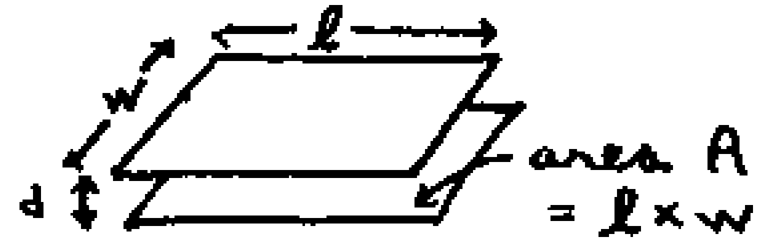
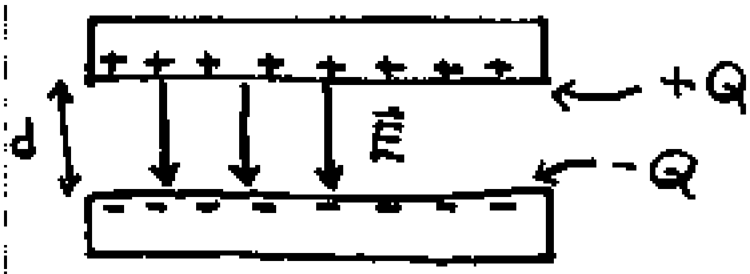
Capacitors

capacitor = any two pieces of metal brought near each other

parallel plate capacitor:



"Charged capacitor" has no net charge:



Always have

$+Q$ on one plate
$-Q$ on other plate
(no net charge)

Charges always on inside surfaces (since + attracts −)

Voltage V of capacitor $= \Delta V$ between plates

Can show that $V \propto Q$ for capacitor:

$$V = \Delta V = E d = \frac{\sigma}{\varepsilon_0} \cdot d = \frac{Q}{A} \cdot \frac{d}{\varepsilon_0}$$

(don't worry about signs)

$$V = \frac{Q}{(\varepsilon_0 A/d)}$$

Capacitance C of capacitor defined as ratio $\frac{Q}{V}$

$$\boxed{C \equiv \frac{Q}{V}}$$

For parallel-plate capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

Showed $V \propto Q$ for special case, but always true, regardless of shape of capacitor:

$$V \quad \propto \quad E \quad \propto \quad Q$$

$$\left(\Delta V = -\int \vec{E}\cdot d\vec{r}\right) \qquad \left(\vec{E} = \sum_i \frac{K q_i}{r_i^2}\,\hat{r}_i\right)$$

$\Rightarrow$ ratio $C = \dfrac{Q}{V}$ is a constant depending only on shape / dimensions of capacitor

Can think of capacitor as device for storing charge and energy

$$[C] = \frac{[Q]}{[V]} = \frac{\text{coulomb}}{\text{volt}} = \frac{C}{V} = 1 \text{ farad (F)}$$

(Don't confuse C = capacitance w/ C = coulomb.)

Big C $\Rightarrow$ get big Q stored for small V.

A farad is a huge capacitance!

Example: $C = \varepsilon_0 A/d$ $\quad A = 1\,m^2$ (big!)
$d = 1\,mm = 0.001\,m$

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85\times10^{-12})(1)}{10^{-3}} \simeq 9\times10^{-9}\,F = 9\,nF$$

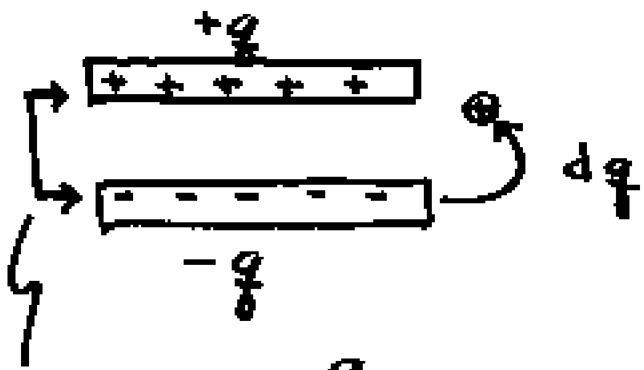
(tiny!)

Multi-farad capacitors in small packages are made by making d very small

$d \approx$ atomic dimensions possible.

Energy stored in a charged capacitor = work done to charge capacitor

Suppose have partially charged capacitor:



Work required to transfer extra bit of charge dq = ΔPE of charge dq =

$V = \Delta V = q/C$ $\qquad dU = V \cdot dq \quad (\Delta PE = q \cdot \Delta V)$

$$dU = V\,dq = \frac{q}{C}\,dq$$

Total work to charge cap. = $U = \int_0^Q \frac{q}{C}\,dq$

$$U = \frac{Q^2}{2C} \underset{(Q=CV)}{=} \frac{1}{2}CV^2 \underset{(C=\frac{Q}{V})}{=} \frac{1}{2}QV$$

(Formula $U = \frac{1}{2}QV$ is almost obvious: work = charge Q × <u>average</u> voltage diff ($= \frac{1}{2}V$))

Where is this energy U? Ans: <u>in</u> the $\vec{E}$-field

$\vec{E}$-fields contain energy.

energy density of E-field = $\frac{\text{energy}}{\text{volume}} = \frac{U}{\text{Vol.}}$

$= \boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$ (small $u = U/_{\text{Vol.}}$)

Proof: $U = \frac{1}{2} C V^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(E\,d)^2$

$$= \frac{1}{2}\varepsilon_0 E^2 \underbrace{(A \cdot d)}_{\text{volume of E-field}} \Rightarrow u = \frac{U}{\text{vol.}} = \frac{1}{2}\varepsilon_0 E^2 \checkmark$$

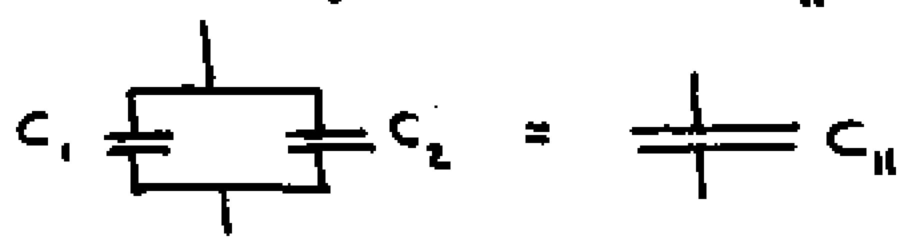
(Proved for special case, but always true!)

Turns out that work done to assemble a collection of charges ($W_{ext} = \Delta U = q\,\Delta V$) is equal to energy in all fields created.

$$U = \int \left(\tfrac{1}{2}\varepsilon_0 E^2\right) dv \quad \text{(volume integral)}$$

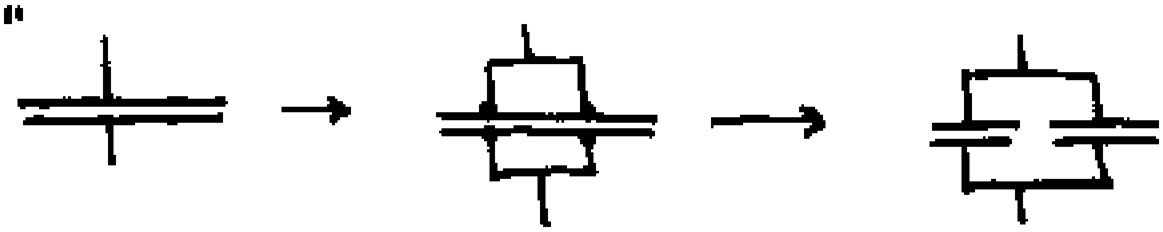
Adding C's in parallel or in series:

For C's in parallel, $C_{\parallel} = C_1 + C_2$



Big C is equivalent to 2 smaller C's side by side

"Proof"



For C's in series: $C_{series} = \dfrac{1}{1/C_1 + 1/C_2 + \ldots}$