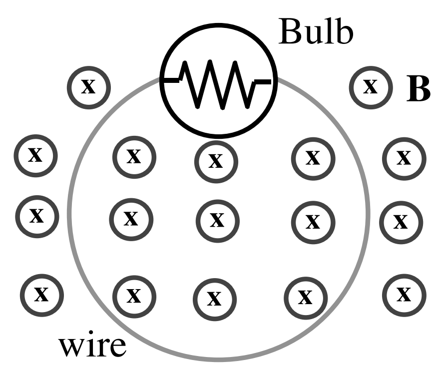
Faraday's Law:

Gauss' law tells us that "charges create **E** fields".
And we know that a steady **E** field pushes charges around, makes *currents* flow.
We've used the word "EMF" for this occasionally, an EMF is any voltage difference capable of generating electric currents.
Think of EMF = ΔV (=$E\,\Delta x$)
(Note: batteries have an EMF, but resistors do NOT. Even though an R can have a voltage difference across it, it is not *generating* it! Resistors don't make currents spontaneously flow, batteries can.)

Michael Faraday, a British physicist (at the same time as Joseph Henry, an American, but Faraday published first) about 180 years ago (1831) discovered a remarkable new property of nature:

Changing magnetic fields (not steady ones) can make EMF's.
In other words, a time-varying **B** field can make currents flow.

Imagine a wire loop sitting in a **B** field, like this:



If the **B** field is steady then there is NO CURRENT, the bulb is dark.

But, if the **B** field *changes* with time, the bulb lights up, a current flows through that wire (!)

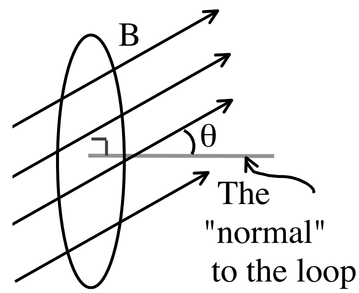
You might do this by e.g. just moving a big magnet closer, or farther away (yes, weakening the **B** field is still a *change*)... or move the coil itself closer (or farther) from the magnet face.

There's no battery here, no external voltage source, but the bulb still glows! This effect is surprising, it's something new...

Faraday spent only 10 days of (intensive) work on these experiments, but they changed the world radically.
This is how most of modern society's electricity is now generated!

Faraday worked out an equation (Faraday's Law) which quantifies the effect (how *much* current do you get?)
But before we can write it down, we need to first define one relevant quantity we haven't seen yet.

Imagine a **B** field whose field lines "cut through" ("pierce") a loop. Define θ as the angle between **B** and the "normal" or "perpendicular" direction to the loop.
We will now define a new quantity, the **magnetic flux through the loop**, as

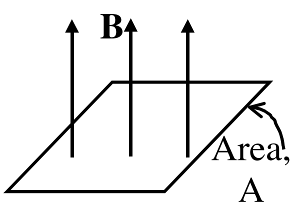


Magnetic Flux, or $\Phi = B_\perp A = B\ A \cos\theta$

$B_\perp$ is the component of B perpendicular to the loop: $B_\perp = B \cos\theta$.
The UNIT of magnetic flux = $[\Phi]$ = T m^2 = Weber = Wb.
If B is not uniform, we find the flux by adding it up over little patches of area, $\Phi_B = \iint \mathbf{B} \cdot d\mathbf{A} \ldots$

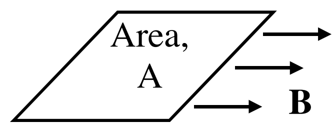
Go back to Ch 24 where we defined *electric* flux, it's all pretty much the same!

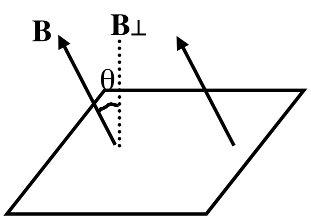
<u>Examples of calculating magnetic flux:</u>



Here (picture to the left) $\Phi = B\ A$,
because B is perpendicular to the area. ($\theta=0$)

Here (picture to the right), $\Phi = 0$, because B is parallel to the area. ($\theta=90$.) No flux: the B field lines don't "pierce" this loop, they "skim" past it...





Here, (picture to the left), $\Phi = B\ A \cos\theta$. The flux is reduced a bit because it's not perfectly perpendicular.

Just like Electric flux - it tells the number of B field lines "poking through" a small loop

Faraday's Law: The induced EMF in any loop is

EMF = - dΦ/ dt .

(Φ is magnetic flux, t is time, this is the rate of change of flux through an area)

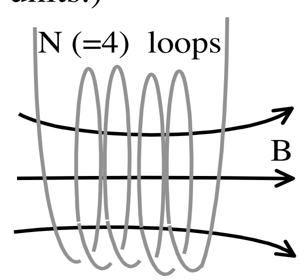
• If you put a loop into a B field, and then *change the flux* through that loop over time, there will be an EMF (basically, a voltage difference) induced.
Current flows, if you have a *conducting* loop.

• EMF is a lot like the battery's voltage, except it's not "localized", it's distributed around the loop we're considering. If you want a formal definition of EMF, it's $EMF = \oint \mathbf{E} \cdot d\mathbf{L}$, which looks a lot like $\Delta V = -\int \mathbf{E} \cdot d\mathbf{L}$

• The formula says it is only the *change* in flux through the loop that matters. A huge B field (lots of flux) does NOT make the EMF, it's the *change* in B with time that does the trick.

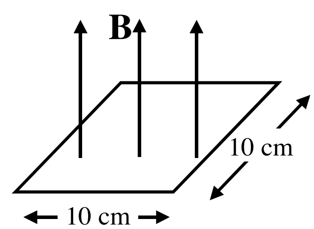
• This equation has not been derived - it's just an experimental fact!

• Units are $\frac{\text{Wb}}{\text{sec}} = \frac{\text{T m}^2}{\text{sec}} = \left(\frac{\text{N}}{\text{A m}}\right)\frac{\text{m}^2}{\text{sec}} = \frac{\text{N m}}{\text{A sec}} = \frac{\text{J}}{\text{C}} = \text{Volt}$ (yikes!
It's a mess, but it works out. The formula gives the correct units.)



• If you were to "pile up" N loops on top of each other, the effective flux will be increased by a factor of N, the formula becomes <u>EMF=-NdΦ/dt</u>. (Do you see why?)

• Since Φ = B A cosθ, *you can change the flux in many ways: you could change B,* or *area,* or *the angle between **B** and the loop.*



Example: **B** is perp. to this loop, θ=0, as shown.
(Remember, θ is the angle from the *normal*)
The area is A= (0.1m)^2 = .01 m^2
Suppose B is 1 Tesla, as shown, and then you turn it off, taking a time of 2 seconds to do so...

Faraday's law says there will be an "induced EMF", or voltage, around the loop,
|EMF|=|ΔΦ/Δt| = [(1 T * 0.01 m^2) cos(0) - 0 / (2 sec) = .005 V

If you had N=1000 coils (loops) of wire, all stacked (coiled) up around that same perimeter, you'd get |EMF|=5 V, enough to light up a small bulb (or perhaps warm up the wire of the loop). But remember, you'd only have this voltage for those 2 seconds while B was changing! Once B reaches 0 (and presumably stays there), there is no more *change*, and so |EMF| goes back to 0.

What's that *minus sign* about in Faraday's law?
Don't plug it in blindly - it's only there as a *reminder*, you must *figure out* the direction of the induced current flow, or voltage difference (the direction of the EMF) by **Lenz's Law:**

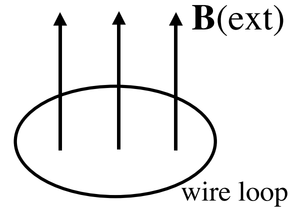
• Induced EMF tries to cause current to flow. If current flows, it will create a new (usually small) **B** field of its own, which we will call **B**(induced). (You'll need to remember what I've called "RHR #1b": i.e. how current in a loop produces **B**)
• I will call the original or "outside" field: **B**(external)

The direction of **B**(induced) *opposes the change in the original* **B**.

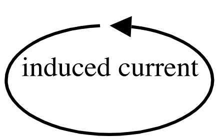
√*Note*: **B**(induced) *does NOT* necessarily oppose **B**(external)(!!)
It is opposite the CHANGE of **B**(external) (or more accurately, the change of flux). **B** is a vector, you really have to *think* about the direction of the *change* of that vector....

Lenz's law is a mouthful! It tells you the direction that the induced current will flow. Nature creates a **B**(induced) to *fight the change*.

Example: Consider a **B**(ext) that is up, and pierces a wire loop, as shown. It might be caused by a big old magnet or something.
If **B**(ext) stays constant, there is no change, and so *no current* spontaneously flows around the loop.

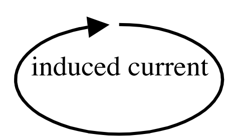


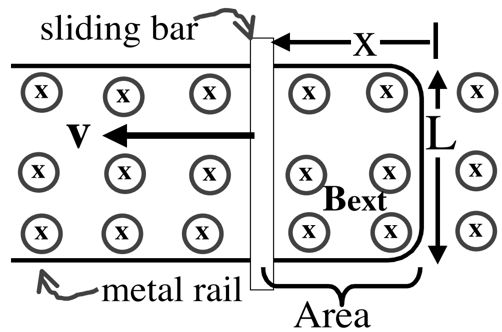
If **B**(ext) starts to decrease, nature will try to *fight that change*.
(Remember, if an "up vector" is decreasing, the change is DOWN)
Lenz's laws says a current will flow (or try to flow) to induce an upwards B field, to try to keep things as they were.
B(induced) may be small: it probably won't succeed, but it *tries*.
The direction of induced current is shown to the left.



B(induced) points up, opposite the *change* in **B**(ext). (Here, this just happens to be the same direction as **B**(ext) was originally, but that's *irrelevant,* it's a *coincidence* here.)

If **B**(ext) instead starts to *in*crease with time, then to fight that change you will induce a *downward* **B**, as shown. (You should look at Knight Fig 33-23 and try to figure out which way i should go. It takes practice to get Lenz's law.)





Example: A metal bar slides along conducting metal tracks in a uniform **B** field pointing *into* the page.

Push the bar to the left (as shown), and consider the conducting loop consisting of rail + slider.

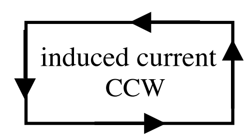
The area inside that loop is increasing, and so flux through the loop (B*A) is also increasing. (A = L*x, and x is increasing with time)

$|EMF| = |\, d\Phi/dt \,| = |B\, dA / dt \,| = B\, L\, dx / dt = B\, Lv$
(Because v = dx / dt is the speed of the sliding bar)
That means current flows around the loop, by Faraday's law.
If you put a light bulb somewhere in that circuit, it'd glow.
The bigger B is (or the faster you slide the rod), the more current.

Now, what *direction* will the current flow, CW or CCW?
Lenz's law will tell us!

The external flux is *into* the page and *increasing* with time. So the change in flux is *into* the page. (Do you see that?)
Lenz's law says current will start to flow to *fight the change*.

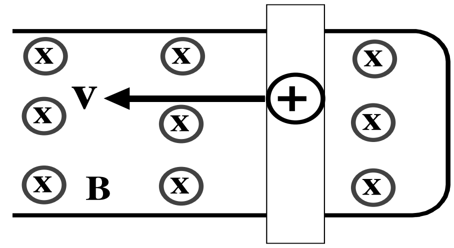
That current will induce a new **B** that points *out* of the page.
By RHR #1b, that means CCW.



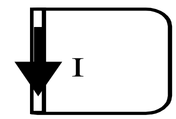
Note: It's not that **B**(induced) points out of the page because **B**(ext) is into the page. That's a coincidence. It's opposite the CHANGE in flux, not oppose the direction of flux.
E.g., If instead you push the slider to the right, **B**(ext) is of course the same, but now the flux is *decreasing* with time, that's opposite: the **B**(induced) will also be opposite, i.e. the current flows CW!!

Knight Fig 33-7 through 9 are similar. Try to figure out the direction of the induced current there, for yourself.

There is a totally different way, kind of "previous chapter style", to reach the same conclusion about the direction of induced current in the previous example.



Consider a small + test charge sitting somewhere in the slider. It (and of course every other atom, electron, etc. too) moves along with the slider to the left, with velocity **v**.
It sits in a uniform **B** field. That means it feels a force (F = q v B), and the direction is given by RHR #2, try it yourself, convince yourself it is DOWN.



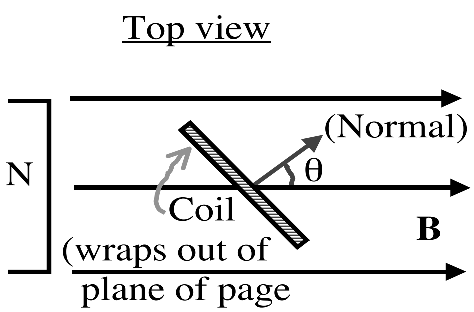
But it's a test charge in a conductor - it's free to move. What that means is the **B** field thus forces test charges down the slider, which means a current I down - exactly the direction we got before (from Lenz's law)
Cool - a rather different way of looking at it, but the same result.

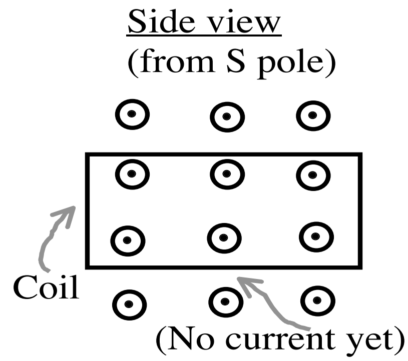
Comment: We just saw there is an (induced) current flowing in the slider, and this current sits in a **B** field. Any current in a **B** field will feel a force F = ILB!
Work out the direction for yourself (!), using RHR #2.
I claim in this example is to the *right*. That means the **B** field tries to slow down the slider. It's kind of like magnetic friction. If you did not continue to push that slider to the left, the induced current feels this force that would slow the slider down to a halt. We call any induced current like this, caused by conductors moving in magnetic fields, ***eddy currents*** (maybe because they look a little like water eddy's in a river?)

Eddy currents always cause slowing or "braking" forces. They behave in some ways like magnetic friction. This effect is *used* to slow down some kinds of trains - it's a "retarding" force proportional to velocity. By changing the resistance in the rest of the track you can change the magnitude of the current, hence the force - and it's easy to control electronically. Eddy currents have many other industrial applications, including stopping trains (or the new hybrid electric cars), or in detecting coins in coin vending machines!

Final note: Suppose the sliding rode had resistance R (and the rest of the rail had negligible resistance). Then:
Total power dissipated here (in resistor) is (EMF^2/R) = B^2 L^2 v^2 / R
Meanwhile, you have to apply a force to make the rod keep moving, which we just argued was F=ILB. So you must do work/time = F*distance/time = ILB v.
But since I = EMF/R = BLv/r. So you supply power B^2L^2v^2/R, which is *precisely* what the resistor is "burning up". You put it in, the resistor dissipates it!

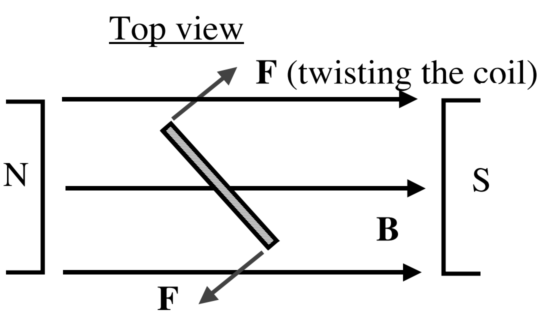
<u>Different example</u>: Put a *coil* of metal into a fixed **B**(external)





We've seen this setup before, in Ch. 29. (Remember, if you run a current I through that coil of wire, RHR #2 says there are forces that twist the loop. That's a MOTOR, putting current into it causes mechanical motion.)

But you can also do the opposite: suppose you (or a waterfall, or a steam engine...) mechanically force the loop to start to rotate. What happens then?



(B and A are both constant!)

The flux through the coil, $\Phi = B\ A \cos\theta$, is *changing*, because θ is now steadily changing.

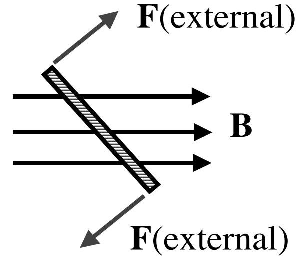
Faraday's law says
$|EMF| = |\ d\Phi/\ dt\ | = B\ A\ d\cos\theta/\ dt.$

There is an induced EMF, a current spontaneously starts to flow in the loop.
If you have wires leading out from the loop (like in the picture of the motor in Ch. 20) current flows to the outside world.

This is an ***electrical generator.*** It's just like a motor, only opposite: mechanical motion causes current. (You can even use the *same apparatus* either way, as a motor or as a generator.)

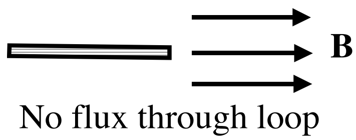
If you rotate it *steadily*, i.e. if $\theta = \omega t$, then we can take the derivative, and get
$EMF = -N(coils)^*\ B\ A\ \omega\ \sin \omega t$
So the resulting EMF varies with time, sinusoidally.

What is the *direction* of the current in the last example?
At the moment shown, the flux through the coil is to the right, and it is *increasing*.



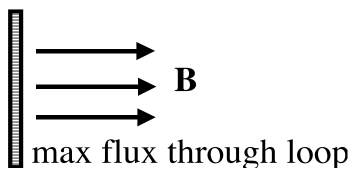
Think about this (it's 3-D spatial imagery)
The B field isn't changing, but it's "piercing" the loop more and more efficiently, around the moment shown.
Let me exaggerate to convince you:

just before :

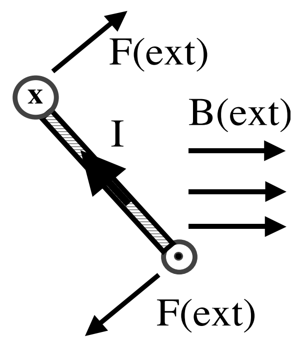


The flux is *increasing to the right*.

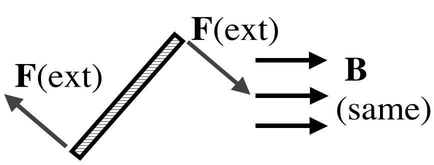
just after :



Lenz' law says the induced current will fight this change, i.e. you will create a **B**(induced) that must point to the *left*.
By RHR #1b,
that means the current at the moment in question flows around the loop as shown here.



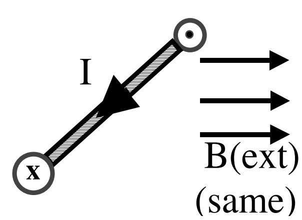
But now let's look again just a little later...



The flux is still, at this moment, to the right. But, now it is *decreasing* with time. (In a moment, there won't be any flux, when the loop is again parallel to the B field)

If flux is "right but decreasing", that means the *change* is leftwards.

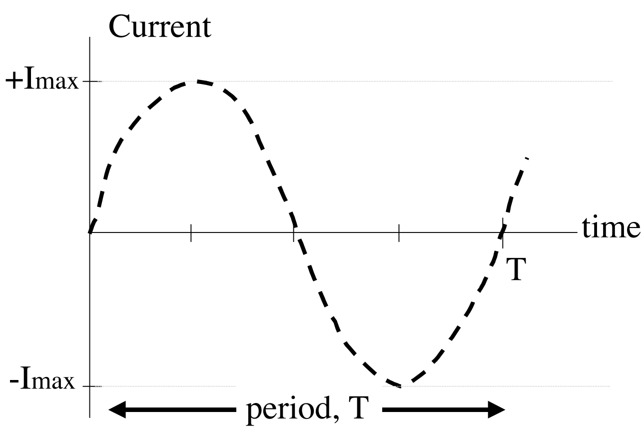
To fight the change, you need to make a current which will make a "*rightwards*" B field, like this:



Notice how the current has flipped its direction.

Bottom line: You do work on the coil to rotate it. In return, you get an electrical current out. This is precisely how our big power plants work - large coils being rotated in a big fixed B field (or sometimes, the magnet gets rotated around a fixed coil.)

The current is flipping direction each partial rotation of the coil (see the previous page for this story). If you graph current coming out of the coil as a function of time, it looks like this: $I = EMF/R = -N(\text{coils})*(B\ A/R)\ \omega\ \sin \omega t$



The frequency is exactly the same as the mechanical frequency of the rotating loop. In the US, that means you must turn that loop 60 times/sec, or 60 Hz.

Suppose you *stopped pushing*. You might imagine that if there was no friction, the loop would keep turning, giving you "free electricity". No such luck. There is still a current flowing, and this current is in an external **B** field, so it feels a force. RHR #2 tells the direction (work it out, looking at the pictures on the previous pages.)

At all times, the resulting forces make a torque that acts to *slow down* the loop. This is the "eddy currents" story again. Induced currents are caused by conductors moving in a **B** field. They will always act to slow things down.

This is a law of nature. If the forces ever acted in the *other* direction, i.e. to speed things up, you'd be getting something for nothing, violating conservation of energy. (Lenz's law, that minus sign in Faraday's law, is basically the statement of conservation of energy)

Induced Electric Fields:

EMF, like any "voltage difference", tells you about work per charge, or $EMF = \oint \mathbf{E} \cdot d\mathbf{L}$. Which means that we can rewrite Faraday's law as:

$$\oint \mathbf{E} \cdot d\mathbf{L} = -\frac{d\Phi_B}{dt}$$

$$= -\frac{d}{dt}\left(\iint \mathbf{B} \cdot d\mathbf{A}\right)$$

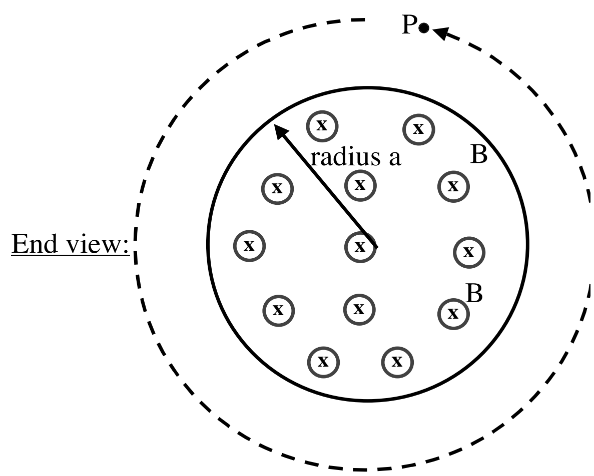
(This reminds me a little of Ampere's law, $\oint B \cdot d\mathbf{L} = -\mu_0 I \ = \ -\mu_0 \frac{dq}{dt}$, something you might think about... but not for now)

Faraday's law says "changing B (in time) creates **E** fields"!

(Once you have E fields, they can drive currents, which is what we've been looking at this whole chapter)

Example: Consider a solenoid which has a uniform B field *inside*. Suppose you increase the current through the solenoid, so B inside changes with time.

Now look *outside* the solenoid, (e.g at point P, shown). where *there is no B field at all, ever*.



Faraday's law says if you follow the dashed loop, and integrate **E** . d**L**, you will not get zero, you get

$$\oint \mathbf{E} \cdot d\mathbf{L} = -\frac{d}{dt}\left(\iint \mathbf{B} \cdot d\mathbf{A}\right) \ = -\pi a^2 \frac{dB}{dt}.$$

This says that although B is (always!) zero out there, the changing B field inside the solenoid *creates* a nonzero **E** field out there. The E field runs in "circles" around the solenoid. The direction will depend on whether B is increasing or decreasing. (In the case shown, if B is increasing, Lenz' law tells us that the E field points counterclockwise around the loop shown. Convince yourself!)

In this last example, since it's totally symmetric, **E** must be the same all the way around the loop at some fixed radius r, which means we can do the "line integral" on the left side of the equation on the last page, to get

$E (2 \pi r) = \pi a^2 dB/dt$.

But recall $B(solenoid) = \mu_0 n I$ (where n is the number of turns per unit length) so we have

$E (2 \pi r) = \pi a^2 \mu_0 n dI/dt$, or

$E = (\mu_0 n a^2 / 2 r) dI/dt$.

(CCW in the case shown. If I was decreasing, it'd be CW)

In general, E that arises from Gauss' law (i.e. from static charges) "diverges" from the charges, whereas E that arises from Faraday's law (i.e. from changing magnetic flux) tends to "run rings" around the changing flux.

This makes for "non-conservative" forces, because if you were to allow a little charge to run around that ring, there would be a net work done on it. That is, the work done on the charge as it moves from place to place depends on the path it takes!

It also means that, although "electrical potential" (which is potential energy/charge, or voltage) is perfectly well defined (and very useful!) for static charge configurations, it really isn't useful or even meaningful when you have time varying magnetic fields. That's why we've starting introducing the term EMF in this chapter.

(For notes on inductors, the last part of this chapter, you'll have to see my handwritten notes)