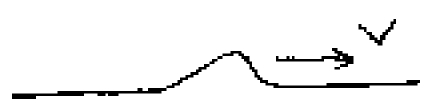
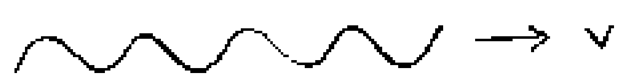
Waves

wave = self-propagating disturbance in a medium

sound wave : medium is air
string wave : medium is string
water " : " " water
E/M wave : medium is nothing! vacuum!

travelling wave carries energy, momentum, information, but not matter!

Travelling wave can be

- impulse  OR
- sinusoidal 

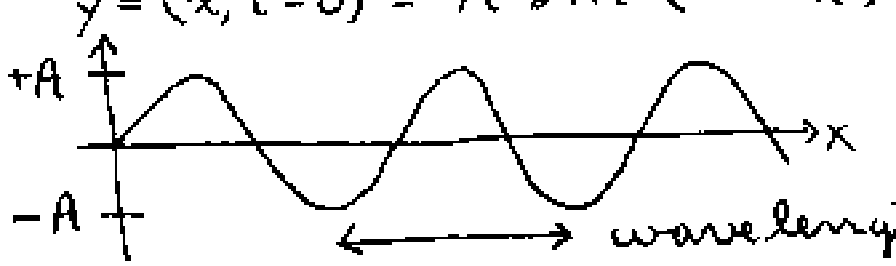
- transverse (displacement of medium $\perp \vec{v}_{wave}$)
  like wave on string or water

  OR
- longitudinal (displacement of medium $\parallel \vec{v}_{wave}$)
  like sound wave

Sinusoidal wave has a wavelength λ and frequency f

displacement of medium from equilibrium =
$y = y(x,t)$

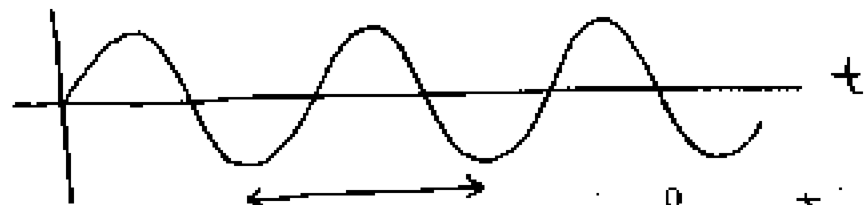
$y = (x, t=0) = A \sin\left(2\pi \frac{x}{\lambda}\right)$ ← snapshot at $t=0$



wavelength λ = length of 1 complete cycle

Now, freeze position x and watch wave go by

$y = y(x=0, t)$



T = period = time for 1 cycle to go by

speed of wave $= v = \frac{\lambda}{T}$ $\quad \left(v = \frac{\Delta x}{\Delta t} = \frac{\lambda}{T} \right)$

frequency $f = \frac{\#\text{ cycles}}{\text{time}} = \frac{1 \text{ cycle}}{T} = \frac{1}{T}$

$\boxed{f = \frac{1}{T}} \Rightarrow \boxed{v = \lambda \cdot f}$

units of f are "hertz" (Hz) = cycles per sec

Usually, v = const independent of λ or f

v depends on properties of medium, not properties of the wave

Example: speed of string wave depends on tension F_T of string and mass per length μ of string

$\left(v_{string} = \sqrt{F_T/\mu} \right)$

$v = \lambda f$ = const $\Rightarrow$ $\lambda \nearrow$ as $f \searrow$
$\lambda \searrow$ as $f \nearrow$

Electromagnetic wave has speed $c = 3 \times 10^8$ m/s (in vacuum) independent of λ or f of E/M wave

Claim: Sinusoidal travelling wave has mathematical form:

$$y(x,t) = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] = A \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} = \text{"wave number"}$$

$$\omega = \frac{2\pi}{T} = 2\pi f = \text{"angular frequency"}$$

$$v = \lambda f = \omega / k \qquad \left(= \frac{2\pi f}{2\pi/\lambda} = \lambda f. \checkmark \right)$$

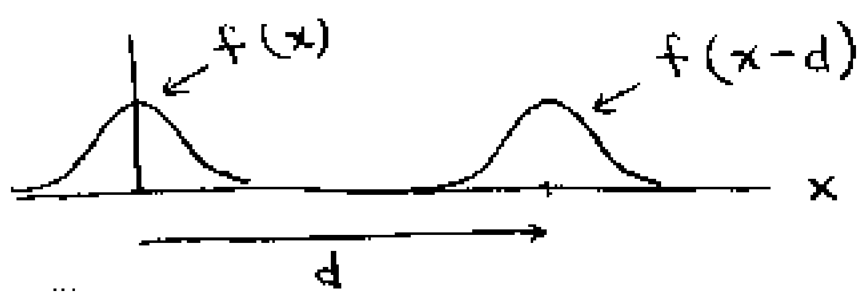
Why?? Any travelling wave (not just sinusoidal) has the form:

$$y(x,t) = \underbrace{f(x - vt)}_{\text{right-going wave } \longrightarrow} \quad \underline{\text{OR}} \quad \underbrace{f(x + vt)}_{\text{left-going wave } \longleftarrow}$$

Consider the "impulse function" $f(x)$

$f_{Max} = f(0)$

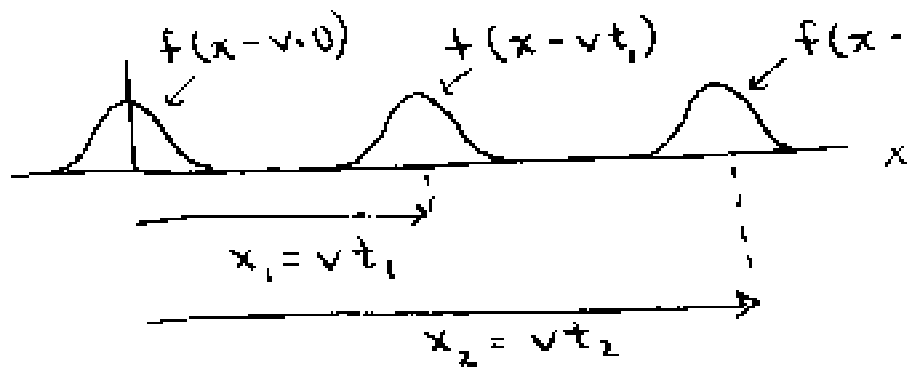
Notice that function $f(x-d)$ looks exactly like $f(x)$ except $f(x-d)$ is shift to the right by distance d



$f(x-d)$ has max at $f(0)$ that is, at $x = d$

Set $d = v \cdot t \Rightarrow y(x,t) = f(x - vt)$ is function shifting to the right at speed v

$f(x-vt)$ is f_{Max} when $(x-vt) = 0$. As t increases, x must increase to keep $(x-vt) = 0$



To make a right-going traveling wave, take any function $f(x)$ and replace x w/ $(x - vt)$

Back to sinusoidal wave:

$$y(x, t=0) = A \sin\left(2\pi \frac{x}{\lambda}\right)$$

$$\Rightarrow y(x,t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

$$v = \frac{\lambda}{T} \Rightarrow y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

(When x changes by λ or t changes by ~~*~~ T, then sinusoid goes thru 1 complete cycle)

$$\boxed{y = A \sin(kx - \omega t)}, \quad k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

$$v = \frac{\lambda}{T} = \lambda \cdot f = \omega / k$$