frac{10^8}{(5\times10^4)^2} \cdot 10 = 0.4 = 40\%$!

$V = 200,000\ V$: $\frac{P_{lost}}{P_0} = \frac{10^8}{(2\times10^5)^2} \cdot 10 = 0.025 = 2.5\%$

(acceptible)

Transformers used to step down voltage in stages so that it is 240 VAC or 120 VAC by the time it enters your house.

Difficult and expensive to transform DC voltages, hence power is generated + distributed as AC.