

Electric Currents and Simple Circuits

Electrons can flow along inside a metal wire if there is an E-field present to push them along ($\vec{F} = q\vec{E}$). The flow of electrons in a wire is similar to the flow of water in a pipe.

Definition: electric current $I = \frac{\Delta Q}{\Delta t}$ = rate of flow of charge

units [I] = coulomb/second = 1 C / 1 s = 1 ampere (A) = "amp"

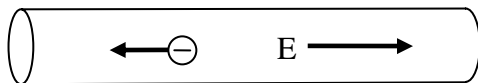
"It's not the voltage that kills you, it the *amps*." About 0.05 A is enough to kill you.

If current $I = 1$ A in a wire, then 1 coulomb of charge flows past any point every second.



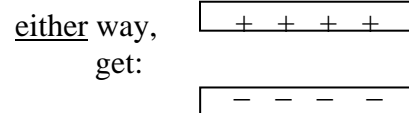
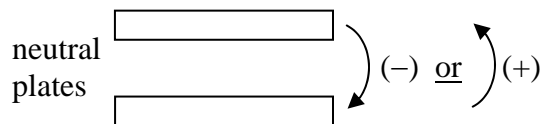
In *electrostatic* problems, $\vec{E} = 0$ inside a metal, but if $I \neq 0$, then the situation is not static, the E-field is not zero.

Electrons flow in metals, not protons, so (-) charges are moving when there is a current. The

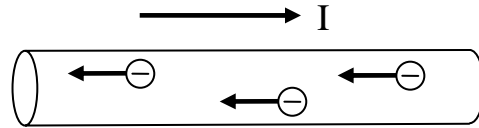


electron feel a force $\vec{F} = -e\vec{E}$ and goes "upstream" against the E-field.

The flow of (-) charge in one direction is electrically equivalent to the flow of (+) charge in the opposite direction:



By convention, we define current I as the flow of imaginary (+) charges, when it is really (-) charges flowing the other way:



(Some texts refer to I as the "conventional current" to distinguish it from the "electron current".)

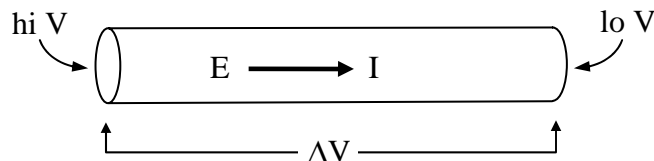
Example: How many electrons flow past per second when the current is 1 A?

$$I = \frac{\Delta Q}{\Delta t} = \frac{\Delta N \cdot e}{\Delta t} \Rightarrow \frac{\Delta N}{\Delta t} = \frac{I}{e} = \frac{1 \text{ A}}{1.6 \times 10^{-19} \text{ C}} = \frac{1 \text{ C/s}}{1.6 \times 10^{-19} \text{ C}} = 5.6 \times 10^{18} \text{ s}^{-1}$$

About 0.01 A = 10 mA flowing through your heart is lethal, yet I could grab a wire carrying 1000A and be safe! Why? Because my body has a much higher *electrical resistance* than the metal. The electrons prefer to flow through the metal wire.

For most materials, the current I is proportional to the voltage difference between the ends.

$$I \propto E \text{ (since } \vec{F} = q\vec{E}\text{)} \text{ and } \Delta V \propto E, \text{ so } I \propto \Delta V$$



From now on, we usually follow the (bad) convention and write "V" when we really mean "ΔV".

$$I \propto V \text{ (really } I \propto \Delta V\text{)} \Rightarrow \frac{V}{I} = \text{constant}$$

Definition of resistance R (of a piece of wire or other material): $R \equiv \frac{V}{I}$

The experimental fact that (for most materials) the ratio $R = V / I$ is a constant, independent of V or I , is called

Ohm's Law : $R = \frac{V}{I} = \text{constant}$, usually written $V = IR$ (R constant)

Units: $[R] = \text{volt} / \text{ampere} = \text{ohm} (\Omega)$ [" Ω " is Greek letter omega]

Ohm's Law should be written $\Delta V = I R$, but the bad convention is to write $V = I R$.

"Ohm's Law" is not really a law, because it is not always true. For many materials, Ohm's Law is approximately true, the resistance R is approximately constant, independent of V or I .

Materials that obey Ohm's Law are called "ohmic materials". But some materials are "non-ohmic"; they do not obey Ohm's Law.

The average speed of electrons in a current-carrying wire results from a competition between two effects: (1) the E-field, which causes an acceleration according to $\vec{F} = q\vec{E} = m\vec{a}$, making the electrons go faster and faster, and (2) the scattering of electrons due to impurities and thermal vibrations, which act like friction, making the electrons slow down.

For typical currents in real wires, the average electron speed (often called the *drift velocity*) is actually quite slow, typically less than 0.1 mm/s. (Incidentally, the term *drift velocity* is incorrect, it should be called the *drift speed*.)

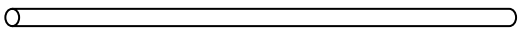
A material with lots of electron scattering has a high resistance:


$$R_{\text{wire}} \ll 1 \Omega, \quad R_{\text{human}} \approx 10^5 \Omega$$

$$I = \frac{V}{R} = \frac{10 \text{ V}}{10^5 \Omega} = 10^{-4} \text{ A (harmless)} \Rightarrow 10 \text{ V } \underline{\text{safe}}, \quad 100 \text{ V } \underline{\text{dangerous}} !$$

$$I = \frac{V}{R} = \frac{100 \text{ V}}{10^5 \Omega} = 10^{-3} \text{ A (painful!)}$$

The resistance R of a piece of material depends on its shape and composition.

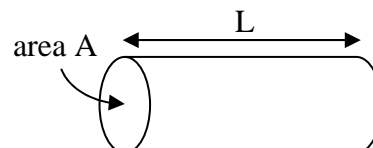
Shape: long and skinny  $\Rightarrow R$ big

short and fat  $\Rightarrow R$ small

— just like the flow of water through a pipe. Long skinny water pipes resist flow of water.

Turns out that $R \propto \frac{L}{A}$,

so big L means big R , big A means small R



$$\boxed{R = \frac{\rho L}{A}}$$

ρ (Greek letter "rho") = resistivity

(We show were this comes in the next section below.)

Resistivity ρ depends on the material composition, not on the shape. ρ is a measure of the scattering of electrons in that material, like viscosity of fluid in a pipe. Big ρ means lots of scattering (friction), big resistance to flow.

Units: $\rho = \frac{A}{\frac{L}{\text{length} \times \text{length}}} \cdot R$ $[\rho] = [R] \times \text{length} = \Omega \cdot \text{m}$

material	ρ	use
Cu	$1.7 \times 10^{-8} \Omega \cdot \text{m}$	house wiring
Al	$2.7 \times 10^{-8} \Omega \cdot \text{m}$	power lines
W (tungsten)	$5.3 \times 10^{-8} \Omega \cdot \text{m}$ (cool) $60 \times 10^{-8} \Omega \cdot \text{m}$ (hot)	light bulb filaments
Fe	$9.8 \times 10^{-8} \Omega \cdot \text{m}$	not used in wiring
glass	$10^{+10} \Omega \cdot \text{m}$	electrical insulator

Microscopic view of Ohm's Law.

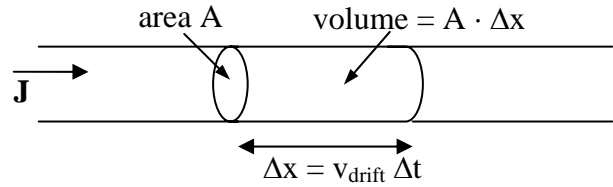
Definition: *current density* $\boxed{J = \frac{I}{A}}$ (current per area in a conductor). We also define current density *vector* \mathbf{J} where direction of \mathbf{J} is the direction of the current.

\mathbf{J} is related to the average speed v_{drift} of the charge carriers (usually electrons) by

$$\boxed{J = n q v_{\text{drift}}},$$

n is the number of carrier per volume, q is the charge per carrier (usually $q = e$).

Proof: Consider a wire with carrier density n (#/volume), cross-sectional area A , and drift speed v_{drift} . In a time Δt , all the charges move an average distance $\Delta x = v_{\text{drift}} \Delta t$.



The charge ΔQ in the length of wire Δx is

$$\Delta Q = \frac{\text{number of carriers}}{\text{volume}} \times \frac{\text{charge}}{\text{carrier}} \times \text{volume} = n q A \Delta x$$

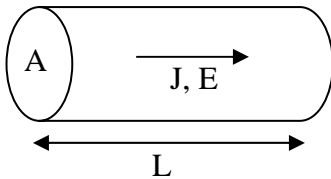
So the current density is $J = \frac{I}{A} = \frac{\Delta Q}{\Delta t} \cdot \frac{1}{A} = \frac{n q A \cancel{\Delta x}}{\cancel{A} \Delta t} = n q v_{\text{drift}}$ Done.

In ohmic materials, the current density J is proportional to the electric field E

$$\vec{J} = \sigma \vec{E}, \quad \text{where the proportionality constant } \sigma \text{ is called the } \textit{conductivity}.$$

The *resistivity* ρ is defined as $\rho = \frac{1}{\sigma}$ and so $E = \rho J$.

$J = \sigma E$ or $E = \rho J$, where $\rho = 1/\sigma = \text{constant}$ is the microscopic version of Ohm's Law. We now show that this is equivalent to $\Delta V = I R$: Consider a cylindrical section of conductor, length L , cross-sectional area A , with current density J , and E-field E .



Start with $E = \rho J$, now multiply both sides by L and write J

as $J = I / A. \Rightarrow EL = \rho L \frac{I}{A}$. Notice that $\Delta V = EL$. So

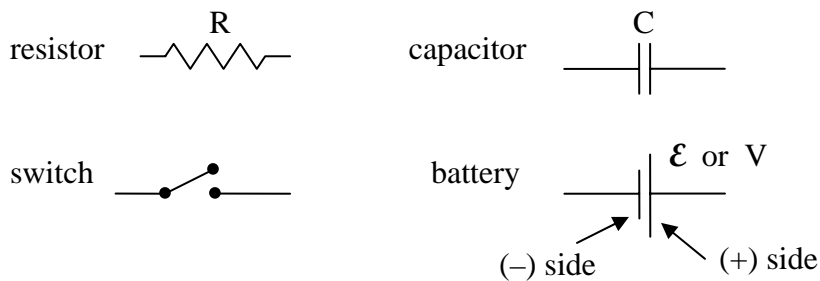
we have $\Delta V = I \frac{\rho L}{A}$, or $\Delta V = I R$, where we have

defined the *resistance* R as $R = \frac{\rho L}{A}$. We have just shown that $E = \rho J$ is the same as

Ohm's Law $\Delta V = I R$, where $R = \frac{\rho L}{A}$.

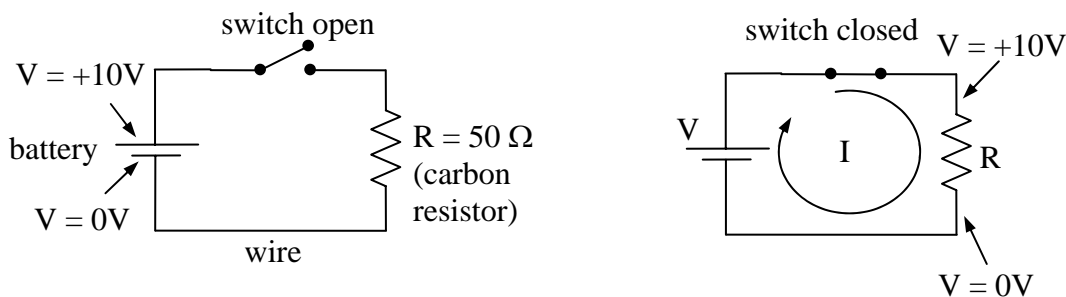
A Simple Circuit

Some electrical circuits symbols:



A battery's job is to maintain a constant voltage difference between its terminals. It acts like a charge pump, pushing (+) charge inside the battery from the (-) side to the (+) side. This is the direction that the charges don't want to go. The battery has to do chemical work to push the charges "uphill" (toward higher electrostatic PE).

"10-volt battery" means voltage *across* battery = \mathcal{E} or ΔV (or just V) = 10 volts = 10 V. Hence the confusing equation: $V = 10 \text{ V}$, meaning "the voltage difference across battery is 10 volts".

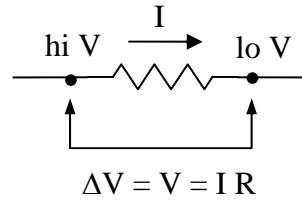
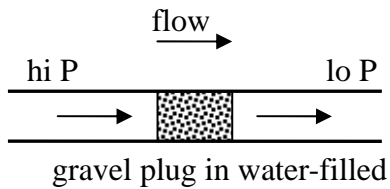


Always assume that the wires connecting circuit elements have zero resistance $R_{\text{wire}} = 0 \Rightarrow$ no voltage change along the wire, since $V_{\text{wire}} = I R_{\text{wire}} = I \cdot 0 = 0$.

$$\text{Current around the circuit is: } I = \frac{V}{R} = \frac{10 \text{ V}}{50 \Omega} = 0.2 \text{ A}$$

Recall that (+) charge tends toward low voltage. Can think of voltage as a kind of "electrical pressure". Water in a pipe flows from high pressure to low pressure. Likewise, (+) charges in a wire flow from hi V to lo V . In the water-pipe analogy, we can think of a resistor as like a porous plug of gravel in the pipe. The gravel plug offers resistance to the flow of water, so we need a large pressure difference to push the water through.

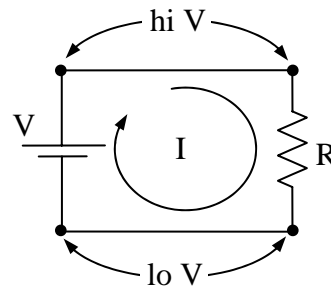
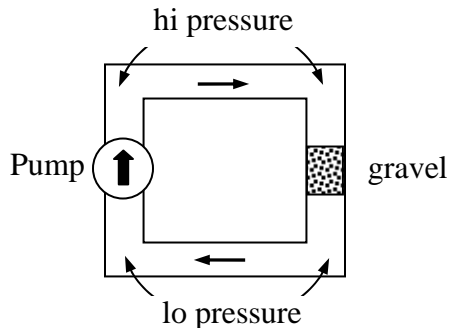
If no pressure difference across a pipe plug, then no water flows.
 If no voltage difference across a resistor, then no current flows.



Inside a resistor, (+) charge flows from hi V → lo V ("downhill")

Inside a battery, (+) charge flows from lo V → hi V ("uphill").

Current I is the same in the battery and the resistor, just as water flow (in gallons per minute) is same through pump and through gravel plug.



Electrical Power

When I is big, R gets hot because the flowing electrons are scattered ⇒ friction.

Recall that power P = rate at which energy is transformed = rate at which work is done

$$P \equiv \frac{\Delta W}{\Delta t}, \quad [P] = \frac{\text{joule}}{\text{second}} = 1 \text{ watt (W)}$$

In a resistor, the rate at which electrostatic PE is converted into thermal energy (heat) is

$$P = I V$$

(really, $P = I \Delta V$)

If I = constant, then the average speed of electrons is constant ⇒ $KE = (1/2)mv^2 = \text{constant}$.

When an amount of charge Q flows through a resistor in a time t , from the high V side to the low V side, the change in PE of that charge is $\Delta PE = Q \cdot \Delta V$. The PE of the charge Q is decreased, but $KE = \text{constant}$. Where did the energy go? Answer: into thermal energy

$$P = \frac{\Delta PE}{t} = \underbrace{\left(\frac{Q}{t}\right)}_I \cdot \underbrace{\Delta V}_V = I \cdot V$$

$$P = I V \text{ and } V = I R \Rightarrow P = I (I R) = I^2 R \text{ or } P = (V / R) V = V^2 / R$$

3 equivalent expressions for power:

$$P = I V = I^2 R = \frac{V^2}{R}$$

"100 W bulb" uses $P = 100 \text{ W}$. Light bulbs and most other appliances (TV, coffee grinder) are designed to work with voltage $V = 120 \text{ V}$ (really $\Delta V = 120 \text{ V}$).

example of electrical power: What is the current through a 100 W light bulb? What is its resistance (when on and hot).

$$I = \frac{P}{V} = \frac{100 \text{ W}}{120 \text{ V}} = 0.83 \text{ A} , \quad P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120 \text{ V})^2}{100 \text{ W}} = 144 \Omega$$