Magnetism! A New Force.

So far, we have studied two forces: gravity and electricity

- Gravity is made by mass + gravity acts on masses
- $\vec{E}$-fields are made by charges ($\vec{E} = \frac{KQ}{r^2}\hat{r}$)

and $\vec{E}$-fields exert forces on charges ($\vec{F}_E = q\vec{E}$)

- There is a different kind of field, called the magnetic field, or $\vec{B}$-field

$\vec{B}$-fields are created by moving charges (currents) and a $\vec{B}$ field exerts a force on moving charges

Remember: current I is to $\vec{B}$ as charge q is to $\vec{E}$

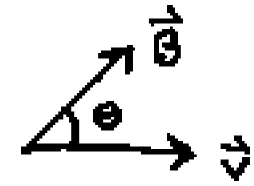
We will see later exactly how $\vec{B}$-fields are made by currents. Now, we will study how $\vec{B}$-fields exert forces on moving charges (currents)

The magnetic force $\vec{F}_B$ exerted on a charge q moving w/ velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\boxed{\vec{F}_B = q\,\vec{v} \times \vec{B}}$$

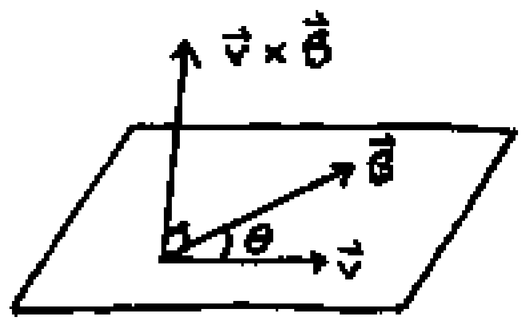
This eq'n is the definition of $\vec{B}$, analogous to eq'n $\vec{F}_E = q\vec{E}$, which defines $\vec{E}$

Magnitude $|\vec{F}_B| = F_B = |q|\, v\, B \sin\theta$



Direction of $\vec{F}_B$ is $\perp$ $\vec{v}, \vec{B}$

"right-hand rule"

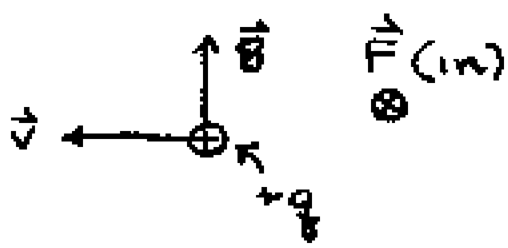


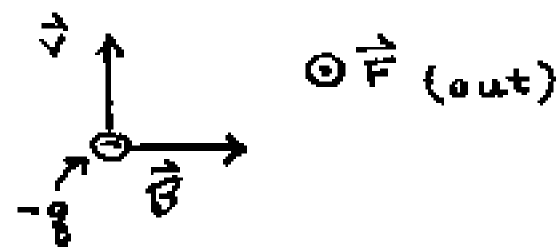
Vector $(\vec{v}\times\vec{B}) \perp \vec{v}$ and $\vec{B}$

Point fingers of right hand in dir. of $\vec{v}$, curl toward $\vec{B}$, thumb points along $\vec{v}\times\vec{B}$.

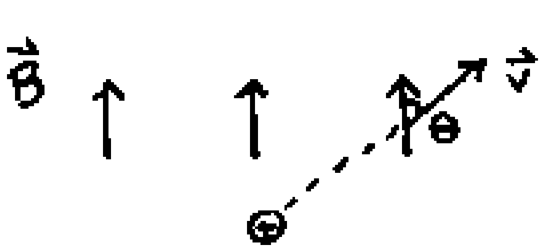
If q is $(+)$, $\vec{F}_B = q\,\vec{v}\times\vec{B}$ same direction as $(\vec{v}\times\vec{B})$

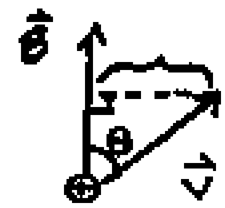
If q is $(-)$, $\vec{F}_B$ opposite dir. of $(\vec{v}\times\vec{B})$





$$|\vec{F}_B| = F_B = |q|\,v\,B\sin\theta = q\,v_\perp B$$





$v_\perp = v\sin\theta$ = component of $\vec{v} \perp$ to $\vec{B}$

$\vec{v} \parallel \vec{B} \Rightarrow \sin\theta = 0 \Rightarrow F_B = 0$ (!)

$\vec{v} \perp \vec{B} \Rightarrow \sin\theta = 1 \Rightarrow F_B = |q|\,v\,B$

$\vec{v} = 0 \Rightarrow F_B = 0$ (Very different from $\vec{F}_E$ or gravity)

Units of B = $[B] = \dfrac{[F]}{[q]\,[v]} = \dfrac{N}{C\cdot m/s}$ = tesla (T)

Earth's magnetic field $\simeq 5\times10^{-5}$ T $= 0.5$ gauss

1 gauss $= 10^{-4}$ T ~old (non SI) unit of B

$1\ T = 10^4$ gauss = a large field

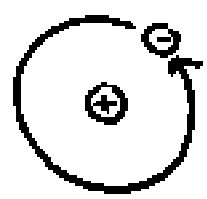
kitchen magnet : 50 - 500 gauss = 0.005 - 0.05 T

iron-core electromagnet : 2T (max)

superconducting magnet : 20 T (max)

Currents make $\vec{B}$-fields, but where's the current in a permanent magnet?
Answer: electron orbiting nucleus is a moving charge, a current, an "atomic current"

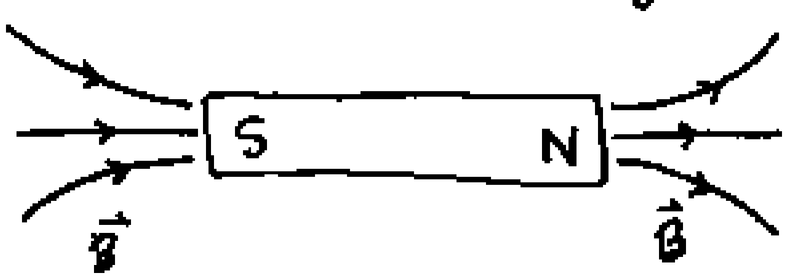
Atom:



In most materials, atomic currents of different atoms have random orientations ⇒

no net current, no B-field

In ferromagnetic materials (Fe, Ni, Cr, some alloys containing these) the atomic currents all orient same way, make a net current.



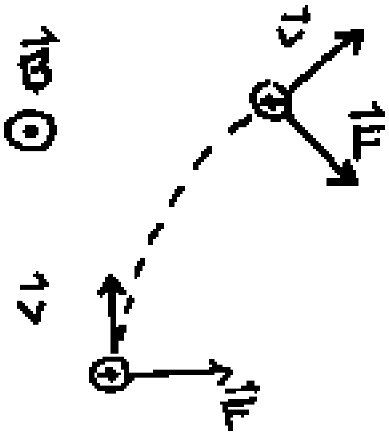
In permanent magnets, always have 2 "poles". B-field points out of north pole, into south pole.

*

Because $\vec{F}_B \perp \vec{v}$ always ⇒ $\vec{F}_B$ can do no work on q

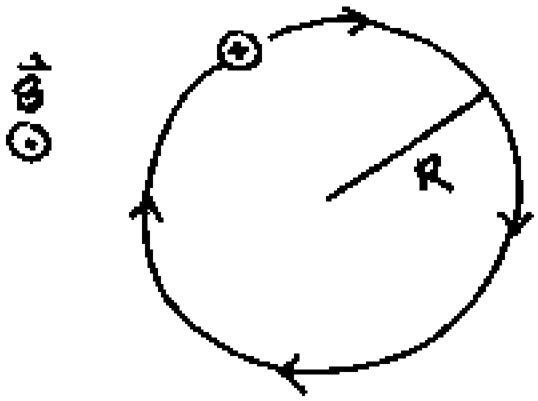
$W_F = \vec{F} \cdot \Delta\vec{r}$ Since $\vec{F}_B \perp \Delta\vec{r} \Rightarrow W = 0$

⇒ $\vec{B}$-field cannot change KE of moving particle (recall $W_{net} = \Delta KE$)



Work done = 0
since $\vec{F} \cdot \Delta\vec{r} = 0$

$\Rightarrow$ moves in a circle w/ constant speed v



$F = ma \Rightarrow q v B = m \frac{v^2}{R}$

$q B = \frac{m v}{R}$

$$\boxed{R = \frac{m v}{q B}}$$

$\Leftarrow$ don't memorize
know how to derive

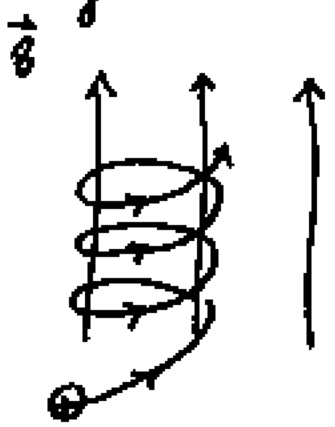
Can solve for frequency f = # revs/sec :

$v = q \frac{R B}{m} = \frac{\text{distance}}{\text{time}} = \frac{2\pi R}{T}$ $\Leftarrow$ period

$$\boxed{f = \frac{1}{T} = \frac{q B}{2\pi m}}$$

= "cyclotron frequency"
independent of R !

(As v grows, R grows in proportion, so time to go around once stays constant.)

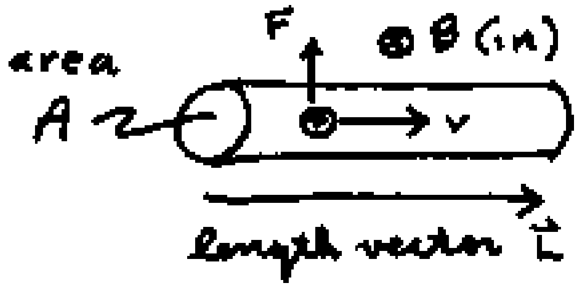


No force along $\vec{B}$ $\Rightarrow$

spiral motion in direction of $\vec{B}$.

Charged particles (protons) from the sun (solar wind) are guided along earth's B-field to artic regions, slam into upper atmosphere
$\Rightarrow$ "northern lights"

Force on a current carrying wire



$$\vec{F}_{\text{on single charge}} = q \vec{v} \times \vec{B}$$

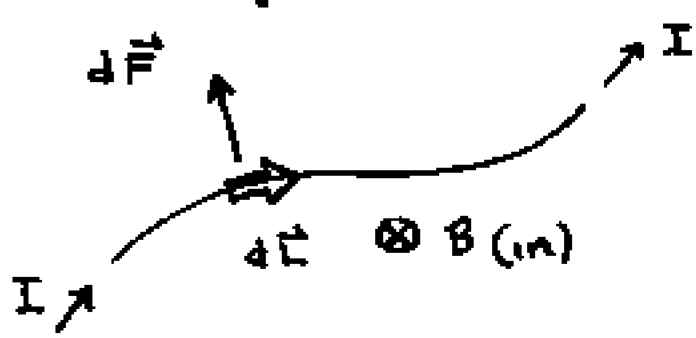
charges in this segment of wire $= N = \underbrace{n}_{\#/\text{vol.}} \cdot \underbrace{A \cdot L}_{\text{vol.}}$

$\Rightarrow$ total force on segment of wire

$$= \vec{F}_{tot} = n A L \,(q \vec{v} \times \vec{B}) = I \vec{L} \times \vec{B}$$

$$\left(\text{since } J = \frac{I}{A} = n q v \Rightarrow I = n q v A\right)$$

If wire not straight or $\vec{B}$ not uniform, break wire up into little segments $d\vec{L}$

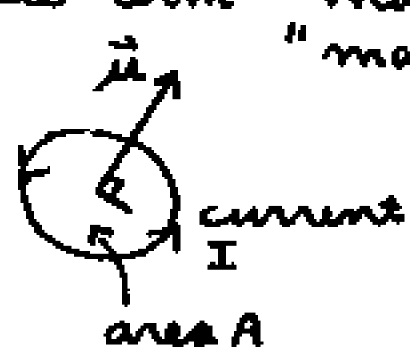


$$\boxed{d\vec{F} = I \, d\vec{L} \times \vec{B}}$$

$$\vec{F}_{tot} = \int d\vec{F} = \int I \, d\vec{L} \times \vec{B}$$

Force on a current loop

New term: magnetic dipole moment or "magnetic moment" = loop of current

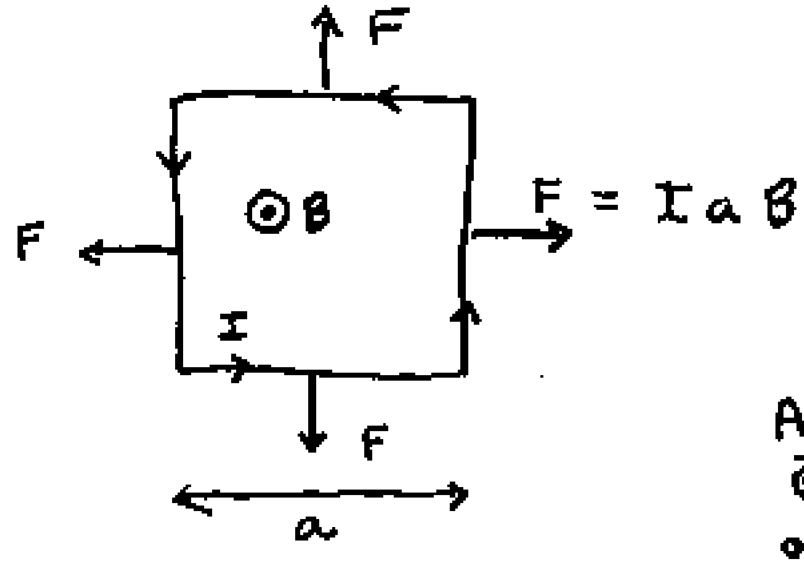


$$\boxed{\vec{\mu} = I \vec{A}}$$

magnetic moment μ = current × area

direction of $\vec{\mu}$ from right-hand rule

Spse loop $\perp \vec{B}$, $\vec{\mu} \parallel \vec{B}$

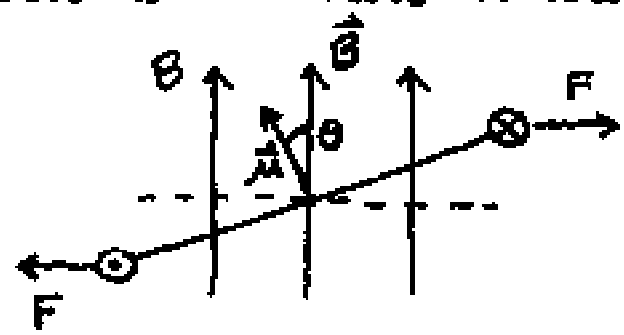


$\vec{F}_{net} = 0$

when $\vec{\mu} \parallel \vec{B}$

Actually, $F_{net} = 0$ if $\vec{B}$ = const, regardless of direction of $\vec{\mu}$

But, if $\vec{\mu}$ not parallel to $\vec{B}$, then there is a torque, tending to twist loop so that $\vec{\mu}$ aligns with $\vec{B}$. Can show $\vec{\tau} = I\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$



← Side view of loop

Note torque ($\vec{\tau} = \vec{r} \times \vec{F}$) tending to twist loop so $\vec{\mu}$ aligns w/ $\vec{B}$

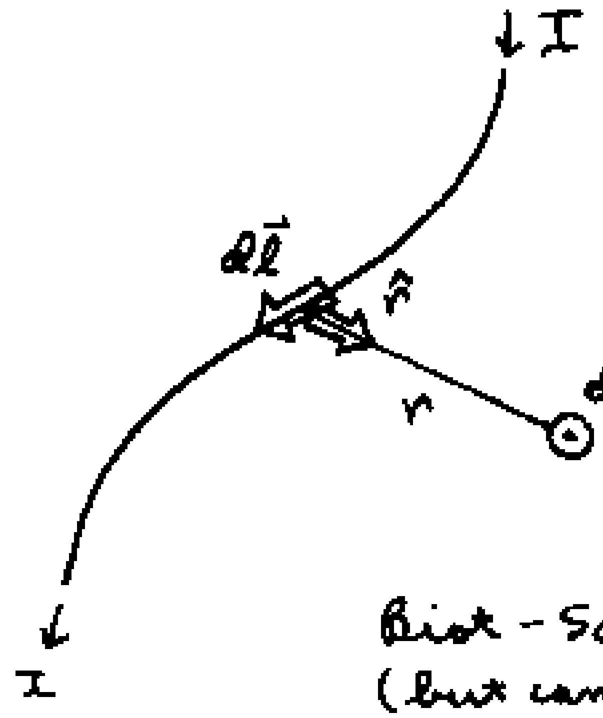
$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$

← principle of operation of galvanometer = device which measures current.

So far, have assumed existence of $\vec{B}$ and described force on moving charge due to $\vec{B}$.

Now, will show how to make $\vec{B}$ with current I.

Biot-Savart Law



$$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}}$$

μ_0 = const = $4\pi \times 10^{-7}$ (SI units)

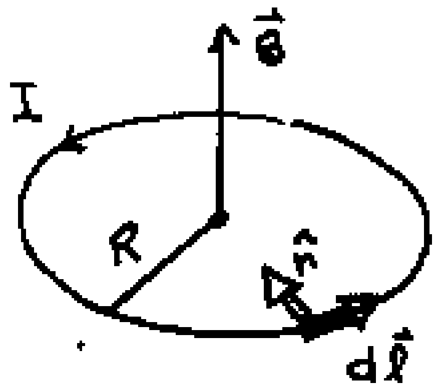
$d\vec{B}$ = field due to element of current $I\, d\vec{l}$

Biot-Savart first discovered experimentally (but can be derived from Maxwell's Eqns)

Biot-Savart similar to $d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r}$

$$\vec{B}_{tot} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}$$ ~ can be very messy integral!

Example: $\vec{B}$ at center of circular loop of current I, radius R



$|d\vec{l} \times \hat{r}| = dl\, |\hat{r}| \sin 90° = dl$

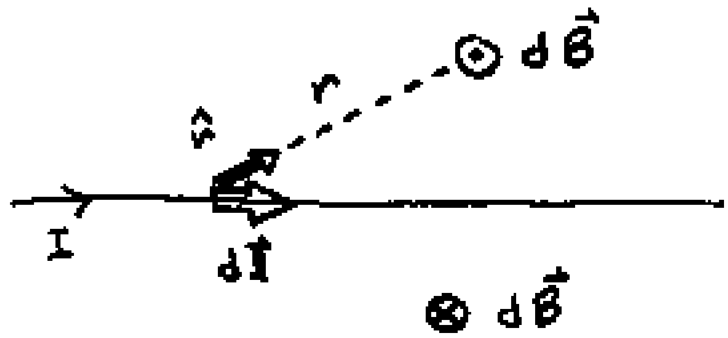
$$\Rightarrow dB = \frac{\mu_0 I}{4\pi} \frac{dl}{R^2}$$

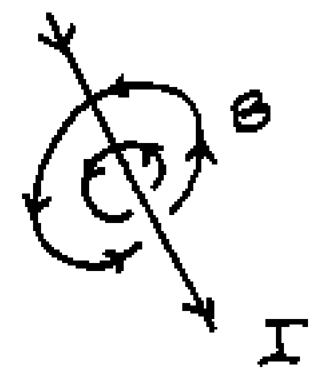
$$B = \int dB = \frac{\mu_0 I}{4\pi R^2} \underbrace{\int dl}_{2\pi R} = \frac{\mu_0 I}{2R}$$

(Able to replace vector integral $\int d\vec{B}$ w/ scalar integral $\int dB$ because all $d\vec{B}$'s are in the same direction.)

Another, more difficult, example:

$\vec{B}$ due to long, straight wire w/ current I.

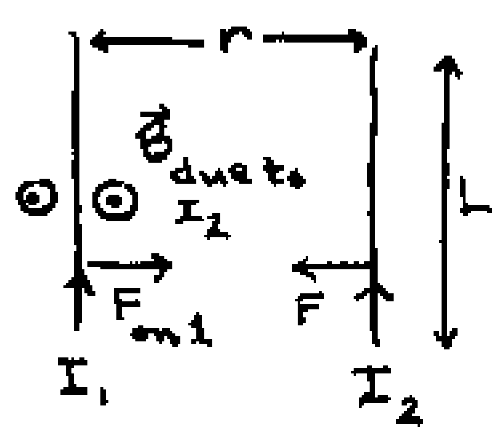




Another "right hand rule":
curl fingers along $\vec{B}$, thumb pts along I

Result of messy integration: $\boxed{B(r) = \dfrac{\mu_0 I}{2\pi r}}$

Force between two current-carrying wires:



$\vec{F}_{\text{on } 1} = I_1 \vec{L} \times \vec{B}_2$
due to $\vec{B}$ from 2

$F_{\substack{\text{on } 1 \\ \text{from } 2}} = I_1 L B_2 = I_1 L \dfrac{\mu_0 I_2}{2\pi r}$

force per length between wires $= \dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$

- parallel currents attract
- anti-parallel currents repel

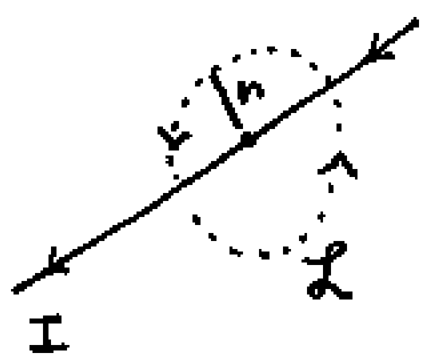
$\boxed{\text{Ampere's Law}}$ For <u>any</u> <u>closed</u> loop $\mathcal{L}$

$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \mu_0 I_{thru} \quad , \quad I_{thru} = \text{current thru loop } \mathcal{L}$$

(Turns out Ampere's Law true only for <u>constant</u> I. If $I \neq$ const, requires modification.)

Ampere's Law for steady currents, like Gauss's Law, is a fundamental law of physics, has no derivation. Notice similarity between Ampere's Law and Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = Q_{inside}/\varepsilon_0$

Can use Ampere to derive $\vec{B}$-field of long straight wire w/ current I:



$\mathcal{L}$ = imaginary circular loop, radius r

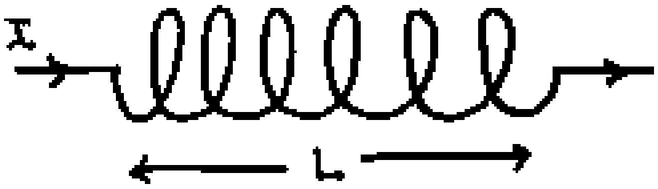
$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \underbrace{\underset{(\vec{B} \parallel d\vec{l})}{\oint B\, dl} = \underset{(B \text{ const})}{B \oint dl}}_{\text{only true because of symmetry}}$$

$\Rightarrow$

$$\oint \vec{B} \cdot d\vec{l} = B \cdot (2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

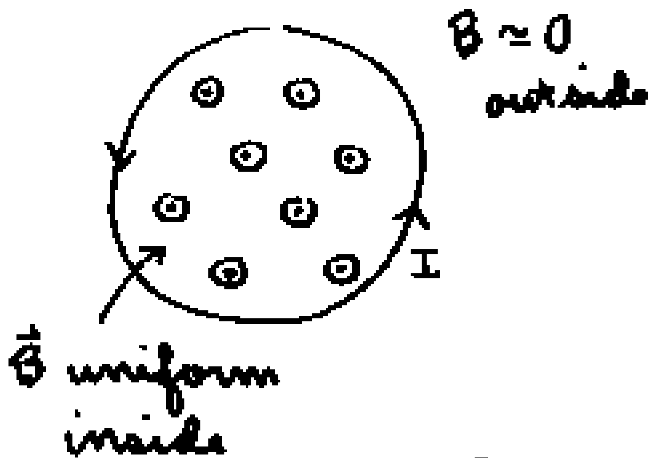
Ampere's Law always true, but only useful for computing B if situation has very high symmetry.

B-field due to <u>solenoid</u> of N turns, length L

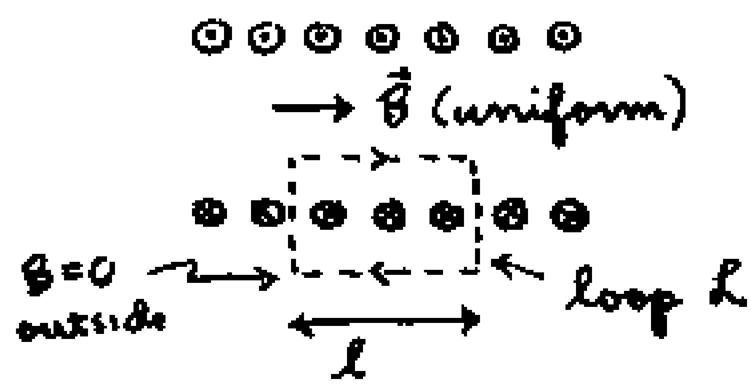


$n = \frac{N}{L}$ = # turns/meter

End View:



Side View:



$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = B \cdot l = \mu_0 I_{thru} = \mu_0 \underbrace{n \cdot l}_{\#\text{ turns thru } \mathcal{L}} I$$

$\Rightarrow$

$$\boxed{B = \mu_0 n I = \mu_0 \frac{N}{L} I}$$ ⟵ B-field inside solenoid

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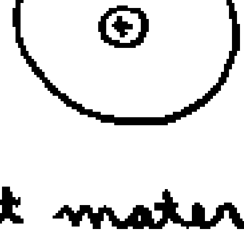
$\vec{B}$-fields are made by currents (according to Biot-Savart and Ampere's Laws)

So where's the current in a permanent magnet?

Atom:



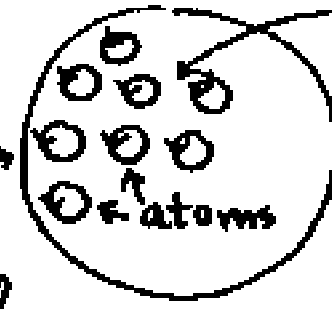
electron orbiting nucleus is a moving charge, a current

In most materials, atomic currents of the atoms have <u>random</u> orientations $\Rightarrow$ no <u>net</u> current

$\Rightarrow$ no B-field

In ferromagnetic materials (Fe, Ni, Co, alloys of these), the atomic currents (the magnetic moments $\vec{\mu}$) can all line up.

Cross-section of magnetized iron bar:



In interior, currents cancel

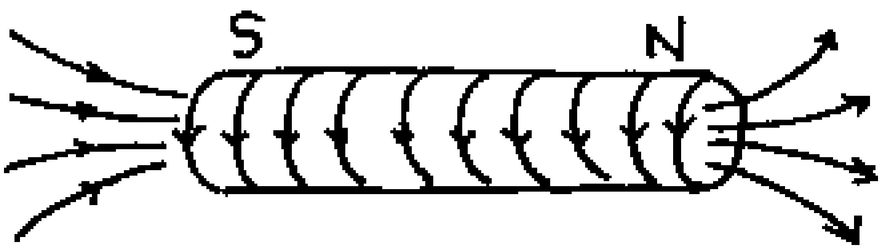
net I = 0

On rim, currents are all in same direction. Currents do not cancel.

Magnetized iron bar acts like loops of current, a solenoid:

net atomic current on rim

Side View:



$\vec{B}$-field lines exit "North Pole"

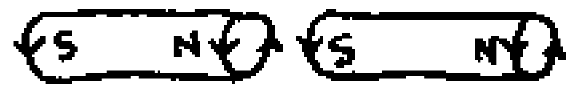
$\vec{B}$ lines enter "South Pole"

If magnet broken in half, each piece still has N/S pair.

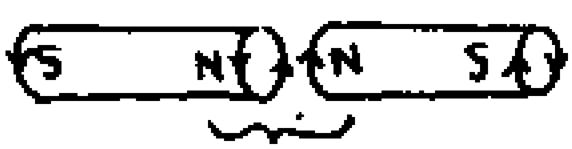
Why do permanent magnetics sometimes attract, sometimes repel?

Recall that parallel currents attract, anti-parallel currents repel.

Two permanents magnets:



parallel currents on ends attract



anti-parallel currents repel

$\Rightarrow$ opposite poles attract like poles repel.