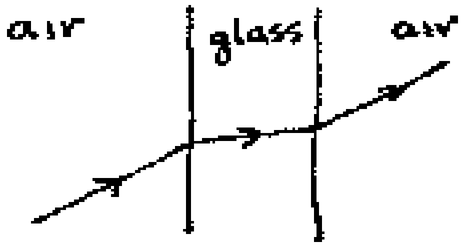
Image Formation

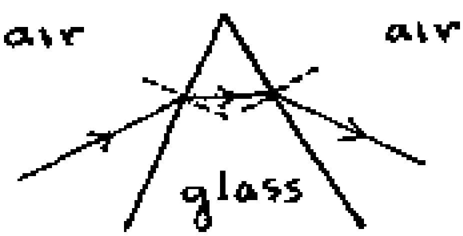
Images can be formed w/ lens or mirrors. The text starts w/ mirrors; we'll start w/ lenses.

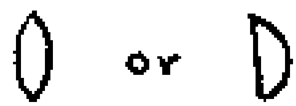
Key ideas in lens design:

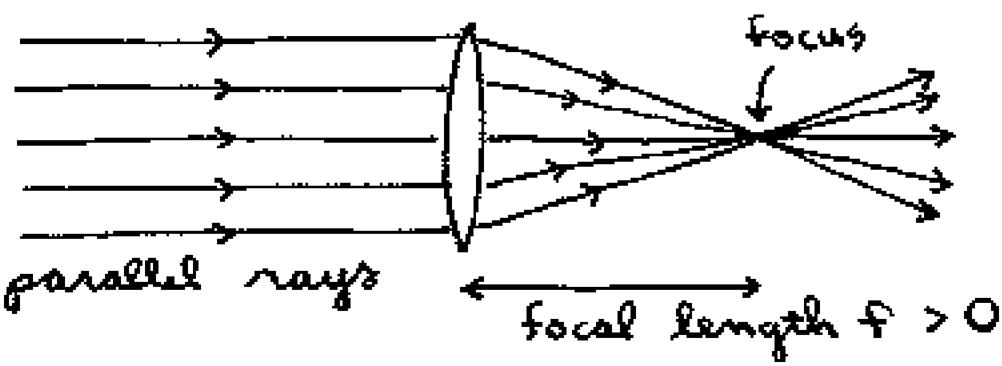
1) For parallel surfaces, ray comes out in <u>same</u> direction:



2) For non-parallel surfaces (like prism), ray is bent toward thicker end:

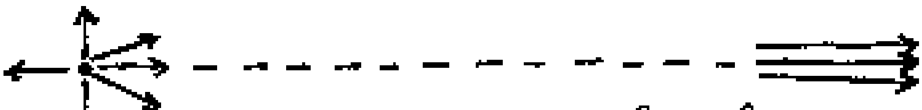


Convex lens = converging lens 

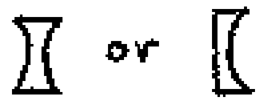


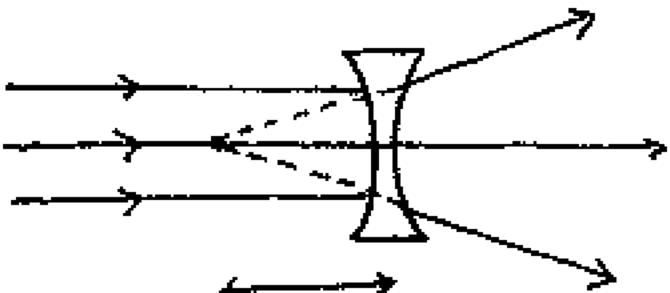
Center ray not bent because surfaces parallel, edge rays are bent toward thicker part.

Parallel rays are produced by distant point source



$\Rightarrow$ light rays from a star are parallel.

Concave lens = diverging lens 



Can form images of distant objects on screen w/ a converging lens:

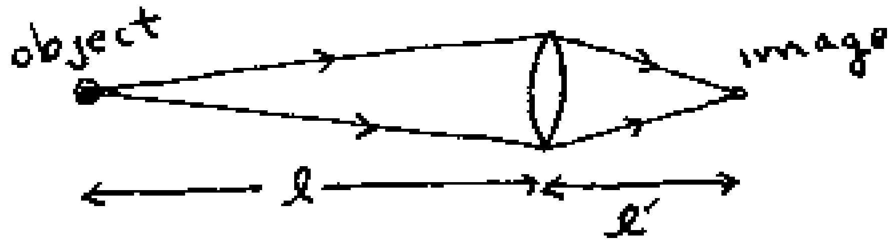
If screen placed at focal plane of lens, will see 2 points of light on screen, the images of stars ① and ②

Real image = image formed by actual convergence of light rays at location of image. Real images can be viewed w/ a screen.

Compare w/ virtual images - which cannot be viewed w/ screen

Converging lens can also form real image of nearby object (if not too close)

ℓ = distance to object, ℓ' = image distance



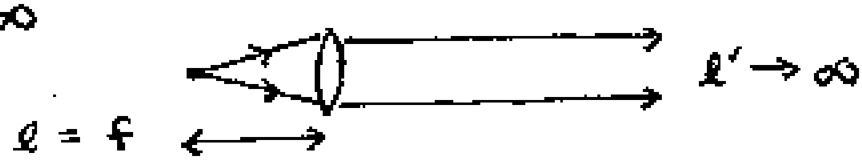
ℓ, ℓ', and focal length f are related by

Lens Equation: $$\boxed{\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}}$$

Note: $\ell = \infty \iff \ell' = f \quad \left(\underbrace{\frac{1}{\infty}}_{0} + \frac{1}{\ell'} = \frac{1}{f} \Rightarrow f = \ell' \right)$

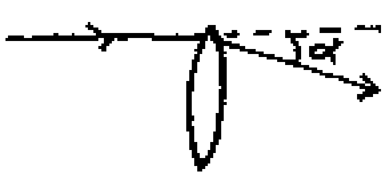
$\ell = 2f \iff \ell' = 2f \quad \left(\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f} \checkmark \right)$

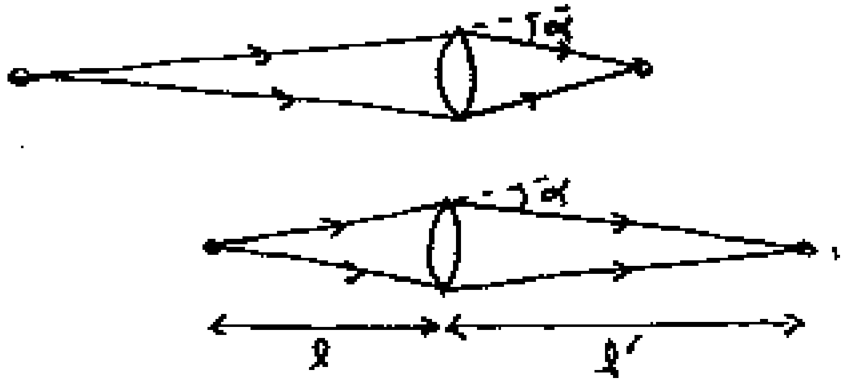
$\ell = f \iff \ell' = \infty$



In general, as object distance ℓ decreases, image distance ℓ' increases.

Why? Converging lens has only so much "bending power". Rays at edge of lens bent by same $\angle\alpha$ regardless of $\angle$ of incoming ray



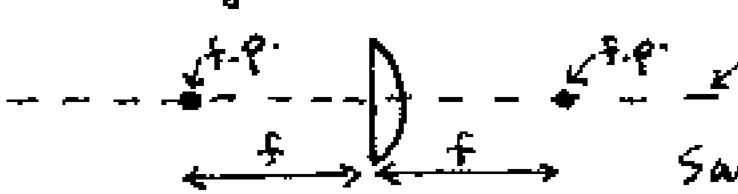


Same ∠α, always

$\Rightarrow l \downarrow, l' \nearrow$

Pt object ⇔ pt image , extended object ⇔ extended image

lens have 2 focal points corresponding to parallel rays on other side

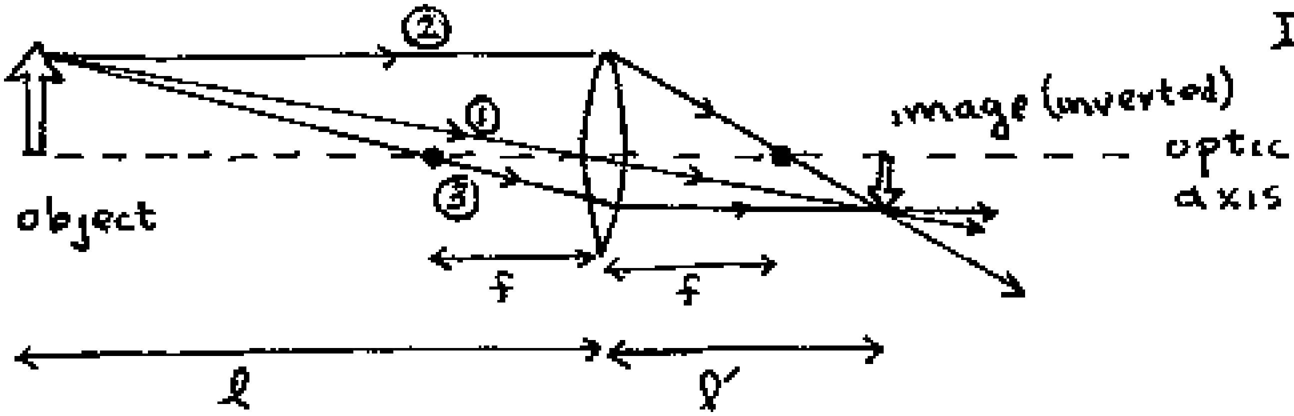


Same f's on both sides.

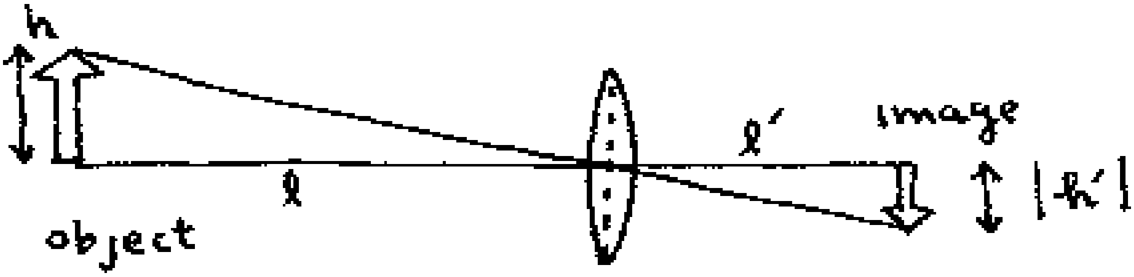
Same f on both sides, even if lens unsymmetric because rays at edge bent same ∠α regardless of ∠ or direction of incoming ray:

Ray Diagrams use 3 rays to get position of image

1. Ray thru lens center is undeviated.
2. Ray ∥ optic axis passes thru focus on far side.
3. Ray thru focus on near side is ∥ optic axis on far side.



Actually only need 2 rays to locate image, 3rd ray is a check



h = object height , $|h'|$ = image height

$h' < 0$, if image <u>inverted</u>.

similar triangles

Lateral magnification $M = \left|\frac{h'}{h}\right| = \frac{l'}{l}$

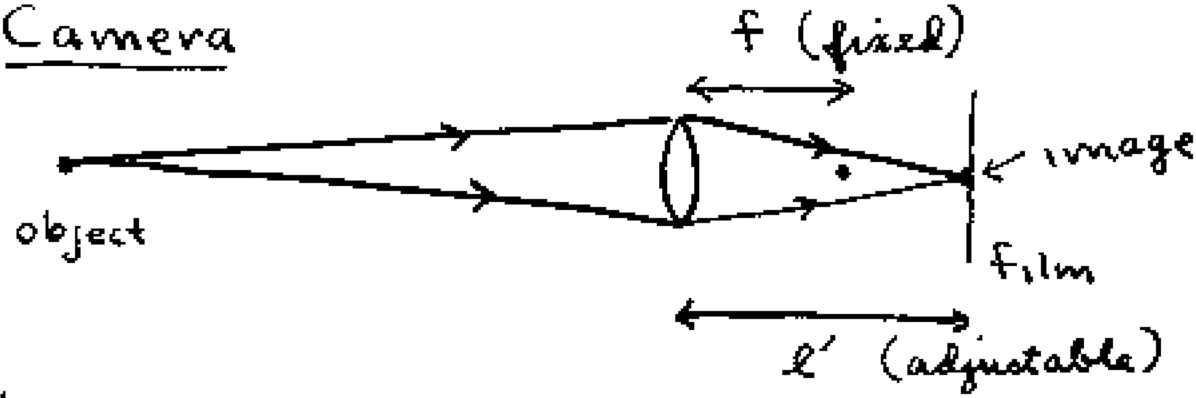
Note: Text defines $M = \frac{h'}{h} = -\frac{l'}{l} \Rightarrow M$ <u>negative</u> if image inverted

But most book define M positive, always.

In a camera, focal length f is fixed. l' = lens/film distance is <u>adjusted</u> as l = object/lens distance varies, in order to maintain focus

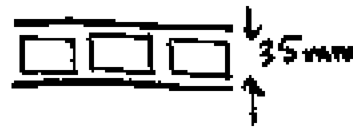
In human eye, "film" is retina. l' = lens/retina distance is <u>fixed</u>. As l = object distance varies, f = focal length of lens is adjusted to keep focus

Camera

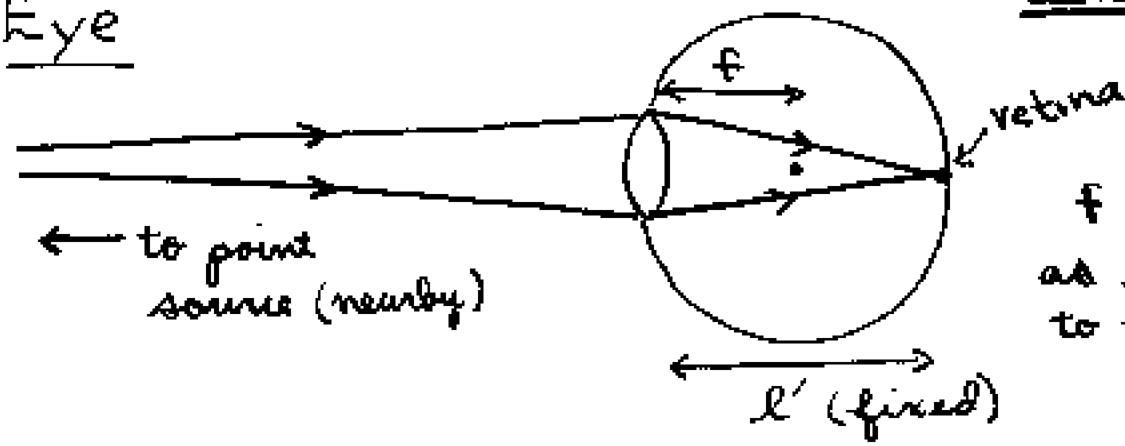


"50 mm lens" mean f = 50 mm

"35 mm camera" means width of film is 35 mm



Eye

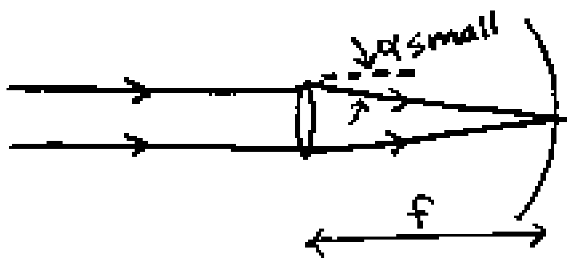


f changes as l changes to keep l′ fixed.

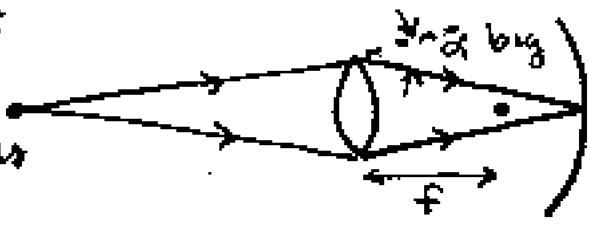
thinner lens bends rays less ⇒ longer f

thicker lens bends rays more ⇒ shorter f

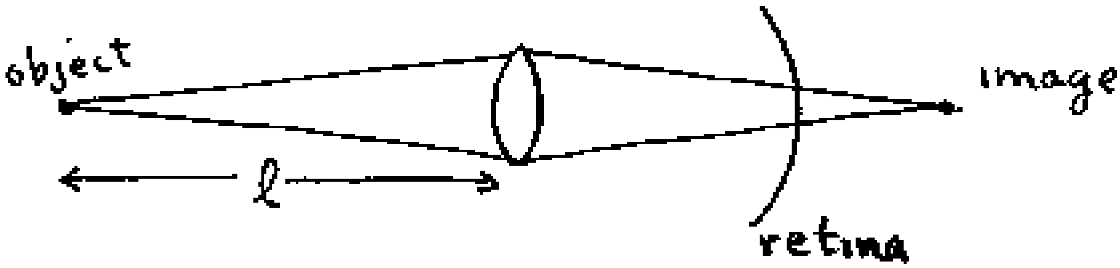
object at $l = \infty$
eye "relaxed",
lens thin, $l' = f$



object near, rays must
bend more to focus
on retina ⇒ fatter lens



If object too close, cannot make lens fat enough (f ~~short~~ short enuf) to focus, see blurry blob, not sharp point.



"Near point" N = minimum l for sharp focus

N = 25 cm for healthy, young people.

Sign conventions for lens equation

$$\frac{1}{l} + \frac{1}{l'} = \frac{1}{f}$$

1. Converging lens or mirror $f > 0$; diverging $f < 0$

~~[illegible]~~ Assume rays travel → (left to right)

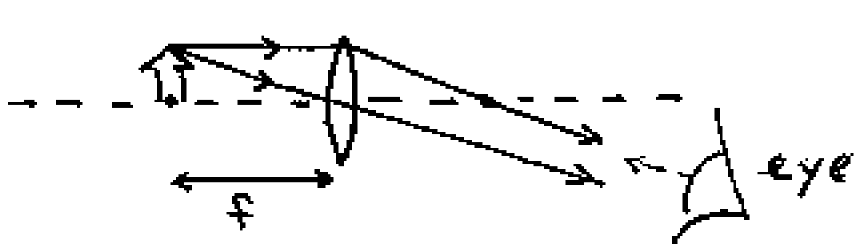
2. l (+) if object on left (almost always)
   l (−) if object on right

3. l' (+) if image on right; l' (−) if image on left (virtual)

(2, 3 reversed for mirrors!)

4) image height h' (+) if image erect,
   (−) if image inverted.

Magnifying Glass. Place object at $\ell = f$



parallel rays enter eye $\Rightarrow$ image at ∞

$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}, \quad \frac{1}{\ell'} = \frac{1}{f} - \frac{1}{\ell} = \frac{\ell - f}{f\ell}, \quad \ell' = \frac{f\ell}{\ell - f}$$

$\ell = f \Rightarrow \ell' = \pm\infty$ Can put object very close to eye: Magnifyingglass puts image at $\infty \Rightarrow$ can see w/ relaxed eye.

For quantitative problems, need only 2 eq'ns:

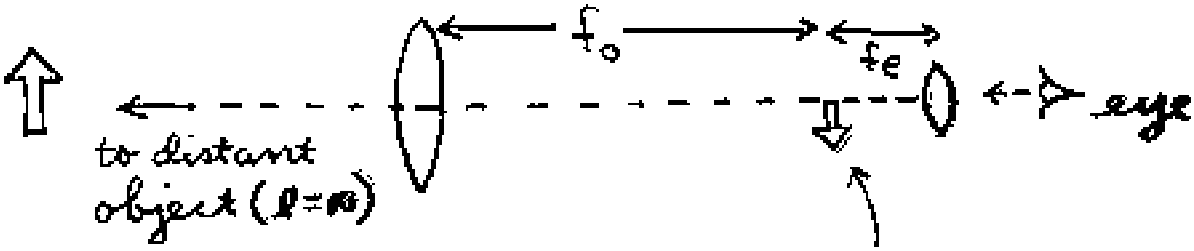
$$\frac{1}{\ell} + \frac{1}{\ell'} = \frac{1}{f}, \quad \frac{h'}{h} = -\frac{\ell'}{\ell}$$

f (+) converging
(−) diverging

ℓ' (+) real
(−) virtual

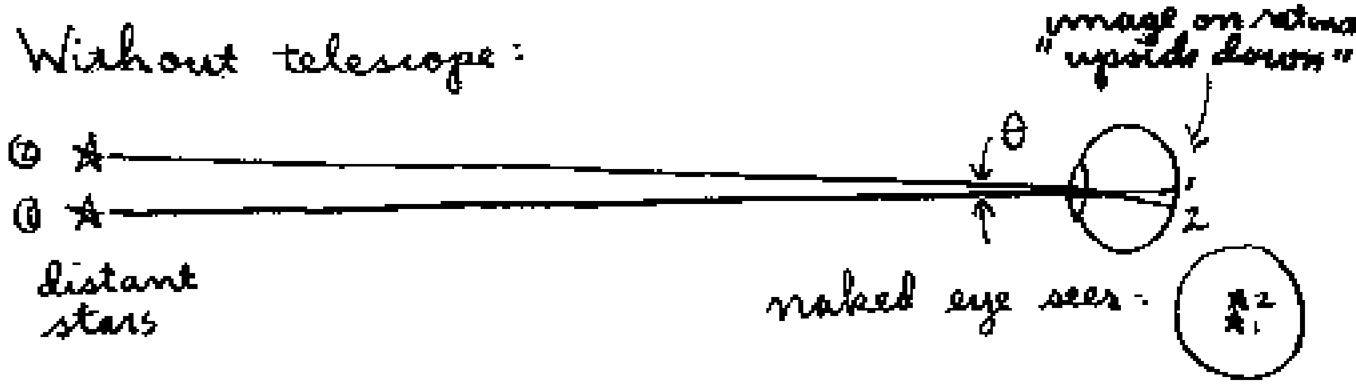
Telescopes are reflecting (uses mirror) or refracting (uses lens)

Refracting, astronomical (Keplerian) telescope uses 2 converging lens

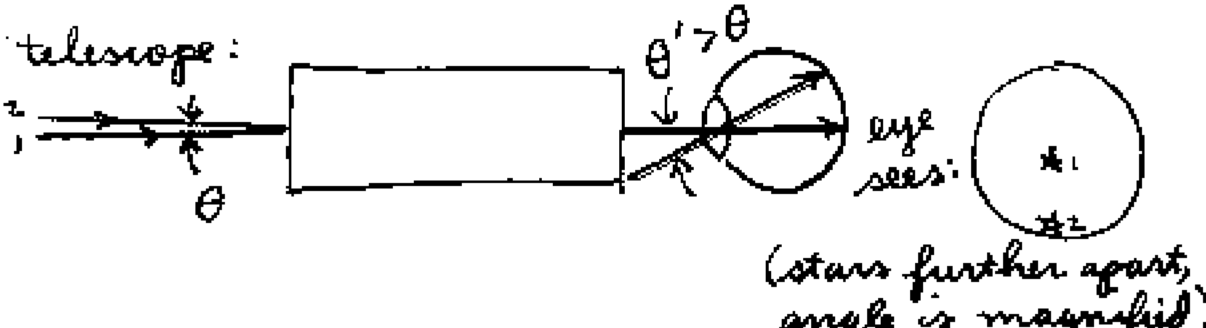


Objective lens forms real image at $\ell' = f_o$. This "intermediate image" is the object for the eyepiece, which is magnifying glass. Eye sees virtual image at ∞. Final image is inverted

Without telescope:



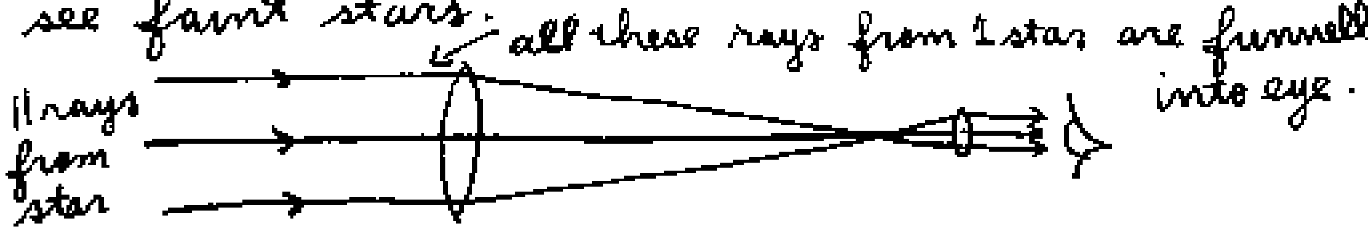
With telescope:



Angular Magnification:

$$m \equiv \frac{\theta'}{\theta} \quad , \quad \frac{\theta'}{\theta} = \frac{f_o}{f_e} \quad \text{(will prove)}$$

Telescopes not only magnify, they gather more light than eye to make image brighter; can see faint stars.



Maximum usable magnification of a telescopes set by 1) atmospheric blurring
2) "diffraction effects" (ch. 37)

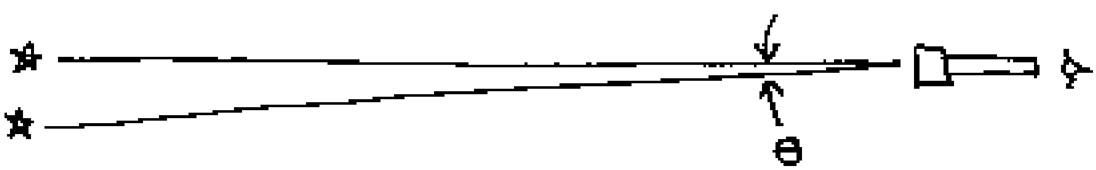
Ray diagrams of geometrical optics indiates that a point source at ∞ will be imaged at a perfect point. But light is a wave, not a ray. Because of wave effects, image is a small patch of light, not a point

Rayleigh Criterion: the minimum angle resolvable by a telescope is

$$\theta_{min} \simeq 1.2 \frac{\lambda}{D}$$

D = diameter of objective lens or mirror

λ = wavelength of light (center of visible $\Rightarrow$ yellow light, $\lambda = 550\,nm$)



Viewer sees:

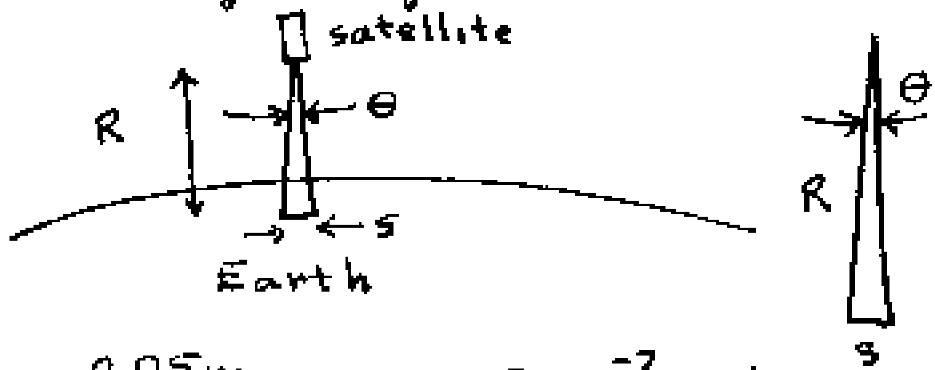
$\theta > \theta_{min}$ stars resolved

$\theta < \theta_{min}$ stars not resolved

Can Spy Satellites read license plates?

R = altitude of spy satellite (<u>lowest</u> orbit)
$\approx$ 200 km

s = feature size on plates $\approx$ 5 cm



$$\theta = \frac{s}{R} = \frac{0.05\,\text{m}}{200 \times 10^3\,\text{m}} = 2.5 \times 10^{-7}\ \text{rad}$$

$= 0.05$ "arc seconds" $(3600'' = 1^\circ)$

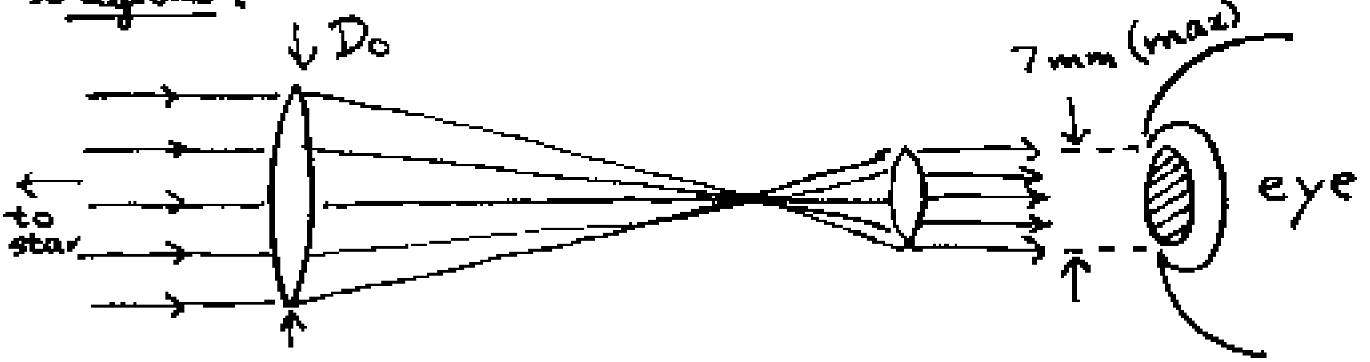
$$\theta = 1.22\,\frac{\lambda}{D} \approx \frac{\lambda}{D}$$

$$D = \frac{\lambda}{\theta} = \frac{550 \times 10^{-9}\,\text{m}}{2.5 \times 10^{-7}} = 2.2\,\text{m}\ !$$

Hubble Space Telescope has D = 2.4 m

Yes, it's possible, but <u>just</u> barely.

Telescope not only magnifies; objective lens collects more light than eye => objects appear <u>brighter</u>



Human eye pupil diameter = 2 mm (daylight)
= 7 mm (night)

Apparent brightness increased by ratio $\dfrac{A_{objective}}{A_{pupil}}$

Area $A = \pi R^2 = \dfrac{\pi}{4} D^2$

$\Rightarrow$

$\dfrac{A_o}{A_p} = \dfrac{D_o^2}{D_p^2}$, $D_p = 0.007\,m$ (fixed)

D_o	brightness ratio
60 mm	73 $(= 60^2/7^2)$
18" = 457 mm	4300
5 m (Hale Telescope) = 200"	510,000
10 m (Keck, largest)	2×10^6

In principle, angular resolution set by diffraction effects. In practice, resolution set by atmospheric blurring at

≈ 1 arc sec $= 1'' = \dfrac{1}{3600}$ deg

Moon's diameter $\cong \frac{1}{2}^{\circ} = 1800''$
planet Jupiter $\simeq 50''$

200" Hale Telescope has same resolution as amateur's 10" on most nights. Hubble Space Telescope ($D_o = 2.4\,m$) has $0.05''$ resolution (set by diffraction effects, not atmosphere.)