Electric Potential (voltage)

Electric fields are made by charges, so is voltage

Voltage = "electric potential" (or just "potential") is a kind of electrical pressure or electrical height. Positive charges want to get away from hi-voltage, get toward lo-voltage (like water wants to move from hi-pressure to lo-pressure; like mass wants to move from big height to lo-height).

Only changes in voltage ΔV have physical meaning. Zero of voltage is arbitrary.

ΔV defined in 2 equivalent ways:

$$\Delta V = \frac{\Delta U}{q} = \text{change in potential energy } \underline{\text{per}}\ \underline{\text{charge}}$$

or

$$\Delta V = -\vec{E} \cdot \Delta\vec{r} \quad (\text{uniform field, } \vec{E} = \text{const})$$

$$= -\int \vec{E} \cdot d\vec{r} \quad (\text{non uniform } \vec{E})$$

[$\Delta\vec{r}$ = change in position]

To understand voltage, must review

Work + Potential Energy (U)

U = stored work

Work done by a (constant) force $\vec{F}$ =

$$W_F = \vec{F} \cdot \Delta\vec{r} = F \, \Delta r \cos\theta$$

W(+) if $\vec{F}$ along $\Delta\vec{r}$

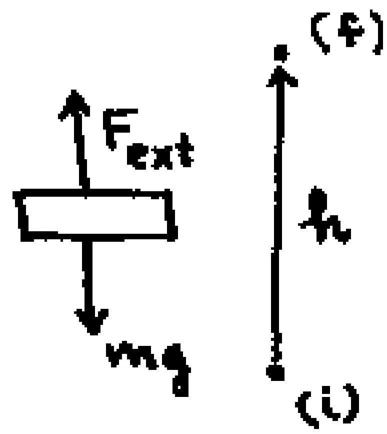
W = 0 if $\vec{F} \perp \Delta\vec{r}$

W(−) if $\vec{F}$ opposite $\Delta\vec{r}$

Lift a book by hand at const velocity

hand = "<u>ex</u>ternal agent"

gravity = "field"



$F_{ext} = mg$

$W_{ext} = F_{ext} \cdot h = mgh$

$W_{grav} = -mgh$

Define $\Delta U = +W_{ext} = -W_{field} = mgh$

When I do (+) work to lift the book, energy leaves me, is stored in grav. PE of book/earth system

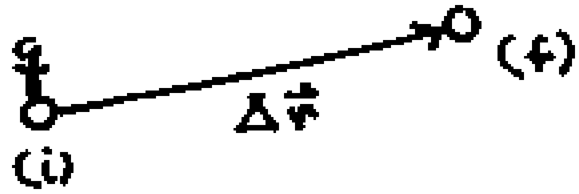
Potential energy is a useful concept because

$$\Delta K + \Delta U = 0 \iff K + U = \text{const} \quad \text{(if no friction)}$$

(K = kinetic energy = $\frac{1}{2}mv^2$)

Can define electrical potential energy in same way as gravitational PE.

Define change in electrical potential energy (ΔU_{elec}) of charge q in moving from $i \rightarrow f$ in <u>const</u> field $\vec{E}$



$$\Delta U = U_f - U_i = +W_{ext} = -W_{field}$$

$$= -\vec{F}_{field} \cdot \Delta\vec{r} = -q\,\vec{E} \cdot \Delta\vec{r}$$

If $\vec{E} \neq$ const, $\Delta U = U_f - U_i = -\int_i^f \vec{F}_{field} \cdot d\vec{r}$

$$\Delta U = -q \int_i^f \vec{E} \cdot d\vec{r}$$

Define change in voltage (= "potential") as

$$\boxed{\Delta V = V_f - V_i \equiv \frac{\Delta U}{q} = -\int_i^f \vec{E} \cdot d\vec{r}}$$

If $\vec{E}$ = const, $\Delta V = -\vec{E} \cdot \Delta\vec{r}$

Def'n $\Delta V = \Delta U/q$ similar to $\vec{E} = \vec{F}/q$:

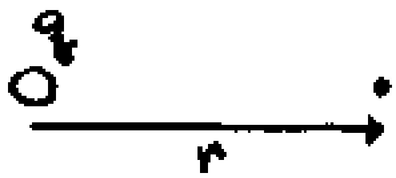
there is a voltage diff. between 2 points even if there is no test charge q moving between the pts.

- $\vec{E}$-field always points from hi V to lo V

$$(\text{hi } V) \xrightarrow[\vec{E}]{\longrightarrow} (\text{lo } V)$$

(If $\Delta\vec{r} \parallel \vec{E}$, then $\Delta V = -\vec{E}\cdot\Delta\vec{r} = (-) \Rightarrow V_f - V_i < 0$
$\Rightarrow V_i > V_f$)

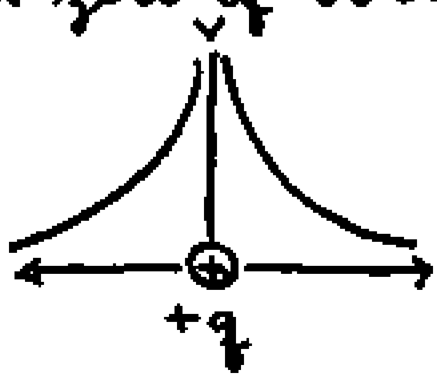
- Voltage near a point charge q

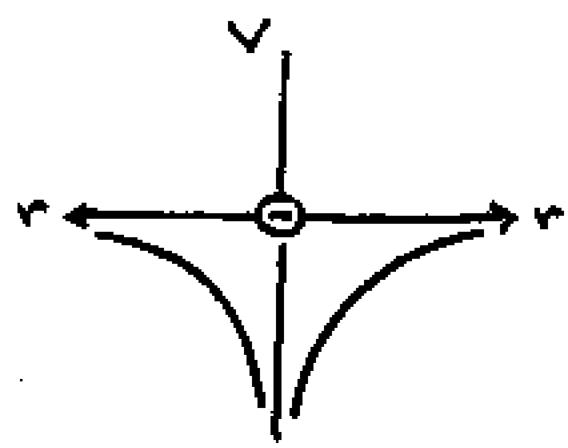


$$\boxed{V(r) = kq/r}$$

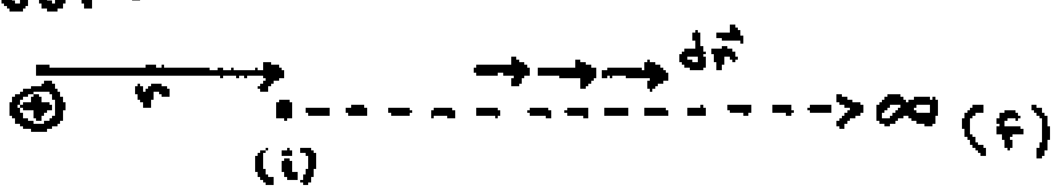
(Not same as $E = k|q|/r^2$)

Convention when dealing w/ pt. charges:
set zero of voltage at $r = \infty$, $V(r=\infty) = 0$





Proof:



$\vec{E} \parallel d\vec{r} \Rightarrow \vec{E}\cdot d\vec{r} = +E\,dr$

$$\Delta V = V_f - V_i = \underbrace{V(r=\infty)}_{0} - V(r) = -\int_r^\infty E\,dr \Rightarrow$$

$$V(r) = +\int_r^\infty E\,dr = \int \frac{kQ}{r^2}\,dr = kQ\left(\frac{-1}{r}\right)\Big|_r^\infty = \frac{kQ}{r} \checkmark$$

Units of voltage = volts (V)

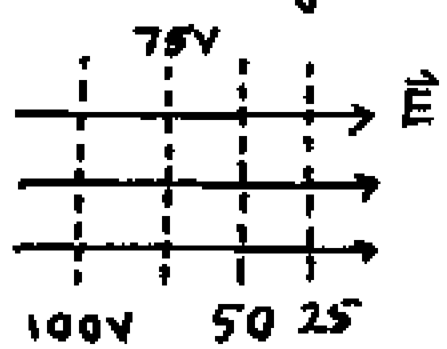
$$\Delta V = \frac{\Delta U}{q} = -\vec{E}\cdot\Delta\vec{r} \Rightarrow 1 \text{ volt} = 1 \, J/_C = 1 \, \frac{N\cdot m}{C}$$

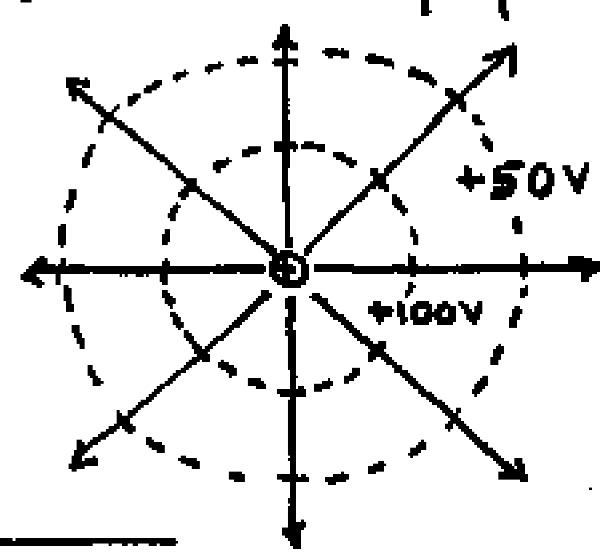
To say that voltage at a point in space is high means it takes a lot of work to put a (+) charge there.

Notice: if $\Delta\vec{r} \perp \vec{E}$, then $\Delta V = -\vec{E}\cdot\Delta\vec{r} = 0$

- No change in V if you move $\perp$ to $\vec{E}$; max change in V if you move $\parallel$ to $\vec{E}$.

$\Rightarrow$ lines or surfaces of const V = "equipotentials" are always $\perp$ $\vec{E}$.





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Voltage due to several charges.

Superposition Principle:

$$V_{tot} = V_1 + V_2 + V_3 + \ldots = \sum_i V_i$$

$$= \sum_i \frac{k q_i}{r_i} \quad \underline{\text{or}} \quad \int \frac{k \, dq}{r}$$

~ <u>much</u> easier than adding vector $\vec{E}$'s.

Proof: $\Delta V = -\int \vec{E}_{tot} \cdot d\vec{r} = -\int (\vec{E}_1 + \vec{E}_2 + \ldots) \cdot d\vec{r}$

$$= -\int \vec{E}_1 \cdot d\vec{r} - \int \vec{E}_2 \cdot d\vec{r} - \ldots = \Delta V_1 + \Delta V_2 + \ldots$$

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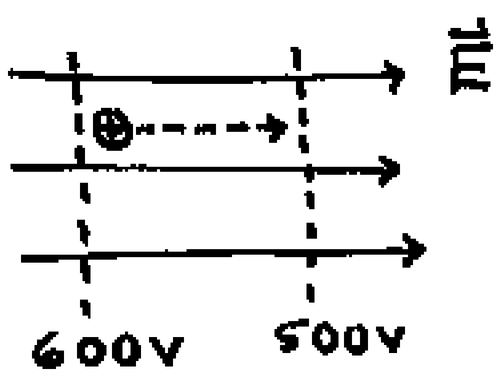
New unit of energy, the electron volt (eV), a non-SI unit.

$\Delta U = q\,\Delta V \Rightarrow$ charge × voltage change = energy

$$1\,eV \equiv (1e) \cdot (1\,\text{volt}) = (1.6 \times 10^{-19} C)(1\,J/C)$$

|charge| of electron $\quad 1\,eV = 1.6 \times 10^{-19}\,J$

Example: a proton ($q = +e$) is released from rest in an $\vec{E}$-field. It falls thru a voltage difference of 100V. What is its final KE?



$KE_{final} = 100\,eV$

No friction $\Rightarrow K + U = \text{const} \Leftrightarrow \Delta K + \Delta U = 0$

$$|\Delta K| = |\Delta U| = |q\,\Delta V| = 1e \cdot 100V = 100\,eV$$

As proton "falls" it loses PE, gains KE.

eV a very convenient unit when dealing w/ charge of e (electron or proton) or multiple of e (ions)

Given $\vec{E}$, we can compute V:

$$\Delta V = -\vec{E}\cdot\Delta\vec{r} \quad \left(= -\int \vec{E}\cdot d\vec{r} \right)$$

Given $V = V(x, y, z)$, how do you get $\vec{E}$?

Spse $\Delta\vec{r} \parallel \vec{E}$ $\xrightarrow{\vec{E}}$ then $\longrightarrow \Delta\vec{r}$ ← (small Δr so $E \approx$ const)

$$\Delta V = -\vec{E}\cdot\Delta\vec{r} = -E\,\Delta r \Rightarrow \boxed{E = -\frac{\Delta V}{\Delta r} = -\frac{dV}{dr}}$$

"E is rate of change of V"

$E = -dV/dr$ only if r-axis is along dir. of $\vec{E}$

More generally, suppose $\Delta\vec{r} = \Delta x\,\hat{x}$, along x-axis, but not necessarily along $\vec{E}$.

$$\Rightarrow \vec{E}\cdot\Delta\vec{r} = \vec{E}\cdot\hat{x}\,\Delta x = E_x\,\Delta x = -\Delta V$$

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{dV}{dx}, \quad E_y = -\frac{dV}{dy}, \quad \text{etc}$$

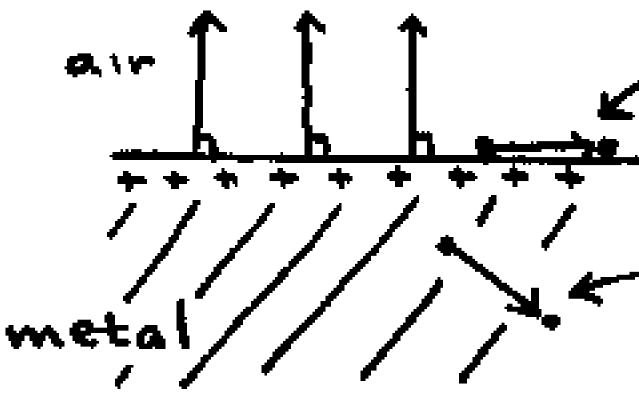
(Actually, must take "partial derivative"

$E_x = -\dfrac{\partial V}{\partial x}$ ← means: hold y, z constant, take derivative w.r.t. x)

Note about units of E:

Since $E = -\dfrac{dV}{dr}$, $[E] = \dfrac{V}{m}$ or $\dfrac{N}{C}$

- Metal objects (conductors) in electrostatic equilibrium are always equipotentials
  (V = const everywhere inside and on surface)



$\Delta V = -\vec{E}\cdot\Delta\vec{r} = 0$ (since $\vec{E}\perp\Delta\vec{r}$)

$\Delta V = 0$ (since $\vec{E} = 0$)

$\Delta V = 0 \Leftrightarrow V = \text{const}$

Bring neutral metal sphere near charged plane. What happens? Sphere becomes polarized (+ repelled, − attracted)