Electromagnetic Waves

Last semester, we studied Classical Mechanics. The fundamental laws (axioms) of Classical Mechanics are called Newton's Laws.

This semester, we are studying a subject called Classical Electromagnetism. There are four fundamental laws of electromagnetism, called Maxwell's Equations (after the Scottish physicist James Clerk Maxwell).

(1) Gauss's Law $\oint \vec{E} \cdot d\vec{a} = \frac{q_{enclosed}}{\varepsilon_0}$ (E-fields are caused by charges.)

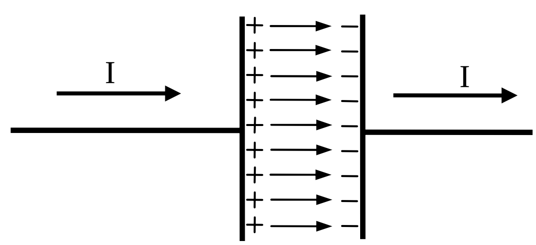
(2) Faraday's Law: $\oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\left(\int_S \vec{B} \cdot d\vec{a}\right)$ (E-fields are also caused by changing B-fields.)

(3) Ampere-Maxwell Law: $\oint_{\mathcal{L}(A)} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed} + \mu_0 \varepsilon_0 \frac{d}{dt}\left(\int_A \vec{E} \cdot d\vec{a}\right)$

(B-fields are caused both by currents and by changing E-fields.)

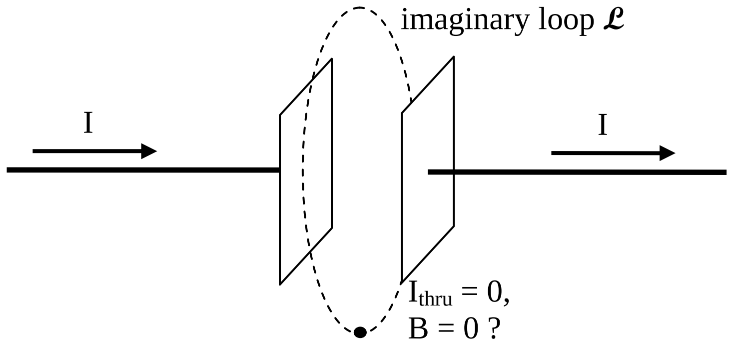
(4) Gauss's Law for B-fields: $\oint \vec{B} \cdot d\vec{a} = 0$ (There are no magnetic monopoles).

Except for the last term in equation (3), all four of these laws had been discovered experimentally before Maxwell started his research in the 1850's. So why do we call them Maxwell's Equations?



Maxwell realized that Ampere's Law,

$\oint_{\mathcal{L}(A)} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}$, was incomplete. He noticed that there are situations in which Ampere's Law fails to give the correct answer. For instance, if a capacitor is being charged up by a steady current, then there must be a B-field around the capacitor, caused by the nearby currents. But according to the original form of Ampere's law, if we consider an imaginary loop circling the capacitor (diagram below), the current through this loop is zero. So Ampere's Law predicts that the B-field is along that loop is zero (since $I_{thru} = 0$). Maxwell noticed that although there is no current through the loop, there is a changing E-field flux through the loop. He saw that he could fix the problem by modifying Ampere's Law with the addition of a new term. The changing electic flux in the capacitor leads to a quantity that has the dimensions of current: $\varepsilon_0 \frac{d}{dt}\left(\int_A \vec{E}\cdot d\vec{a}\right)$.

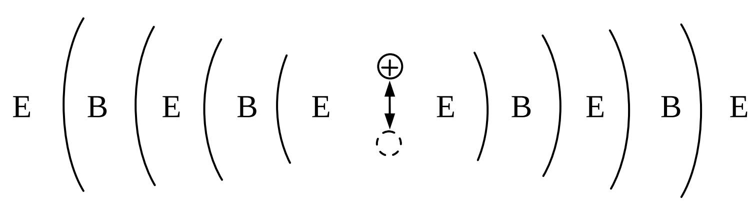


Notice that, from Gauss's Law, the quantity $\varepsilon_0 \int_A \vec{E}\cdot d\vec{a}$ has the dimensions of charge. So, $\varepsilon_0 \frac{d}{dt}\left(\int_A \vec{E}\cdot d\vec{a}\right)$ has the dimensions of current. Maxwell called this new quantity the *displacement current*. By replacing the current I in Ampere's Law $I + \varepsilon_0 \frac{d}{dt}\left(\int_A \vec{E}\cdot d\vec{a}\right)$, he was able to resolve the problem.

This new form of Ampere's Law (now called the Ampere-Maxwell Law) appealed to Maxwell's sense of aesthetics. There was now a pleasing symmetry in the equations:

- changing B-fields create E-fields (Faraday's Law)
- changing E-fields create B-fields (Ampere-Maxwell Law)

Maxwell realized that because of this symmetry, the equations predicted a peculiar kind of self-sustaining interaction between E and B fields. Maxwell thought: Suppose you have a charge q and you *shake* it, back and forth. The q creates an E-field, but when you shake the charge, you are changing the E-field in the space around it. This changing E-field creates a B-field. But now you just created a B-field where there was none before, so you have a changing B-field. This changing B-field will create an E-field, and that newly created E-field will create a B-field, which will create an E, which will create a B, which will … (the process will go on, forever). Maxwell showed that the equations predicted the existence of an *electromagnetic wave* which travels outward from the shaking charge:



Maxwell computed the speed of this strange, new electromagnetic wave and found that the speed was given by a simple formula: speed $v = c = \dfrac{1}{\sqrt{\varepsilon_o \mu_o}} = 3.0\times10^8$ m/s .

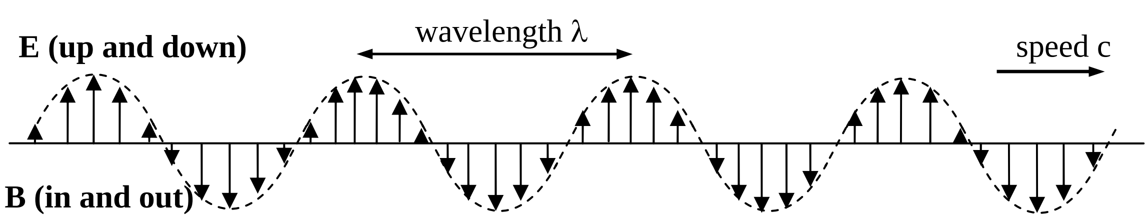
This number is the same as the speed of light! Maxwell had shown that light was an electromagnetic wave! Before Maxwell, scientists had no clear idea what light is. This was a great synthesis, a bringing together of previously separate fields of physics: electricity, magnetism, and optics. Before Maxwell, no one knew what light was. It was known that light was some kind of wave (we will see the evidence for this later), but no one knew what kind of wave it was. Maxwell figured it out.

Light is an electromagnetic wave which is created by accelerating electric charge.

Wave speed is $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{\text{1 wavelength}}{\text{time for 1 }\lambda\text{ to go by}} = \dfrac{\lambda}{T}$ $\qquad v = \dfrac{\lambda}{T} = \lambda f$

For light waves, speed v = c, this is written $\boxed{c = \lambda f}$

EM waves are transverse waves: the E- and B-field vectors are both perpendicular to the direction of the wave. Drawing an EM wave in space is quite difficult; the E and B-fields are everywhere and intimately mixed. The figure here shows the E-field along a particular line, at a moment in time.



All EM radiation is caused by shaking (accelerating) electric charge. The more rapidly the charge is shaken (the higher the frequency of the shake), the shorter the wavelength of the light, since $\lambda = \dfrac{c}{f}$. Now we can understand why all things glow (give off light) when they get hot.

When something is very hot, its atoms are jiggling furiously. Atoms are made of charges (electrons and protons), and the jiggling charges emit EM radiation.

Different wavelength ranges are given names:

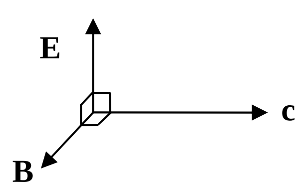
Wavelength λ	**Name**	**Use/occurrence**
< ≈ 0.01 nm	Gamma-rays	Radioactivity
≈ 0.01 nm → ≈ nm	X-ray	medical
≈ nm → 400 nm	Ultraviolet(UV)	Sunburns, "black" lights
400nm → 700 nm	Visible	Human seeing
700nm → ≈ 1mm	Infrared (IR)	"Heat rays"
≈ cm	microwave	Communications, microwave ovens
≈ m → km → ∞	radio	Radio, TV

Electromagnetic radiation (light) can have *any* wavelength. But our eyes are sensitive only to a narrow range of wavelengths between 400 nm and 700 nm. Different wavelengths in this range of *visible light* correspond to different colors. Wavelength = 700 nm light appears red to us, 400 nm light appears violet, and the wavelengths in between correspond to all the colors of the rainbow (ROYGBIV). All wavelengths outside this narrow band are invisible to human eyes.

Some important facts about EM waves:

- EM waves are transverse:

The E and B-field are perpendicular to each other are each perpendicular to the direction of propagation, like so:



Radio receiving antennas must be oriented correctly in order to function: the wire antenna must be parallel to the E-field in order for the electrons in the wire to be accelerated along the wire.

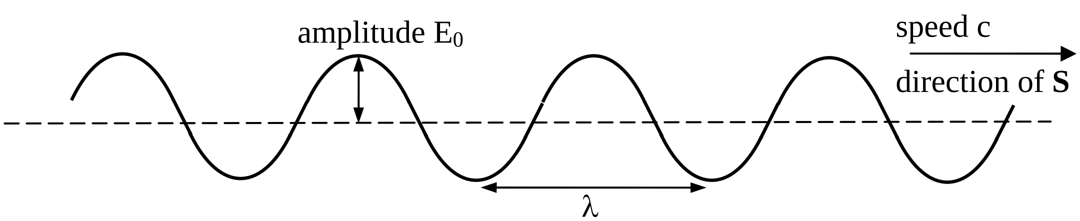
- The E and B fields are in phase: E reaches max at same time/place as B does. The amplitudes of the E and B fields are related by $B_0 = \frac{E_0}{c}$. (Notice that units are OK: $F = qE = qvB \Rightarrow E = vB$)

- EM waves carry energy. In an EM wave, the energy density of the E-field ($u_E = \frac{1}{2}\varepsilon_0 E^2$) is the same as the energy density of B-field ($u_B = \frac{1}{2\mu_0} B^2$).

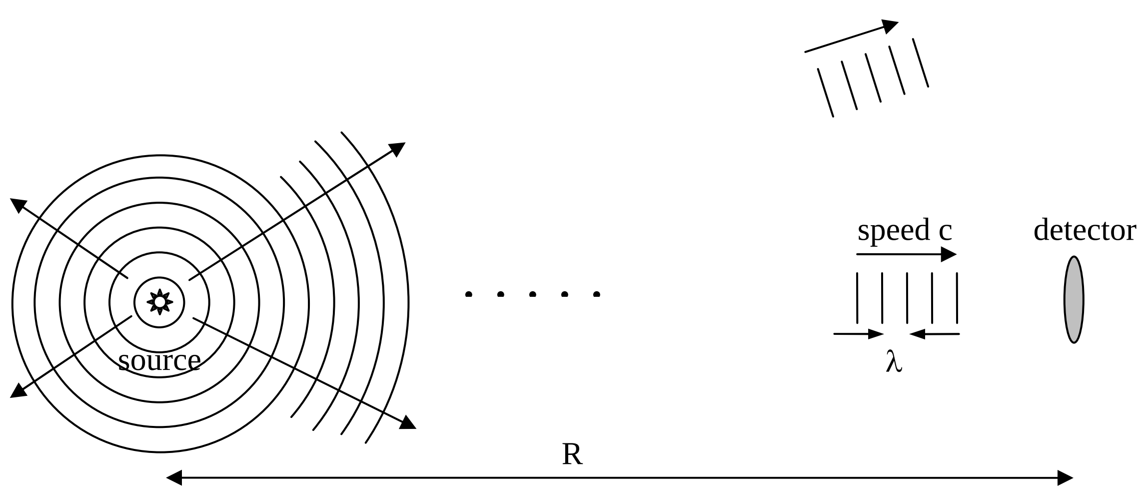
You can feel the energy of an EM wave hitting your face when you face the sun. The *intensity* I is defined as the energy per time *per area* impinging on a surface:

$$\text{intensity} = \frac{\text{power}}{\text{area}}, \qquad \boxed{I = \frac{P}{A}} \qquad \text{units}[I] = W/m^2$$

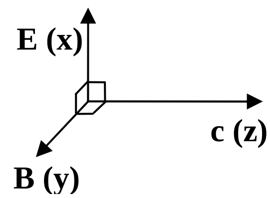
Intensity is also called brightness. The intensity of an EM wave is proportional to E^2. The intensity is described by the Poynting vector: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$. The instantaneous value of the intensity is $S = EB/\mu_0$. The average intensity is $S_{rms} = E_{rms}\, B_{rms} / \mu_0 = E_0\, B_0 / (2\mu_0) = E_0^2 / (2\, c\, \mu_0)$



Consider a small source of monochromatic (single-wavelength) light, emitting energy at a rate P (in J/s). If the source sends light out in all directions uniformly, then the *wavefronts* are expanding spherical shells. A small detector (like an eyeball), very far from the source, will sense a small portion of the wavefronts. Over the small region that is detected, the wavefronts are nearly flat planes. Such a beam of light is called a *plane wave*.



A plane EM wave, traveling in the z-direction, has the mathematical form:



$$\vec{E} = E_0 \sin(kz - \omega t)\,\hat{x}$$
$$\vec{B} = B_0 \sin(kz - \omega t)\,\hat{y}$$

Notice that E and B are in phase.

Example of intensity calculation: Suppose a small source of EM radiation emits power P_o isotropically (uniformly in all directions). What is the intensity of the light at a distance R from the source? Answer: Consider the energy U_o emitted from the source during a very brief time interval t_o. When that energy is carried by the EM wave a distance R from the source, the energy is spread out uniformly over a sphere of area $A = 4\pi R^2$, and the energy per area is $U_o / (4\pi R^2)$. The energy per time per area, the intensity, is then $I = \dfrac{U_o / t_o}{A} = \dfrac{P_o}{4\pi R^2}$. Notice that the intensity falls as $1 / (\text{distance})^2$. The more distant a light source, the dimmer it appears.

Example of power detected from a distant source: What is the power entering an observer's eye from a 100-W tungsten-filament lightbulb a distance R = 20 m away? Only about 3% of the power from an incandescent light bulb comes out as visible light (the rest is heat). The diameter of a human eye pupil is about 2 mm.

Answer: The visible light power from the bulb is $P_o = 3$ W. At R = 20 m, the intensity is

$I = \dfrac{P_o}{4\pi R^2} = \dfrac{3\,\text{W}}{4\pi\,(20\,\text{m})^2} = 6.0 \times 10^{-4}\ \text{W}/\text{m}^2$. The power entering a detector of area A is

$$(\text{power detected}) = \frac{\text{power}}{\text{area}} \cdot (\text{detector area}) = \text{intensity} \cdot (\text{detector area}) = \frac{P_o}{4\pi R^2} \cdot A\ .$$

In this case, A = area of human iris = $\pi r^2 \approx 3 \cdot (1\text{mm})^2 = 3 \times 10^{-6}\ \text{m}^2$. So the power detected is $(6 \times 10^{-4}\ \text{W/m}^2)\,(3 \times 10^{-6}\ \text{m}^2) = 1.8 \times 10^{-9}$ W (not much power – the eye is extremely sensitive!)

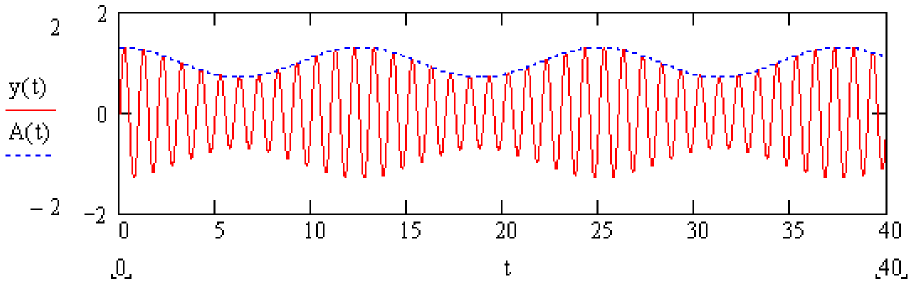
AM vs. FM

TV transmissions are in the radio range of wavelengths: λ_{TV} = 1 to 5 m, frequency $f \approx 10^8$ Hz = 100 MHz. On TV, different channels corresponds to different frequencies. For instance, channel 6 is allotted the frequency range f = 82 – 86 MHz (wavelength range λ = 3.4 – 3.7 m)

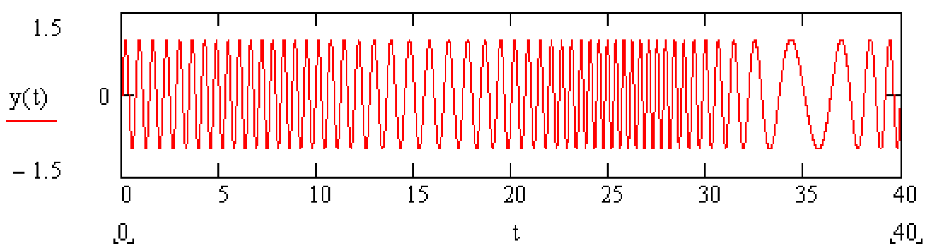
The signal (audio and video information) is carried by a range of frequencies (Δf = "bandwidth") centered on a "carrier frequency" f_o.

In audio-only radio transmissions, the signal's information is encoded either as

Amplitude Modulation (AM)



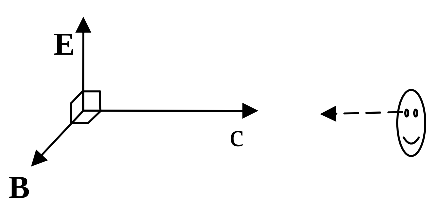
or Frequency Modulaiton (FM)



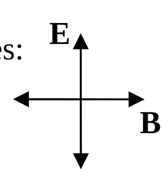
Analog television signals (which are no longer used as of 2009) are always sent as FM. Digital TV signals are sent in an entirely different format: the picture is encoded as a series of numbers (0's and 1's).

Polarized Light

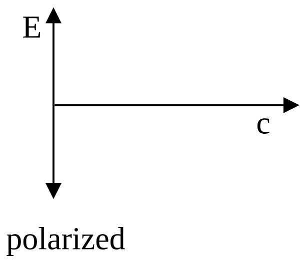
EM wave always has direction of **E** ⊥ direction of travel:

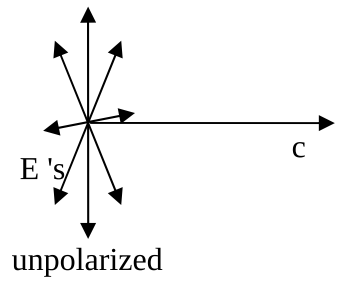


observer sees:

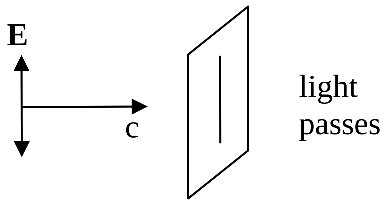


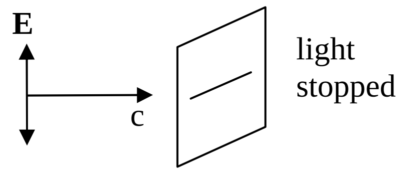
Ordinary light (from the Sun or a lightbulb) is unpolarized, a mixture of waves with E-field in random directions but always ⊥ direction of travel.



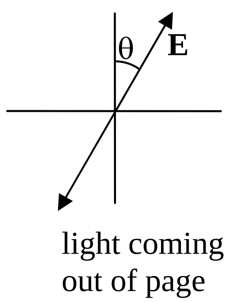


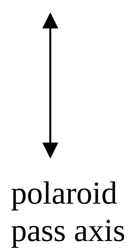
Polarizer = polaroid filter = filter that passes light with the E-field along the "pass axis" of the filter.

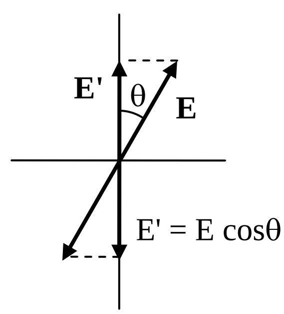




If **E** is not parallel to the pass axis, then only the *component* of **E** along the pass axis gets through.







The intensity of the light is proportional to E^2 , so $I_{trans} = I_o \cos^2 \theta$.

For unpolarized light, with is a mixture with θ random, $\langle \cos^2 \theta \rangle_{avg} = \frac{1}{2}$, so $I_{trans} = \frac{1}{2} I_o$.

