Faraday's Law

Faraday's Law is one of 4 basic equations of the theory of electromagnetism, called Maxwell's Equations. We have said before that

- charges makes electric fields. (Gauss's Law)
- currents make magnetic fields. (Ampere's Law)

This is the truth, but not the whole truth. Michael Faraday (British physicist, c.1850) showed that there is a second way to make an electric field:

- a *changing* magnetic field makes an electric field. (Faraday's Law)

Around 1860, James Maxwell(Scottish physicist) showed that there is a second way to make a magnetic field:

- a *changing* electric field makes an magnetic field. (modification of Ampere's Law)

Before stating Faraday's Law, we must define some new terms:

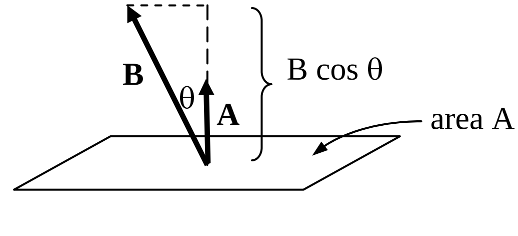
Definition: emf , $\mathcal{E}$, is (roughly speaking) a voltage difference ($\Delta V = E\,d$) capable of doing continuous useful work.. Think of emf as a battery voltage. Batteries have an emf, but resistors do not, even though a resistor R can have a voltage difference across it ($\Delta V = I\,R$)

Technically, the emf around a *closed* loop $\mathcal{L}$ is defined as $$\mathcal{E} = \oint_{\mathcal{L}} \vec{E}\cdot d\vec{\ell}$$

Recall that voltage difference was defined as $\Delta V = -\int_A^B \vec{E}\cdot d\vec{r}$. For the case of E-fields created by charges, the voltage difference when we go around a *closed* loop is zero, since voltage depends only on position, not on path: $\Delta V = -\int_A^A \vec{E}\cdot d\vec{r} = 0$

Definition: **magnetic flux** through some surface S,

$$\Phi_B = \int_S \vec{B}\cdot d\vec{A} \underset{\text{if } \mathbf{B} \text{ const and A flat}}{=} \vec{B}\cdot\vec{A} = BA\cos\theta$$

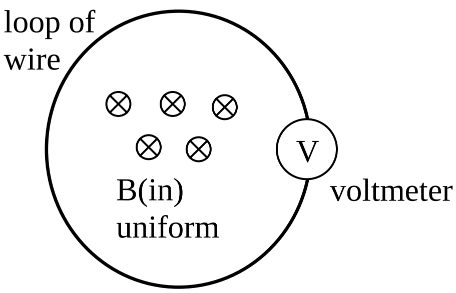


Units $[\Phi] = T\cdot m^2$ = weber (Wb)

Faraday's Law (in words): An induced emf ($\mathcal{E}$) is created by <u>changing</u> magnetic flux.

Faraday's Law (in symbols):

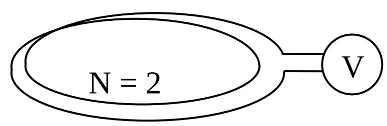
$$\boxed{\mathcal{E}_{(1\text{ loop})} = -\frac{d\Phi_M}{dt}}$$



If **B** = constant $\Rightarrow$ emf $= \mathcal{E} = 0$

If **B** is changing with time $\Rightarrow$ $|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| \neq 0$.

If have several loops, $\mathcal{E}_{(N\text{ loops})} = -N\frac{d\Phi}{dt}$



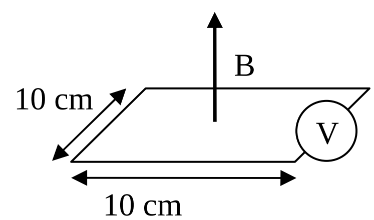
We can change the magnetic flux Φ in several ways:

1) change B (increase or decrease magnitude of magnetic field)

2) change A (by altering shape of the loop)

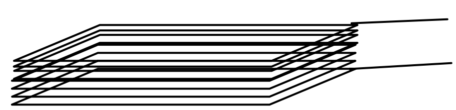
3) change the angle θ between **B** and the area vector **A** (by rotating the loop, say)

Example of Faraday's Law: We have a square wire loop of area A = 10 cm × 10 cm, perpendicular to a magnetic field B which is *increasing* at a rate $\frac{dB}{dt} = +0.1\,\text{T/s}$. What is the magnitude of the emf $\mathcal{E}$ induced in the loop?



Answer: $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt} = (0.01\text{m}^2)(0.1\text{T/s}) = 10^{-3}\,\text{V} = 1\text{mV}$

What is the emf if N = 1000 loops?



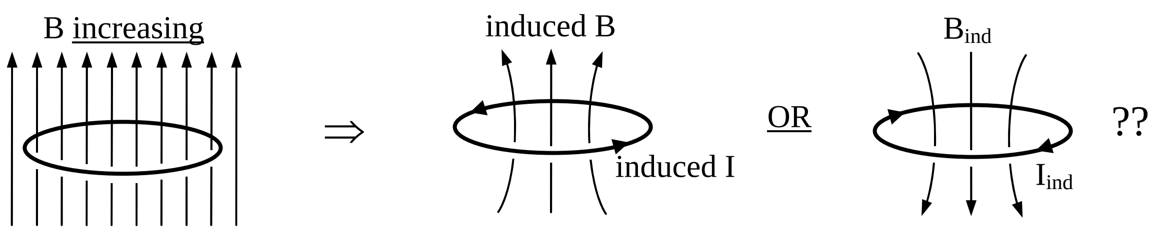
$$|\mathcal{E}| = N\frac{d(BA)}{dt} = 1000 \times 10^{-3}\,\text{V} = 1\text{V}$$

Lenz's Law

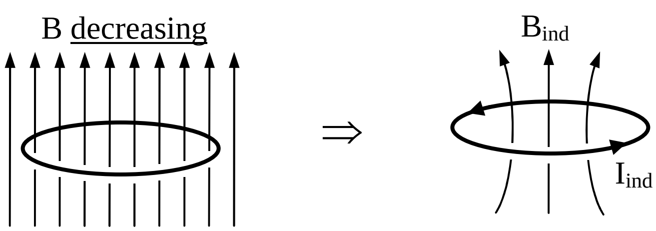
The minus sign in Faraday's law is a reminder of ..

Lenz's Law: the induced emf $\mathcal{E}$ induces a current that flows in the direction which creates an induced B-field that *opposes the change* in flux.

Example: a loop of wire in an external B-field which is increasing like so



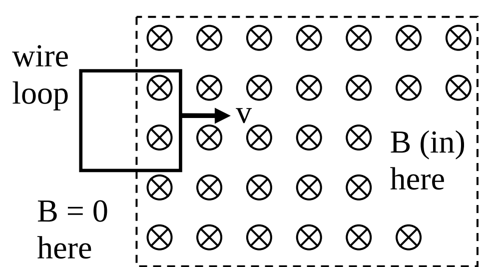
Answer: $B_{induced}$ <u>downward</u> opposes the increase in original B.



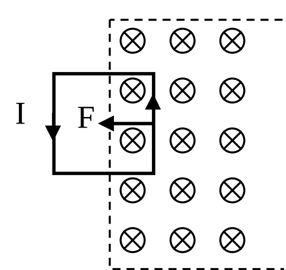
Here, induced B is <u>upward</u> to oppose the *decrease* in the original B.

Lenz's Law says "Change is bad! Fight the change! Maintain the status quo."

Example of use of Lenz's Law A square loop of wire moving to the right enters a region where there is a uniform B-field (in). What is the direction of the current through the wire: CW or CCW? Answer: CCW



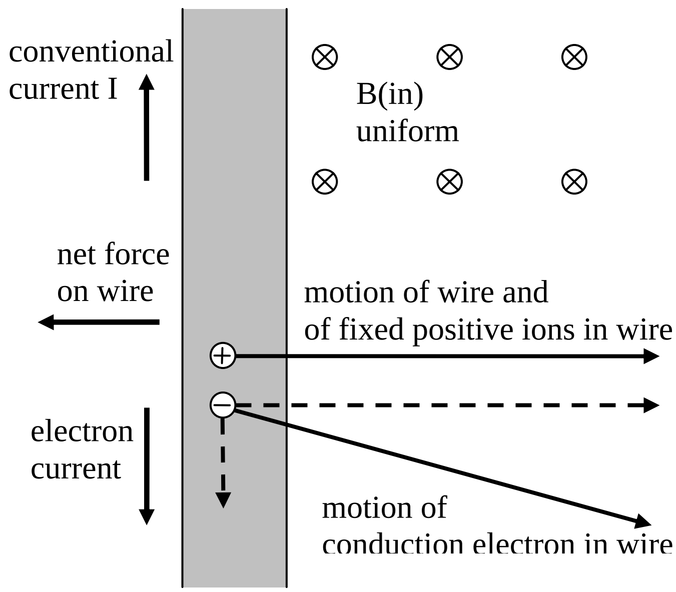
The flux is increasing as the loop enters the field. In order to <u>fight</u> the increase, the induced B-field must be out-of-the-page. An induced CCW current will produce a B-field pointing out.



Does the magnetic field exert a net force on the loop as it enters the field? Answer: Yes. The upward current on the right side of the loop will feel a force to the left (from $F_{wire} = ILB$ and

R.H.R.). Notice that the direction of the force on the wire loop will slow its motion.

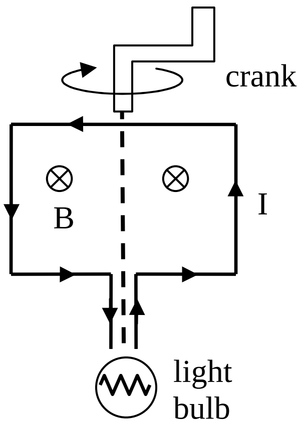
Aside: There is a subtlety in this problem that we have glossed over. To get the direction of the force on the right-hand side of the wire, we assumed that the direction of the (imaginary positive) moving charges in the wire is upward, along the direction of the current, and not to the right, along the direction of the motion of the entire loop. Now, it is really the negative conduction electrons that are moving within the wire, but we still have the problem of understanding which velocity **v** we should pick when we apply the force law **F** = q **v** × **B**. Should we pick the direction of the electron current (downward, parallel to the wire), the direction of the motion of the loop (to the right), or some combination of these directions? The conduction electrons in the right half of our wire are actually moving both downward and to the right. But only the downward motion



matters, because the motion to the right is effectively canceled by the motion of the positive charges within the wire. Remember that the wire is electrically neutral; there are as many fixed positive ions in the wire as there are mobile negative electrons. The force on the electrons due to their rightward motion is exactly canceled by the force on the positive charges, which have exactly the same rightward motion. But the force on the conduction electrons due to their downward motion is not canceled out, and this is the cause of the net force on the wire.

Electrical Generators

Convert mechanical energy (KE) into electrical energy (just the opposite of motors). A wire loop in a constant B-field (produced by a magnet) is turned by a crank. The changing magnetic flux in the loop produced an emf which drives a current.



Eddy Currents

If a piece of metal and a B-field are in relative motion in such a way as to cause a changing Φ through some loop within the metal, then the changing Φ creates an emf $\mathcal{E}$ which drives a current I. This induced current is called an *eddy current*. The relative direction of this eddy current I and the B-field are always such as to cause a magnetic force ($\vec{F} = I\vec{L}\times\vec{B}$) which <u>slows</u> the motion of the metal .

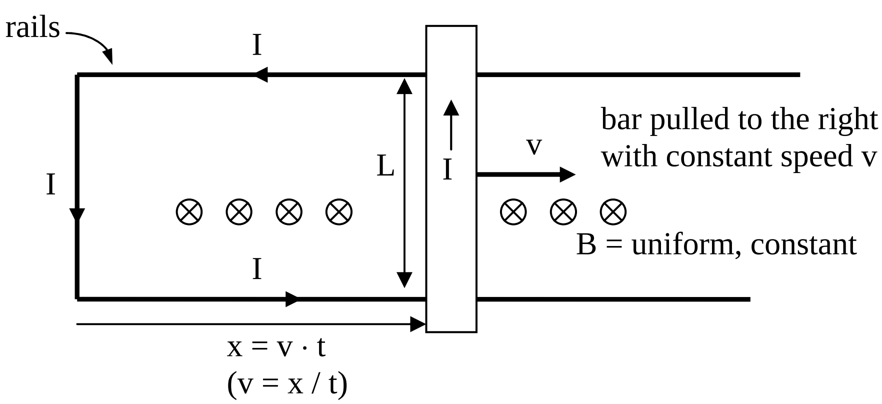
Again, if metal moving in a B-field makes a changing Φ $\overset{\text{(Faraday)}}{\Longrightarrow} \underbrace{I}_{\text{eddy current}} \Rightarrow F_{\text{on metal}} (= I L B)$

and the direction of the force <u>always</u> <u>slows</u> the motion.

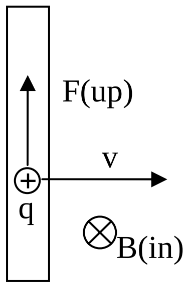
If the eddy current force did not slow the motion, but instead aided the motion, then we would have runaway motion ⇒ free energy ⇒ violation of energy conservation.

Motional emf

Consider the following contraption. A metal bar sliding on metal rails.



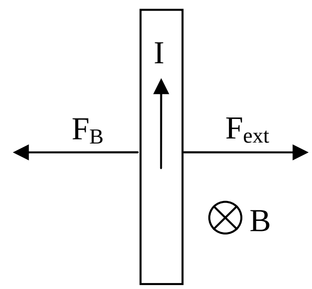
The bar+rails form a wire loop of area A = L·x = L·v· t. The magnetic flux Φ = B·A is increasing, which produces an emf, which drives a current I (an eddy current!) which is CCW by Lenz' Law. $|\mathcal{E}| = \frac{d\Phi}{dt} = \frac{d(BA)}{dt} = \frac{d(B\,L\,v\,t)}{dt} = B\,L\,v$



This is an example of a "motional emf". The motion of the bar creates the emf.

There is another way to see why there is current in the circuit with the current flowing upward in the bar: The charges in the bar are moving in a B-field, so they feel a force $F_{\text{on q}} = q\,v\,B$ (upward) ⇒ I (upward)

Because of this current I, there is a magnetic force on the bar $F_B = I\,L\,B$ (Recall $\vec{F}_{\text{on wire}} = I\vec{L}\times\vec{B}$). An external force F_{ext} must act on the bar to balance the magnetic force F_B and keep the bar moving with constant speed v: $F_{ext} = F_B$. The power delivered by F_{ext} must equal the electrical power delivered by the emf in the circuit, which is dissipated as heat in the resistance of the rails+bar.



$$P_{ext} = \frac{dW}{dt} = \frac{d}{dt}(F_{ext}\cdot x) = F_{ext}\cdot v \ , \quad P_{electrical} = \mathcal{E}\cdot I \quad \text{(Recall P = I V)}$$

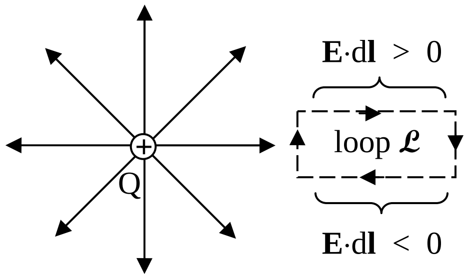
$$F_{ext}\cdot v \overset{?}{=} \mathcal{E}\cdot I \Rightarrow F_B\cdot v = \mathcal{E}\cdot I \Rightarrow I\,L\,B\cdot v = B\,L\,v\cdot I \quad \text{(it checks!)}$$

Two different E-fields

Faraday's Law $\mathcal{E} = -\frac{d\,\Phi}{d\,t}$ can be written:

$$\oint_{\mathcal{L}} \vec{E}\cdot d\vec{\ell} = -\frac{d}{d\,t}\left(\int_S \vec{B}\cdot d\vec{a}\right) \qquad \leftarrow \text{loop } \mathcal{L} \text{ encloses surface S.}$$

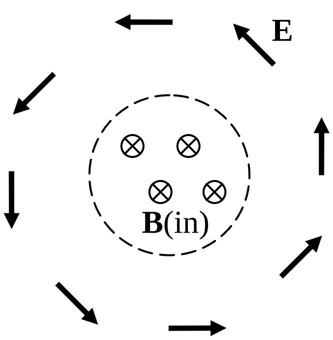
Written this way, we see that Faraday's Law says that a changing B-field makes an E-field. In *electrostatics*, there is no current, and so, no **B** (since no I to make B), and, so, no *changing* **B**. So, in the case of stationary charges (electrostatics) we have $\oint_{\mathcal{L}} \vec{E}\cdot d\vec{\ell} = 0$, for any loop $\mathcal{L}$.



(+) part cancels (–) part

$$\Rightarrow \oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} = 0$$

By contrast, when there are moving charges that make a changing B-field, then the changing B-field makes an E-field, which has $\oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} \neq 0$, says Mr. Faraday.



B increasing $\Rightarrow$ **E** CCW

B decreasing $\Rightarrow$ **E** CW

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} \neq 0$$

So there are two qualitatively-different kinds of E-fields:

1) E-fields due to stationary charges. For this kind of E-field, we can define a voltage: $\Delta V = -\int_A^B \vec{E} \cdot d\vec{\ell}$. It is only possible to define a voltage if the line-integral of the E-field is path-independent, and this is only possible if $\oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} = 0$. In this case, when a charge moves around any closed loop, the work done by the E-field is zero.
2) E-fields due to changing B-fields. For this kind of E-field, $\boldsymbol{\mathcal{E}} = \oint_{\mathcal{L}} \vec{E} \cdot d\vec{\ell} \neq 0$, and we cannot define a voltage. In this case, when a charge moves around a closed loop, the E-field does non-zero work on the charge, just like a battery does.

