$\boxed{\text{Faraday's Law}}$

Have said before that...

$\vec{E}$ fields are created by charges.
$\vec{B}$ fields are created by currents.

~ true, but not the whole truth.

$\vec{E}$ fields can also be created by a <u>changing</u> $\vec{B}$ field

$\vec{B}$ fields can also be created by a <u>changing</u> $\vec{E}$ field.

Faraday's Law relates changing B field to $\vec{E}$ field

New terms:

emf ($\mathcal{E}$) = (roughly speaking) a voltage difference capable of generating power. Batteries have an $\mathcal{E}$. Resistors do not, even though $\Delta V = IR$ is a voltage difference.

Technically, $\mathcal{E}$ around a loop $\mathcal{L}$ is defined as

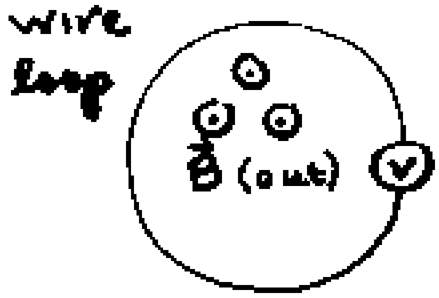
$$\mathcal{E} = \oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} \qquad \left(\text{Recall } \Delta V = -\int \vec{E} \cdot d\vec{l}\right)$$

magnetic flux Φ_m over some area is

$$\Phi_M = \int \vec{B} \cdot d\vec{A} \underset{(\text{if } \vec{B} \text{ const, A flat})}{=} \vec{B} \cdot \vec{A}$$

(Like electric flux $\Phi_E = \int \vec{E} \cdot d\vec{A}$)

Faraday's Law: $\boxed{\mathcal{E} = -\dfrac{d\Phi_M}{dt}}$

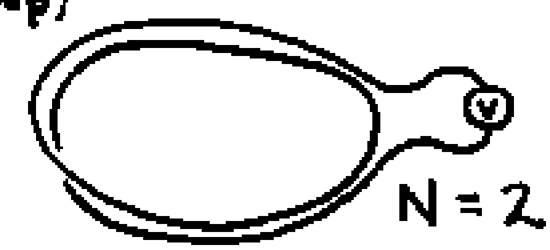


voltmeter measures $\mathcal{E}$ around loop

if $\vec{B}$ = const
$\Rightarrow \Phi$ = const
$\Rightarrow \mathcal{E} = 0$

if $\vec{B}$ changing $\Rightarrow \Phi$ thru loop changing $\Rightarrow$
$|\mathcal{E}| = |d\Phi/dt| \neq 0$

$$\mathcal{E}_{N\ \text{loops}} = -N \frac{d\Phi_{(1\ \text{loop})}}{dt}$$



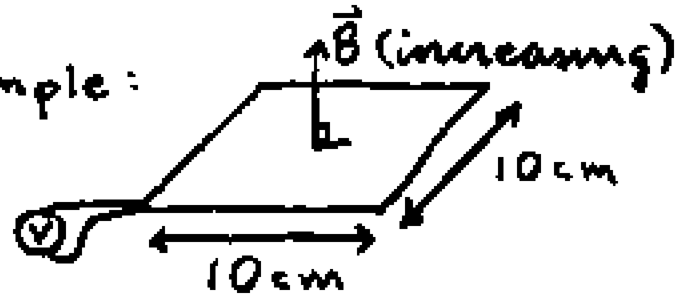
Can change $\Phi = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A}$ thru loop in several ways:

1) change B

2) change area A (by altering area of loop)

3) change $\theta = \measuredangle\, \vec{B}, \vec{A}$ (by turning loop)

Example:



$A = (0.1\,m)^2 = 0.01\,m^2$
$dB/dt = +0.1\ T/s$

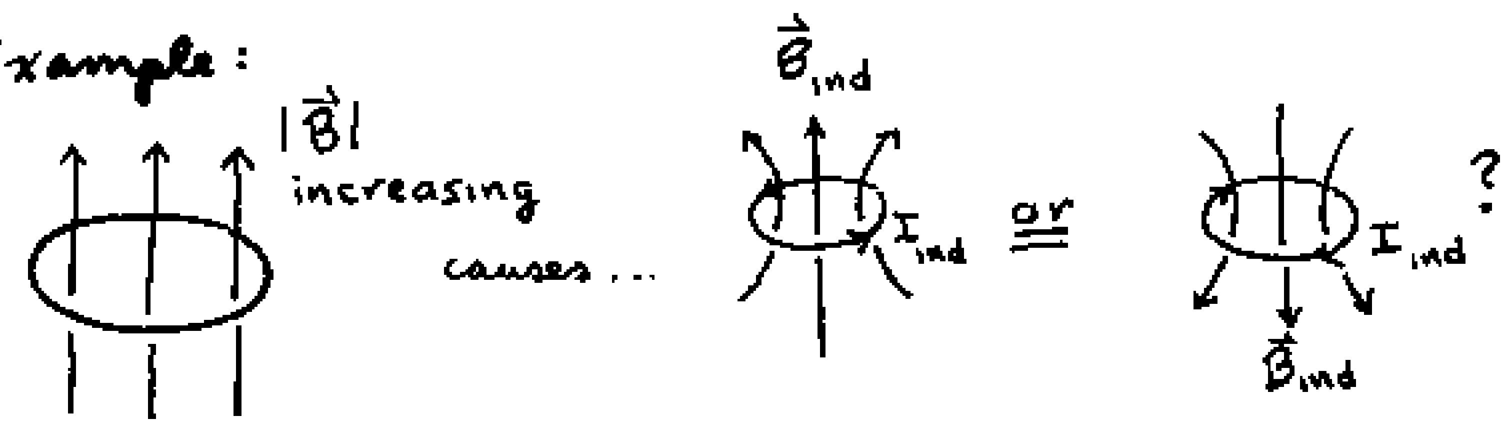
$$|\mathcal{E}| = d\Phi/dt = \frac{d}{dt}(BA) = A\frac{dB}{dt} = (0.01)(0.1) = 10^{-3}\,V$$

If N = 1000 turns of coil of wire $\Rightarrow |\mathcal{E}| = 1\,V$

The (−) sign in Faraday's Law is a reminder of ...

Lenz's Law: the induced emf tends to induce a current in the direction which <u>opposes</u> the <u>change</u> in flux.

Example:



Answer: $\vec{B}_{ind}$ <u>down</u> fights the change in flux

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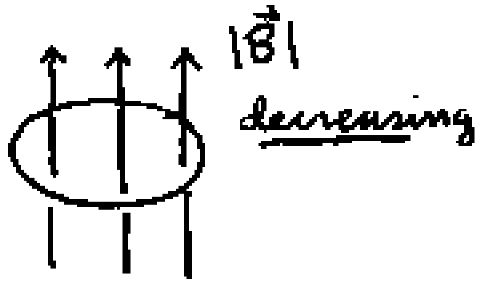
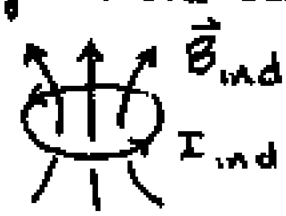
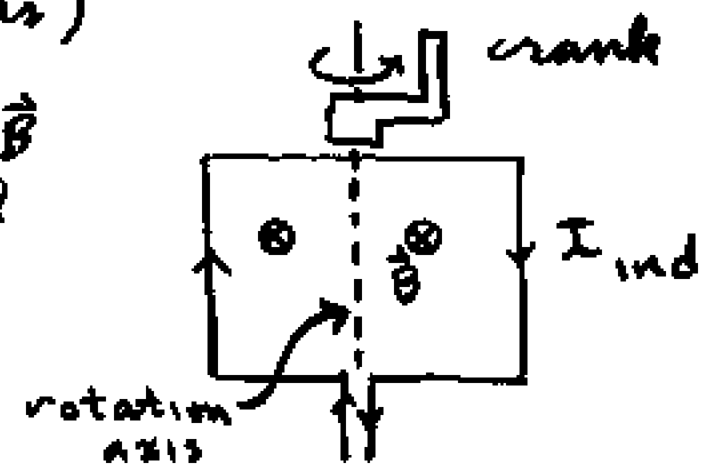


Now, must have $\vec{B}_{ind}$ <u>up</u> to oppose the decrease

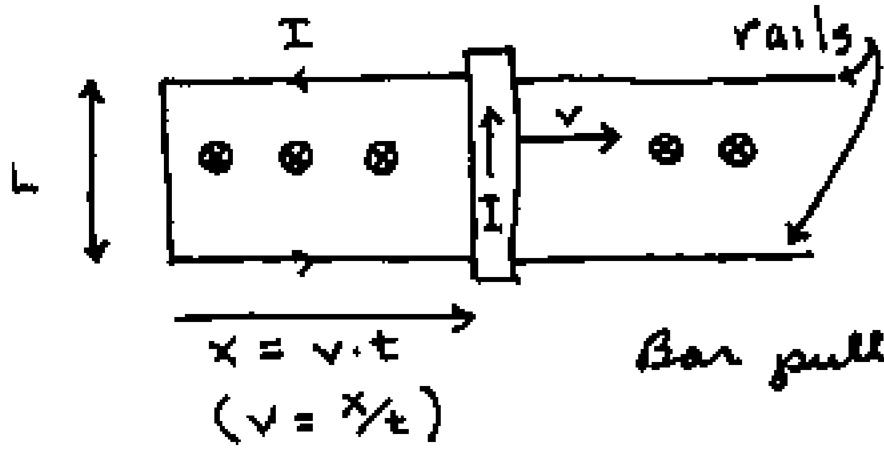


<u>Generators</u> use Faraday's Law to convert mechanical energy into electrical energy (opposite of motors)

wire loop in const $\vec{B}$ field (from external magnet) turned w/ hand crank



Sliding metal bar on metal rails:



B = const, uniform

Bar pulled to right at const v

Bar + rails form wire loop of area $A = L \cdot x = L v t$

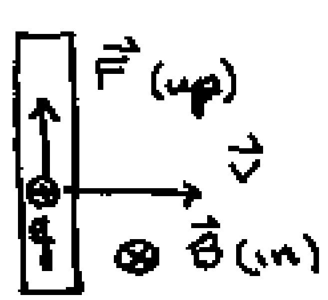
$\Phi = B \cdot A$ increasing $\Rightarrow \mathcal{E} \Rightarrow$ current I induced (ccw)

$$|\mathcal{E}| = d\Phi/dt = \frac{d}{dt}(BA) = \frac{d}{dt}(BLvt) = BLv$$

~ example of "motional emf" (motion of bar causes emf)

There is another way to see why I is induced in bar in upward direction:

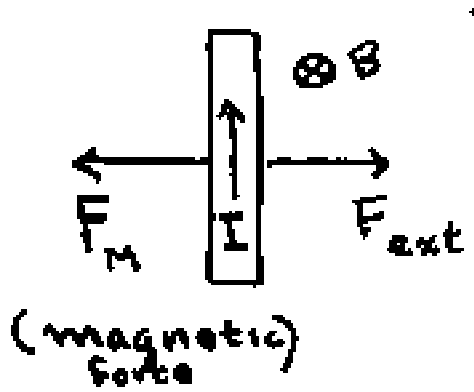
Charges in bar are moving w/ bar thru $\vec{B}$



$F_{\text{on } q} = q v B$ (upward)

$\Rightarrow I$ upward

Current I in bar $\Rightarrow$ magnetic force on bar



$F_M = F_{\substack{\text{on bar} \\ \text{due to } B}} = I L B$

(recall $\vec{F}_{\substack{\text{on} \\ \text{wire}}} = I \vec{L} \times \vec{B}$)

$\leftarrow$ must exert external force to overcome magnetic force

To maintain const v, must have $F_{ext} = F_M$

Power delivered by F_{ext} must equal electrical power dissipated in wire

$$P_{ext} = \frac{dW}{dt} = \frac{d}{dt}(F_{ext} \cdot x) = F_{ext} \cdot v$$

$$P_{electrical} = \mathcal{E} \cdot I \qquad (\text{Recall } P = IV)$$

$$F_{ext} \cdot v \overset{?}{=} \mathcal{E} I \Rightarrow F_M \cdot v = \mathcal{E} \cdot I$$

$$\Rightarrow ILB \cdot v = BLv \cdot I \quad \checkmark \quad (\text{it checks!})$$

Current I in bar is example of Eddy Current

If metal object and source of $\vec{B}$-field are in relative motion so that there is changing Φ thru some loop within metal, then

Faraday $\Rightarrow \mathcal{E} \Rightarrow I$ (eddy current)

If metal moving, then direction of I always such as to cause force ($\vec{F}_{wire} = I\vec{L} \times \vec{B}$) which opposes motion, slows metal.

If eddy current force aided motion, instead of slowing motion, then would get runaway motion $\Rightarrow$ free energy $\Rightarrow$ impossible!

In case where loop is stationary and B is changing, then there is no $\vec{F}_m = q\vec{v} \times \vec{B}$

magnetic force on charges in wire, and the only way to explain induced current in wire is to admit creation of $\vec{E}$-field.

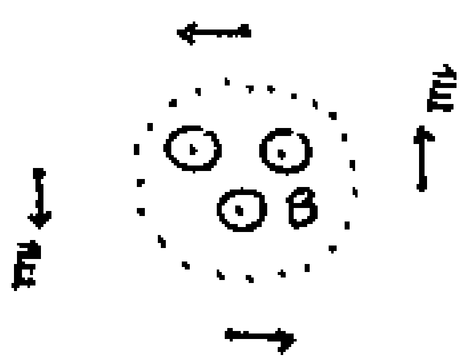
emf $\mathcal{E}$ around a (stationary) closed loop is defined as $\mathcal{E} = \oint_{\mathcal{L}} \vec{E} \cdot d\vec{l}$.

Faraday's Law $\mathcal{E} = -d\Phi/dt$ can be written as

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\left(\int_A \vec{B} \cdot d\vec{A}\right)$$

← loop $\mathcal{L}$ encloses area A

⇒ a changing $\vec{B}$-field creates an $\vec{E}$-field

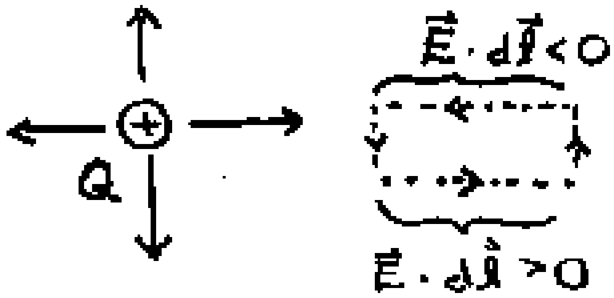


B decreasing ⇒ $\vec{E}$ is CCW (Lenz's Law)

B increasing ⇒ $\vec{E}$ is CW

The (non-Coulomb) $\vec{E}$-field created by $d\Phi/dt$ has different properties from Coulomb $\vec{E}$-field created by charges:

The electrostatic (Coulomb) $\vec{E}$ created by charges always has $\oint \vec{E} \cdot d\vec{l} = 0$ for any loop



(+) part cancels (−) part

If stationary charges only, no currents, no B

$\Rightarrow$ no Φ $\Rightarrow$ $d\Phi/dt = 0$ $\Rightarrow$ $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = 0$

Inductors (inductor = coil of wire)

Important fact: magnetic flux Φ_M ($= \vec{B} \cdot \vec{A}$) is proportional to current I making the Φ_M.

All our formulas for $\vec{B}$ show $B \propto I$:

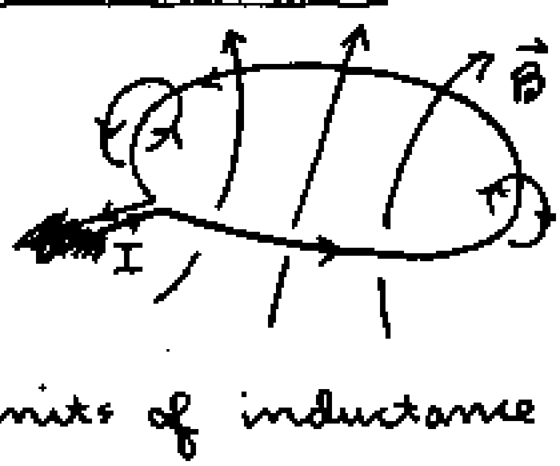
Biot Savart:
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2},$$

Ampere:
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{thru}$$

$\Rightarrow$ flux $\Phi \propto B \propto I$ $\Rightarrow$ $\Phi \propto I$

Definition: Self-inductance L of coil of wire

$$\boxed{\Phi_M \equiv L I}$$ ($L \equiv \Phi/I$ = const independent of I)



current $I \Rightarrow B \Rightarrow \Phi_M$

units of inductance $[L] = \frac{[\Phi]}{[I]} = \frac{T \cdot m^2}{A} = \frac{V \cdot s}{A}$

$= 1$ henry (H)

An inductor is a coil of wire. One or a few loops of wire has $L \simeq 1\,\mu H$ (usually insignificant); a coil w/ thousands of turns has $L \simeq 1\,H$ (big!)

$$\Phi = L \cdot I \Rightarrow \frac{d\Phi}{dt} = L\frac{dI}{dt} = -\mathcal{E}$$

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

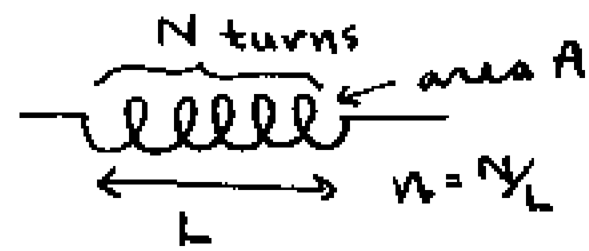
Changing the current in an inductor creates an emf which <u>opposes</u> the change in I.

Induced emf often called "back emf"

⇒ it is difficult (requires big external voltage) to change quickly the current in an inductor.

⇒ The current in an inductor <u>cannot</u> change instantly. (If it did (or tried to) would have infinite $\mathcal{E}$, infinite $\vec{E}$ to fight change.)

<u>Self-inductance</u> of <u>solenoid</u>:



If coil long, $B = \mu_0 n I$

$$\Phi = NBA = N\mu_0 n I A$$

$$L \equiv \Phi/I = \mu_0 N n A = \mu_0 n^2 A L$$

inductance (don't confuse the L's) length

Computing the inductance of a single turn coil (or a few turn coil) is quite messy because the $\vec{B}$-field in a loop of wire is non-uniform. So computing the magnetic flux $\Phi_M = \int \vec{B}\cdot d\vec{A}$ is difficult (messy).

Magnetic Energy Density

Recall that for capacitor C, stored energy $\mathcal{U} = \frac{1}{2} C V^2$

energy is stored in E-field, energy density =

$$u_E = \frac{\mathcal{U}}{\text{vol.}} = \frac{1}{2} \varepsilon_0 E^2$$

For an inductor L, with current I, stored energy is

$\mathcal{U} = \frac{1}{2} L I^2$. The energy is stored in the B-field.

energy density is $u_B = \frac{\mathcal{U}}{\text{vol.}} = \frac{1}{2} \frac{B^2}{\mu_0}$

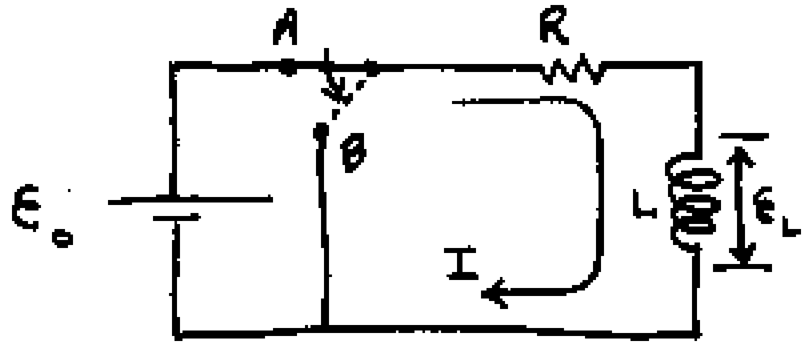
It takes work to get current flowing in an inductor (must work against the back emf which opposes any change in current). That work $= \mathcal{U} = \frac{1}{2} L I^2$ is stored in the inductor's magnetic field.

LR circuits (circuits w/ L's + R's)

3 things to remember:

- $|\mathcal{E}_L| = L \left|\frac{dI}{dt}\right|$ ~ the inductor acts like a battery if its current is changing
- I thru L <u>cannot</u> change instantly. (because that would cause infinite $\mathcal{E}$)
- In steady state (after long time) $I = \text{const} \Leftrightarrow \mathcal{E}_L = 0$

$\mathcal{E}_L = 0 \Rightarrow$ inductor acts like short (zero-R wire)



Switch at A for long time:

$I = \text{const}, \ \mathcal{E}_L = 0$

$I = \mathcal{E}_0 / R$

At $t=0$, switch $\rightarrow$ B:

At $t = 0+$, $I = I_0 = \mathcal{E}_0/R$

$\mathcal{E}_L$ keeps current going!

Loop Law: $IR = |\mathcal{E}_L| = -L \frac{dI}{dt}$

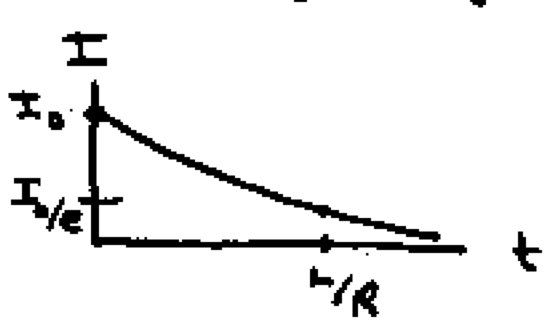
$$\left[dI/dt < 0 \Rightarrow -L \, dI/dt > 0 \right]$$

$\frac{dI}{dt} = -\left(\frac{R}{L}\right) I$ ~ a differential eq'n w/ exponential sol'n

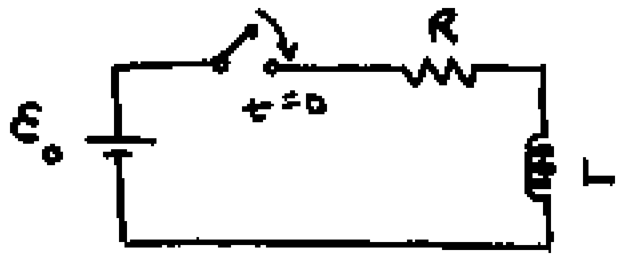
$$\Rightarrow I = I_0 e^{-\left(\frac{R}{L}\right) \cdot t} = I_0 e^{-t/(L/R)}$$

$\frac{L}{R}$ = time constant of LR circuit

= time for anything to change by factor of e



Another LR circuit:



$t = 0+$

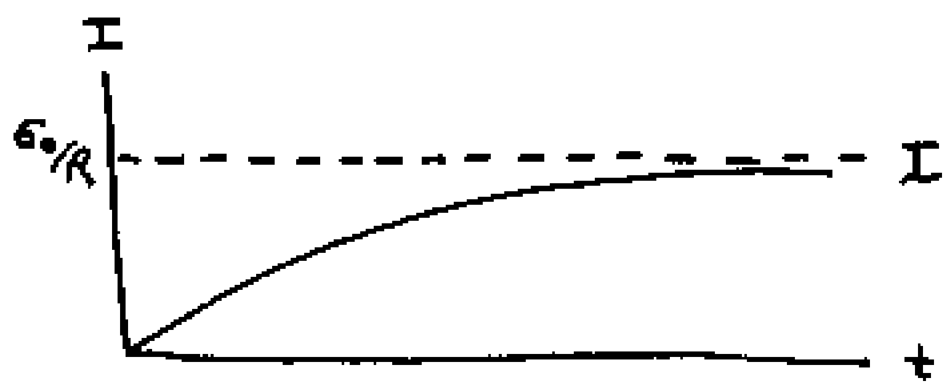
$I = 0, \quad |\mathcal{E}_L| = \left|L \frac{dI}{dt}\right| = \mathcal{E}_0$

Since $\mathcal{E}_0 = IR + L\frac{dI}{dt}$ (by Loop Law)

and $I = 0$ (at $t = 0+$)

As $t \nearrow$, $I \nearrow$, $V_R = IR \nearrow$, $|\mathcal{E}_L| = |L\, dI/dt| \searrow$

As $t \to \infty$, $|\mathcal{E}_L| \to 0$, $\mathcal{E}_0 = IR$



$I(t) = \frac{\mathcal{E}_0}{R}\left[1 - e^{-t/(L/R)}\right]$