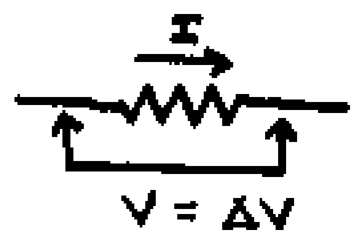
Electric Circuits

power $P = \dfrac{\text{energy}}{\text{time}}$ = rate of energy conversion

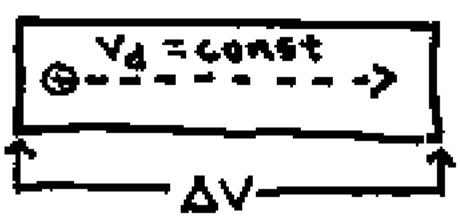
$[P] = J/s$ = watt (W)

Inside current-carrying resistor, electrostatic potential energy is converted into thermal energy (heat) at a rate:

$$\boxed{P = I V}$$



inside resistor:



$\Delta PE = \Delta U = q \Delta V$ (PE ↓)

Where did PE go? Not into KE.

KE = const, since drift velocity v_d = const.
PE converted into heat

ΔN q's pass in time Δt $\Rightarrow I = \dfrac{\Delta Q}{\Delta t} = \dfrac{\Delta N q}{\Delta t}$

$P = \dfrac{\Delta U}{\Delta t} = \dfrac{\Delta N \cdot q \Delta V}{\Delta t} = I \cdot \Delta V$ ✓

$$P = I V \underset{(V = IR)}{=} I^2 R \underset{(I = V/R)}{=} V^2/R$$

"100 W" lightbulb → 100 J/s converted to $\underbrace{\text{heat}}_{95\%}/\underbrace{\text{light}}_{5\%}$

For all household appliances: $\Delta V = 120\,V$

$P = \dfrac{V^2}{R} \Rightarrow$ IF V = const = 120V, then

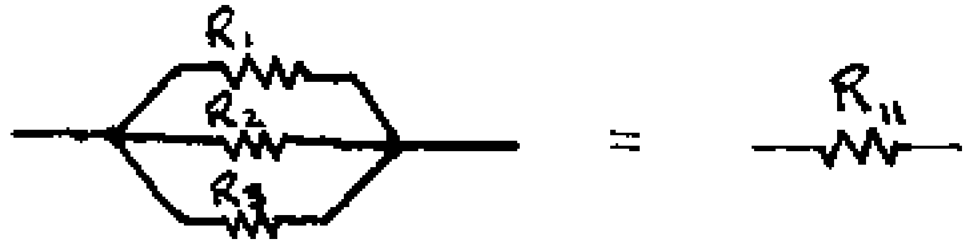
P big $\Leftrightarrow$ R small

Combinations of resistors

in series

(current same for all)

in parallel



(ΔV same for all)

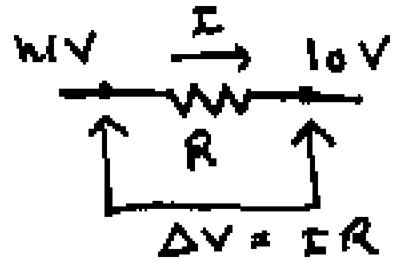
$$R_{series} = R_1 + R_2 + R_3$$

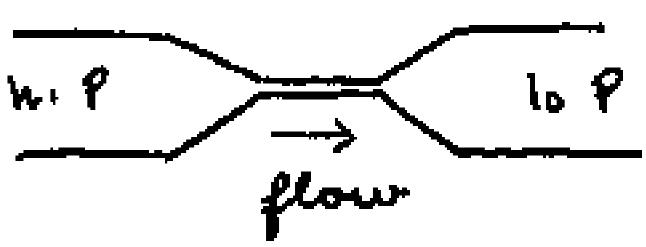
$$\frac{1}{R_{\parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \iff R_{\parallel} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Adding R in series always increases R_{total}.

Adding R in parallel always decreases R_{tot}.

Charge/water analogy

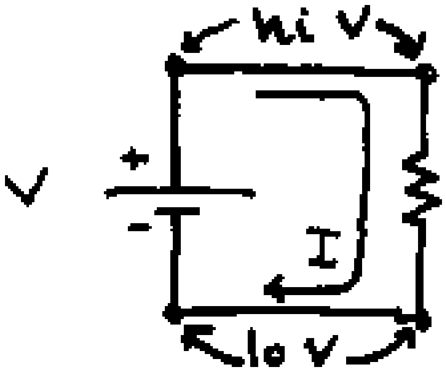


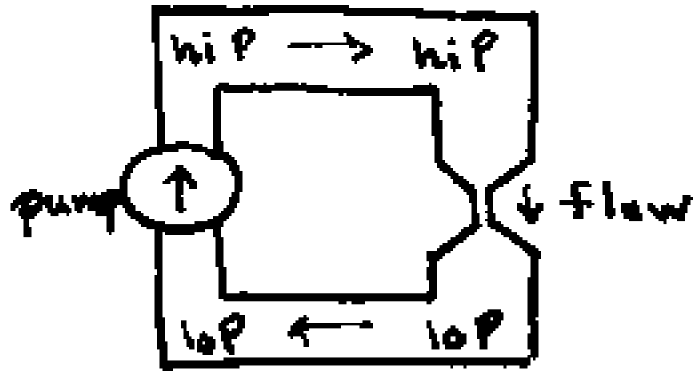


Think of voltage as "electrical pressure"
(+) charges want to flow from hi V to lo V, just as water in a pipe wants to flow from hi P to lo P.

A battery acts like a "charge pump",

Battery pumps (+) charges "uphill" in the direction they don't want to go, from (−) terminal to (+) terminal.





- inside resistor, (+) charges flow "downhill" hi V → lo V
- inside battery, (+) charges are pumped "uphill" lo V → hi V

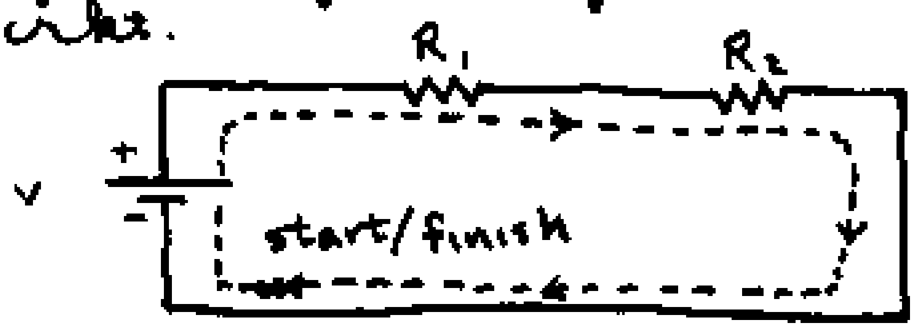
To analyze circuits need Kirchhoff's 2 Laws

- K1 : Loop Law (Voltage Law)

For any closed loop in a crkt,

sum of voltage rises = sum of voltage drops

~ must be so, since must return to same voltage after complete loop returning to original spot in crkt.



Chose to go around loop in same dir. as current.

$$\underbrace{V}_{\text{rise}} = \underbrace{IR_1}_{\text{drop}} + \underbrace{IR_2}_{\text{drop}} = I(R_1 + R_2)$$

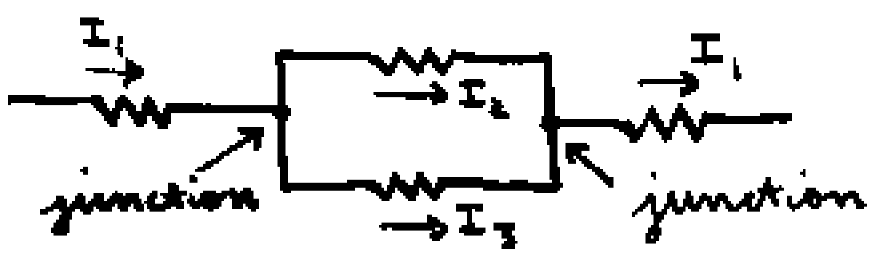
$I = \frac{V}{R_1 + R_2}$, Voltage across R_2 = V_2

$= V_2 = I R_2 = V \cdot \left(\frac{R_2}{R_1 + R_2}\right)$

"voltage divider"

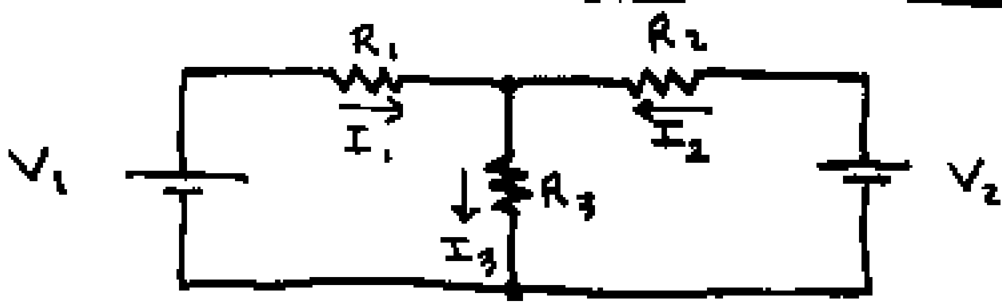
- K2 Junction Law (Current Law)

For any junction (place where 3 or more wires meet), total current in = tot current out.



$I_1 = I_2 + I_3$

$\boxed{\text{Crkts w/ multiple loops and batteries}}$



Know R's, V's

Seek I's

3-step procedure:

I. Guess direction of I thru each R.

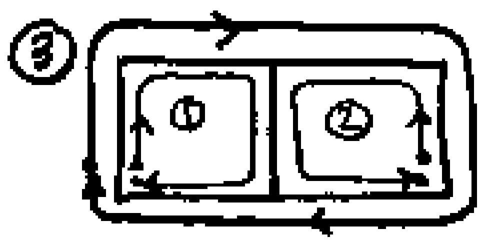
Draw I arrows, label each. Direction of I not always obvious, so just guess. If guess is wrong, current I will come out w/ negative value after numbers are plugged in

II. Junction Law (K2) gives 1 or more eq'ns

Eq'n (1) $I_1 + I_2 = I_3$

(3 unknowns: $I_1, I_2, I_3 \Rightarrow$ need 3 eq'ns)

III. Loop Law (Voltage Law) gives eq'n for each loop



Only need 2 more eq'ns (in this example) $\Rightarrow$ only 2 of 3 loops needed.

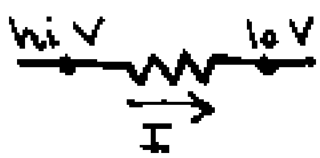
Loop ① $\underbrace{+V_1}_{\text{rise}} \underbrace{- I_1 R_1}_{\text{drop}} \underbrace{- I_3 R_3}_{\text{drop}} = 0$ (Eq'n 2)

Loop ② $+V_2 - I_2 R_2 - I_3 R_3 = 0$ (Eq'n 3)

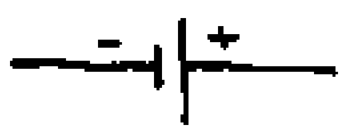
Loop ③ (Don't need because already have 3 eq'ns)

$$\underbrace{+V_1}_{\text{rise}} \underbrace{- I_1 R_1}_{\text{drop}} \underbrace{+ I_2 R_2}_{\text{rise!}} \underbrace{- V_2}_{\text{drop!}} = 0$$

Remember:



$\Leftarrow$ if traveling "upstream" (against I), then V rises, ΔV is (+)



$\Leftarrow$ if traveling from (+) to (−) terminal, V drops, ΔV is (−)

3 eq'ns in 3 unknowns:

(1) $I_3 = I_1 + I_2$

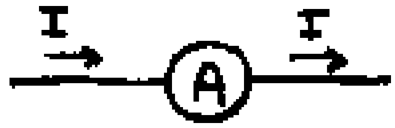
(2) $V_1 = I_1 R_1 + I_3 R_3$

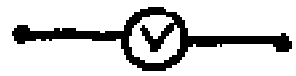
(3) $V_2 = I_2 R_2 + I_3 R_3$

[$I_1 = (I_3 - I_2)$ plug into (2) $\Rightarrow$ 2 eq'ns in 2 unk's: I_2, I_3)

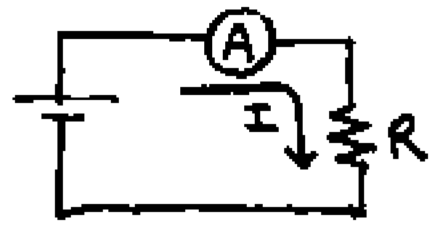
Circuit probes:

Ammeter measures current thru itself

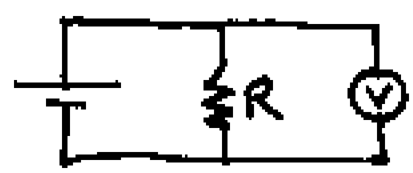


Voltmeter measures voltage diff between its terminals 

To measure current thru R, must place ammeter <u>in</u> <u>series</u>

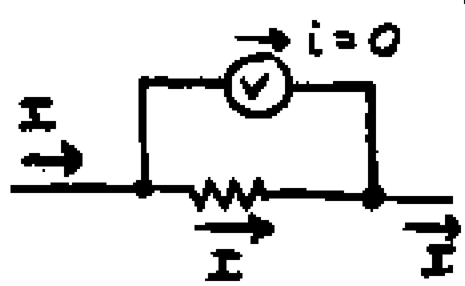


To measure voltage across R, must place voltmeter <u>in</u> <u>parallel</u>



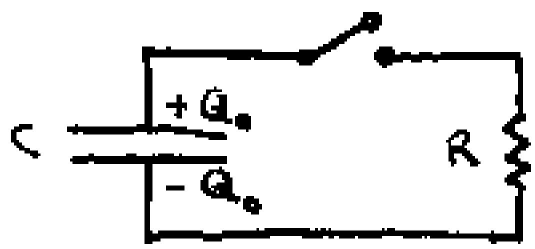
I<u>deal</u> ammeter has $r_{internal} = 0$ so measured current is not affected by presence of ammeter:

Ideal voltmeter has $r_{internal} = \infty$, so <u>no</u> I flows thru voltmeter $\Rightarrow$ currents, voltages in rest of circuit are not affected



RC Circuits

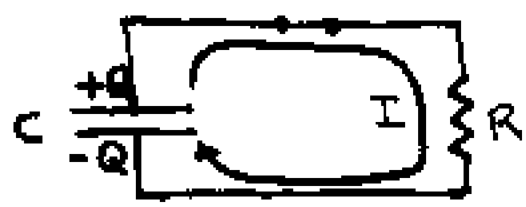
Simplest RC circuit: capacitor C, charged to voltage $V_0 = \frac{Q_0}{C}$, attached to resistor R, w/ switch.



$C = \frac{Q}{V} \Rightarrow V_0 = \frac{Q_0}{C}$

"initial"

Close switch at $t = 0$, current I starts to flow.



At $t = 0^+$, $I_0 = \frac{V_0}{R}$

$I = -\frac{dQ}{dt}$ (− sign since Q is decreasing)

$V_{\text{across } C} = V_{\text{across } R}$, $V_C = V_R$, $\frac{Q}{C} = IR$,

$\frac{Q}{C} = -\frac{dQ}{dt} \cdot R$, $\boxed{\frac{dQ}{dt} = -\frac{1}{RC} \cdot Q}$ ★

RC = "time constant" τ, has units of time

$[R][C] = \frac{\cancel{V}}{A} \cdot \frac{\text{coulombs}}{\cancel{V}} = \frac{\text{coul}}{\text{coul/s}} = s$ (seconds)

★ is a differential eq'n of the form: $\frac{dx}{dt} = \underbrace{a}_{\text{const}} \cdot x$

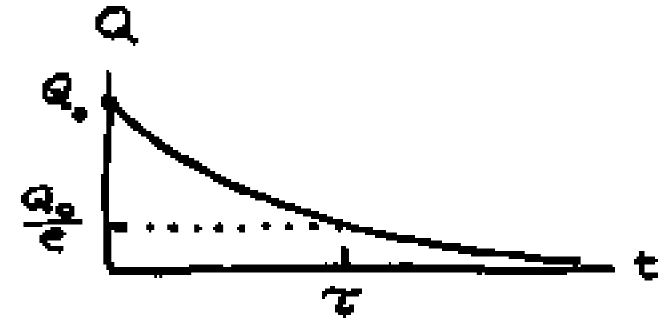
rate of change of $x \propto x \Longleftrightarrow$ exponential sol'n

$x = x_0 e^{at}$ Check: $\frac{dx}{dt} = a \cdot x_0 e^{at} = a \cdot x$ ✓

$a > 0 \Rightarrow$ exponential growth

$a < 0 \Rightarrow$ exponential decay

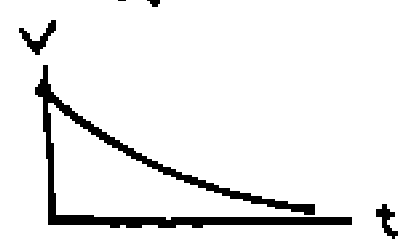
$$Q(t) = Q_0\, e^{-t/RC}$$
$$= C V_0\, e^{-t/\tau}$$



In time τ, Q falls by factor of $1/e \simeq 0.37$
In time 2τ, Q falls by factor of $(1/e)(1/e) \simeq 0.14$
etc.

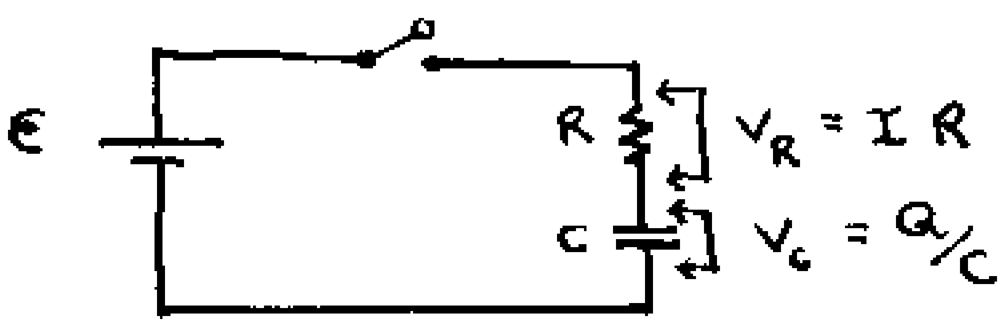
$\Rightarrow$ Q approaches zero asymptotically, so does V, I

$V = \frac{Q}{C}$



$\left(I = -\frac{dQ}{dt} = +\frac{V_0}{R} e^{-t/\tau}\right)$

More complex RC circuit: Charging C w/ battery



Before switch closed, $I = 0$, $Q = 0$

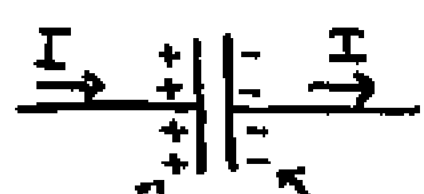
Close switch at $t = 0$. Now $\mathcal{E} = V_R + V_c$

Charge Q and voltage $V_c = Q/C$ of capacitor cannot change instantly (takes time for Q to build up).

At $t = 0+$, $Q = 0$, $V_c = 0$, $\mathcal{E} = V_R = I R \Rightarrow I_0 = \mathcal{E}/R$

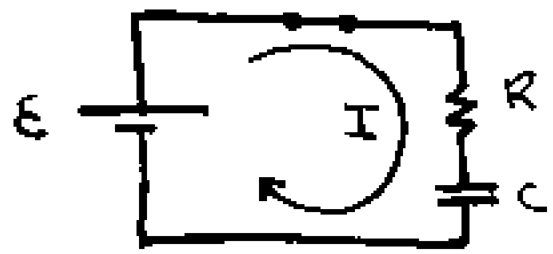
(Although Q on capacitor cannot change instantly, current $I = dQ/dt$ can.)

"Current thru a capacitor"



~ even though no charge passes between plates.

+Q/−Q increasing as I flows



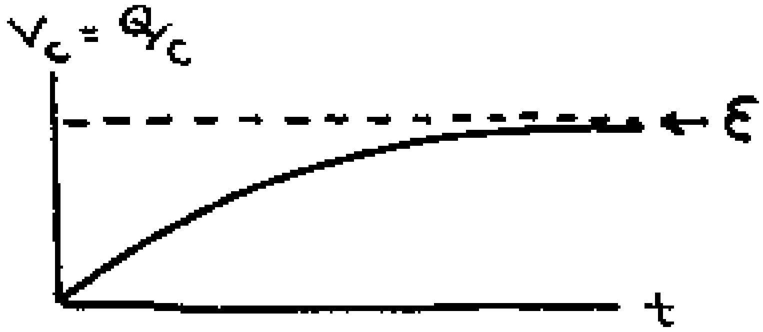
$$\mathcal{E} = V_R + V_C$$

$$\mathcal{E} = IR + Q/C$$

$$\mathcal{E} = +\frac{dQ}{dt}\cdot R + Q/C$$

As $t \nearrow$, $Q \nearrow$, $V_C = \frac{Q}{C} \nearrow$, $V_R \searrow$, $I = \frac{V_R}{R} \searrow$

After long time, $t \gg \tau = RC$, $I = 0$, $V_C = \mathcal{E}$, $Q = C\cdot\mathcal{E}$



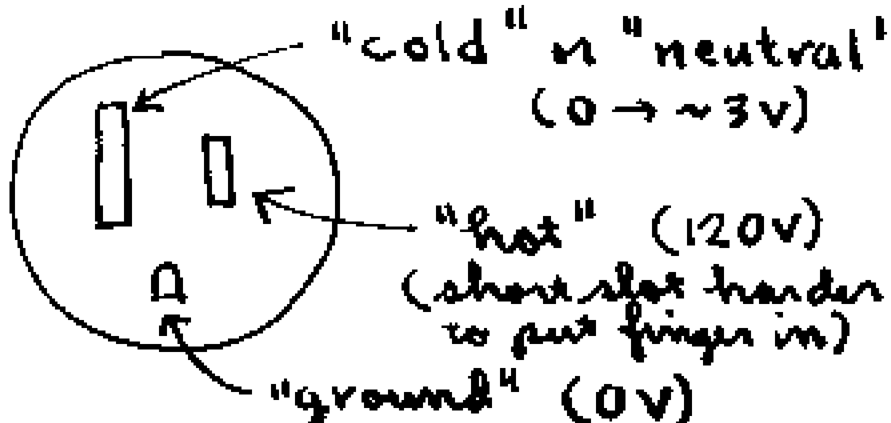
$$V_C(t) = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

Things to remember:

- Unchanged capacitor acts like a "short": $V_C = \frac{Q}{C} = 0$
- After long time, when capacitor is fully charged, it acts like "open-circuit". Must have $I_C = 0$ eventually, otherwise $Q \to \infty$.

House Wiring

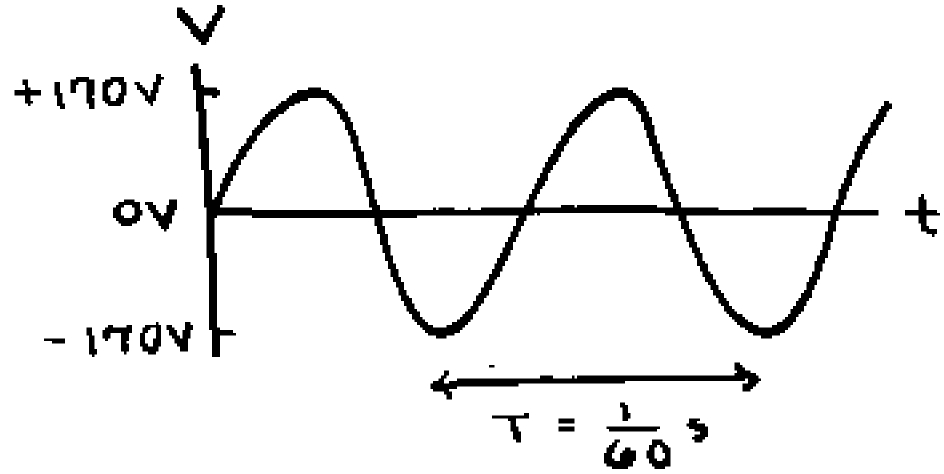
Wall socket = 3-prong plug



"hot" voltage is AC, not DC

Battery voltage is DC = const in time

Wall socket voltage is AC = sinusoidal in time



$$\sqrt{(V^2)_{avg}} = 120\text{ V}$$

freq $f = \frac{1}{T} = 60\text{ Hz}$

AC voltage is more economical to distribute from power plant to homes than DC.

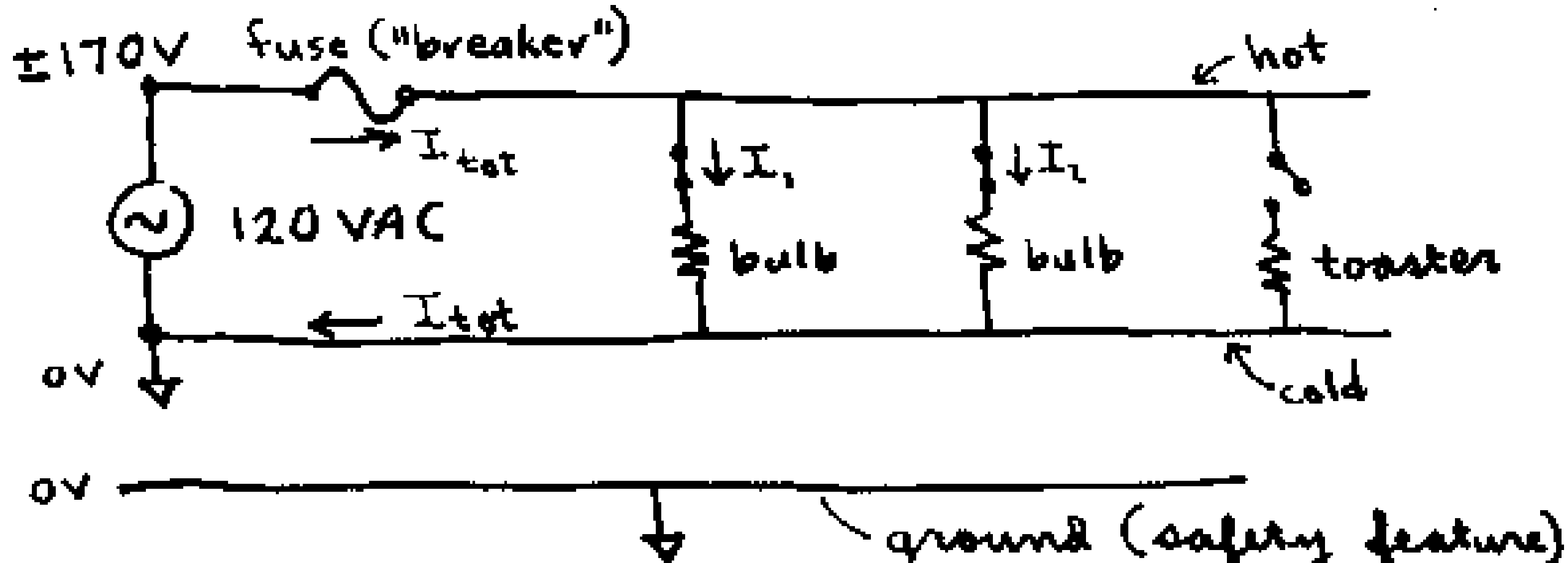
Standard electrical wiring colors:

black = hot (burnt black)

white = cold (like snow)

green = ground (grass)

Never assume house wiring is correct – always test!



If $I_{tot} > 15\text{ A}$, breaker opens, preventing wires from over-heating

Chassis case of appliances are connected to ground as safety precaution.