Current, Voltage, and Resistance : Ohm's Law

Electrons can flow along inside a metal if there is an $\vec{E}$-field inside the wire to push them along (since $\vec{F} = q\vec{E} = -e\vec{E}$) Similar to flow of water in a pipe.

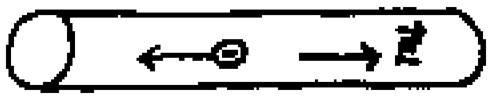
electric current $I \equiv \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$ = rate of flow of charge

unit $[I] = \frac{\text{coulomb}}{\text{sec}} = \frac{1\,C}{1\,s} = 1$ ampere (A) "amp"

In metal, in electrostatic equilibrium: $\vec{E} = 0$, $I = 0$

But now, $I \neq 0$, situation not static, $\vec{E} \neq 0$.

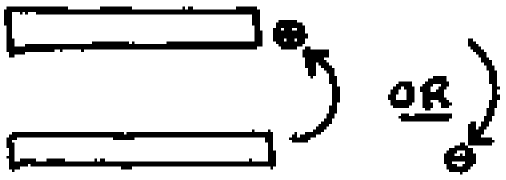
Electrons flow in metals, not protons, so (−) charges are moving.



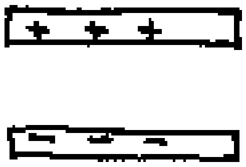
$\vec{F} = q\vec{E} = -e\vec{E}$ ⇒ electrons go "upstream" against $\vec{E}$

Flow of (−) charge one direction is equivalent to flow of (+) charge in opposite direction.

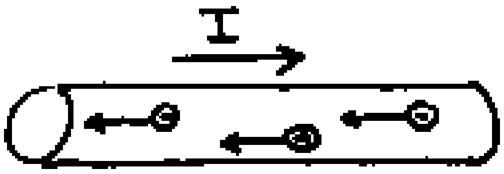
Capacitor plates:



either way, get



By convention, we define direction of current I as direction of flow of (imaginary) (+) charges, when it's really (−) charges flowing other way.



How many e's flowing by when $I = 1A$?

$e = 1.6 \times 10^{-19}\,C$, $Q = Ne$, $\Delta Q = \Delta N \cdot e$

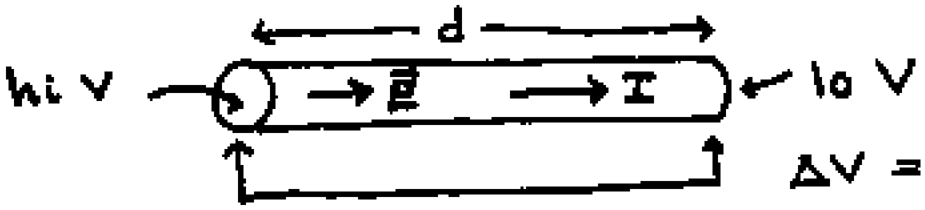
$$I = \frac{\Delta Q}{\Delta t} = e\frac{\Delta N}{\Delta t}, \quad \frac{\Delta N}{\Delta t} = \frac{I}{e} = \frac{1\,C/s}{1.6 \times 10^{-19}\,C}$$

$$\frac{\Delta N}{\Delta t} = 5.6 \times 10^{18}\,s^{-1} \quad (\text{for } I = 1A)$$

$I = 10\,mA = 0.01\,A$ is LETHAL, yet I could grab a wire carrying 1000 A and be safe!

Why? My body has a much higher electrical <u>resistance</u> than metal. e's prefer to flow thru wire.

Most materials are <u>ohmic</u>. In ohmic materials, current $I \propto \vec{E}$ inside $\propto \Delta V$ = voltage diff between ends.



$\Delta V = E \cdot d \quad (\Delta V = -\vec{E} \cdot \Delta\vec{r})$

From now on, use V when we mean ΔV

$$I \propto \Delta V \quad (I \propto V) \quad \Rightarrow \quad \frac{V}{I} = \text{const}$$

Resistance R of piece of wire (or other material)

$$R \equiv \frac{V}{I} = \text{const}$$

Def'n of R

"Ohm's Law"

$$\boxed{V = IR}$$

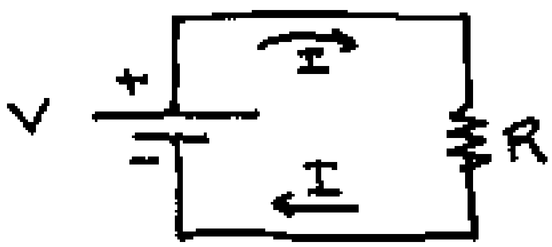
$V = \Delta V$!

High resistance ⇔ lots of scattering (internal friction)

Inside metal, electrons have net movement when $\vec{E} \neq 0$

$$\left.\begin{array}{l} \vec{F} = q\vec{E} = m\vec{a} \Rightarrow \text{electrons speed up} \\ \text{scattering} \Rightarrow \text{electrons slow down} \end{array}\right\} \begin{array}{l} \text{e's reach} \\ \text{"terminal velocity"} \\ = v_d = \text{drift velocity} \end{array}$$

Simple Circuit:

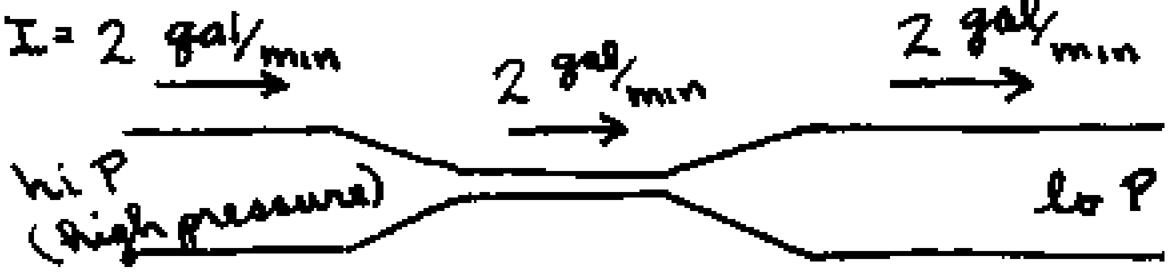


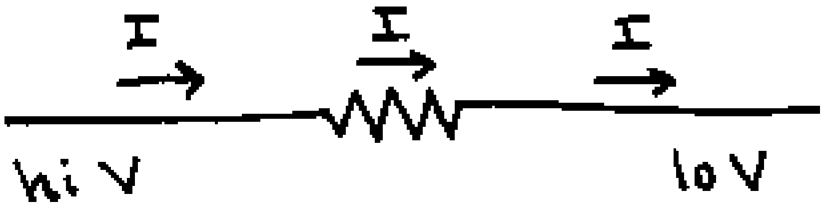
ideal wire: $R_{wire} = 0$

$\Rightarrow \Delta V_{wire} = I R_{wire} = 0$

ideal battery: $R_{battery} = 0$

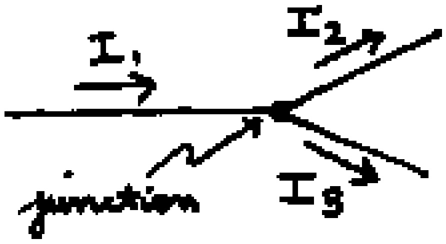
Flow of e's in wire or resistor is like flow of water in a full pipe (no bubbles or leaks)





I constant along wire

Junction Rule: $I_{in} = I_{out}$



$$\underbrace{I_1}_{I_{in}} = \underbrace{I_2 + I_3}_{I_{out}}$$

In steady state, must have $I_{in} = I_{out}$, otherwise, charge is building up somewhere, which cannot happen in steady-state.

Resistance R of piece of conductor depends on

1) composition
2) shape + dimensions

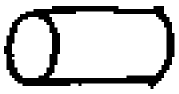
$$\boxed{R = \rho \cdot \frac{L}{A}}$$

(will show why later)

ρ = resistivity = measure of internal friction, depends on composition

factor of $\frac{L}{A}$ depends on dimensions

$R \propto \frac{L}{A}$ ⇒ long + skinny:  <u>hi</u> R

short + fat:  <u>lo</u> R

current density $J = \frac{I}{A} = \frac{\text{current}}{\text{area}}$ $\left(\vec{J} = \frac{\vec{I}}{A}\right)$

$\vec{J}$ caused by $\vec{E}$. Turns out that $J \propto E$ (usually)

$$J = \underset{\text{conductivity}}{\sigma} E = \frac{1}{\underset{\text{resistivity}}{\rho}} E$$

In <u>ohmic</u> materials, $J \propto E \iff \sigma, \rho$ are <u>const</u> (ind. of E)

Will now show that:

$$E = \rho J \iff \Delta V = IR, \text{ where } R = \rho \frac{L}{A}$$

(microscopic and macroscopic forms of Ohm's Law)