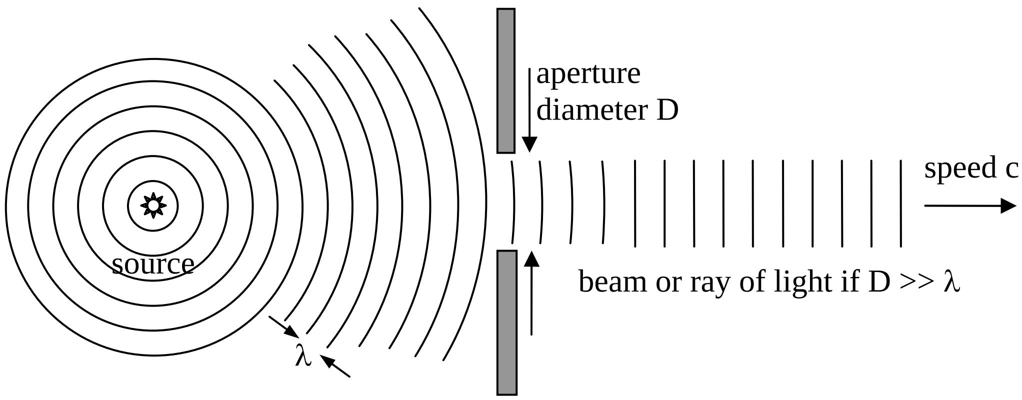
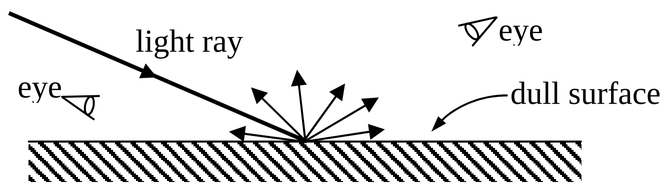
Ray Optics (or Geometrical Optics)

In many circumstances, we can ignore the wave nature of light and assume that light is a stream of particles that travel in straight lines called **rays**. For instance, if a light wave from a point source passes through an aperture (a hole) that is very large compared to the wavelength of light, then a beam or ray of light is produced. (We'll see later why you need the hole diameter large compared to the wavelength).



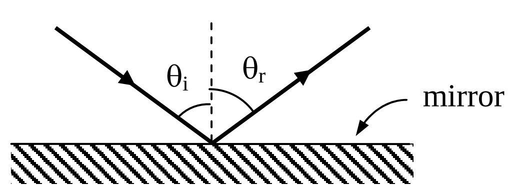
Mirrors

When light reflects from a dull surface, the rays scatter in all directions, so observers can see reflected rays from all directions. Why does light scatter from the surface of a material? The incident light ray is an electromagnetic wave. The oscillating electric field of the EM waves shakes the charges (electrons and protons) in the surface of the material. The shaking charges create new electromagnetic waves which radiate outward in all directions.

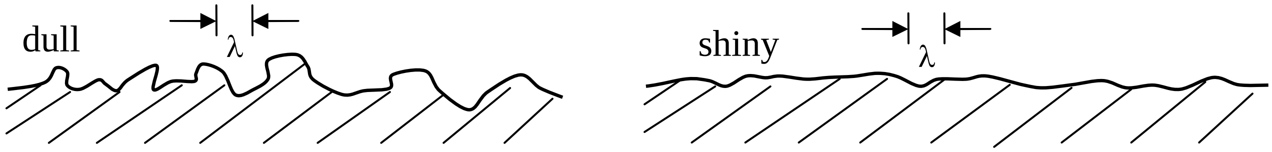


When a ray of light scatters from the surface of a smooth **mirror**, the ray is reflected in one direction only. The incident angle is equal to the reflected angle

$$\theta_i = \theta_r$$

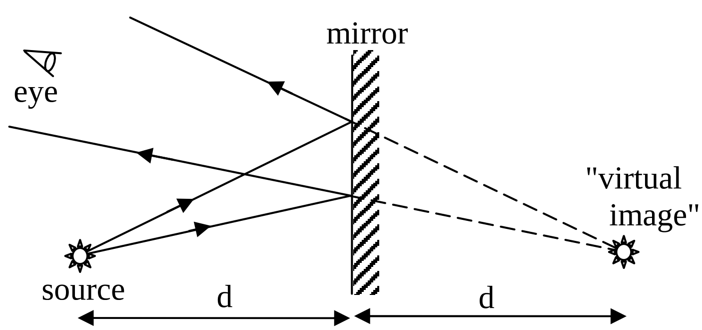


Why does the light reflected from a smooth surface scatter in one direction only? When you study interference and diffraction, you will see that when the surface is smooth, the scattered rays interfere destructively (cancel) in all directions, except the one direction for which $\theta_i = \theta_f$.



Any surface is shiny and mirror-like if it is smooth *compared* to the wavelength of light $\lambda_{visible} \approx 500$ nm. A surface that is dull in visible light can be shiny in the infrared.

Rays from a point source, reflected from a mirror, appear to be coming from a point behind the mirror. A "virtual image" occurs when rays appear to be coming from a point in space, but are not really. Here's the trick to analyzing mirror problems: redraw the incident & reflected rays as straight lines.



Refraction and Snell's Law

Any transparent medium (air, water, glass, etc) can be characterized by a dimensionless number called the index of refraction $\boxed{n = \frac{c}{v}} = \frac{\text{speed of light in vacuum}}{\text{speed of light in the medium}}$.

The speed of light in a vacuum, c, is an absolute maximum speed; the speed of light in a medium is always less than c.

So in a medium $v < c$, always $\Rightarrow$ $n = c / v > 1$, always.

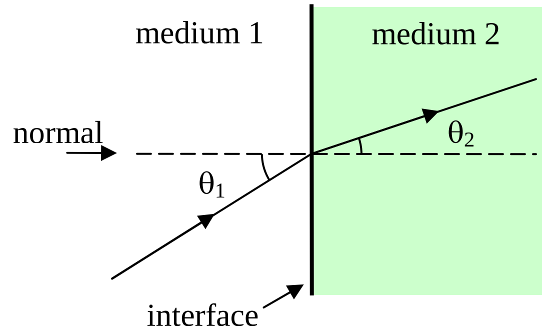
material	**index n**
vacuum	1
air	1.0003 ≅ 1
water	1.33
Lucite	1.51
glass	1.45 – 1.75
diamond	2.42

When a ray of light passes from 1 medium to another, the ray is bent or refracted according to ..

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

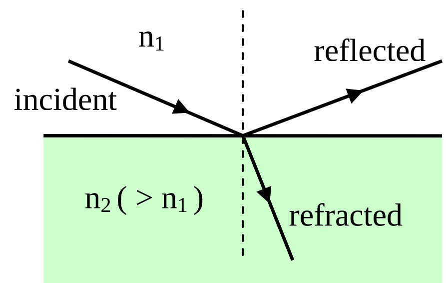
In optics, angles are <u>always</u> measured with respect to the normal (perpendicular) direction.

Notice that the ray is closer to the normal in the medium with the larger index n. The larger the <u>change</u> in n, the more the ray is bent.

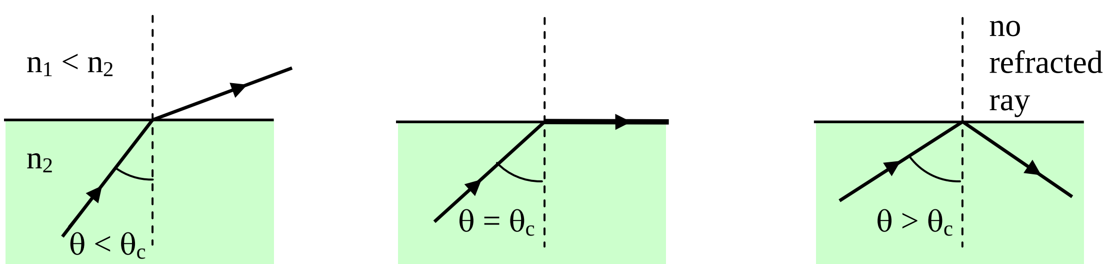


Why does light slow down and change direction when passing from vacuum into a medium? The oscillating E-field of the incident EM waves shakes the charges in the medium. The shaking charges create a new EM wave which interferes with the original wave to make a new net wave that moves more slowly, and in a different direction.

In general, when a ray is incident on an interface, there are both reflected and refracted rays.



If the ray passes from a higher n to a lower n material, the ray is bent away from the normal. If the incident angle is large enough, you get **total internal reflection**, and no refracted ray



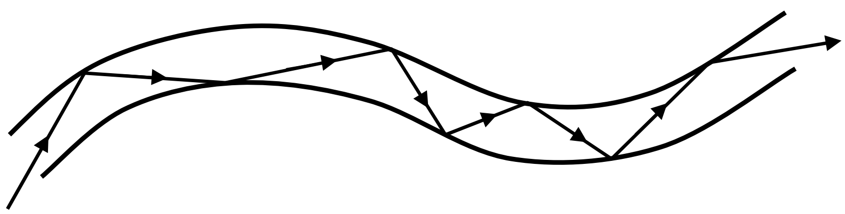
Example of total internal reflection. What is the critical angle θ_c for a light ray in water at an air/water interface? Medium 1 = water, medium 2 = air. Index of water = $n_1 = n_w = 1.33$. Index of air = $n_2 = n_a \cong 1$. We start with Snell's Law:

$$n_1 \ \sin\theta_1 \ = \ n_2 \ \sin\theta_2$$

$$n_w \ \sin\theta_c \ = \ (1) \ \underbrace{\sin(90^o)}_{1} \ = \ 1$$

$$\sin\theta_c \ = \ \frac{1}{n_w} \ \Rightarrow \ \theta_c \ = \ \sin^{-1}\left(\frac{1}{n_w}\right) \ = \ \sin^{-1}\left(\frac{1}{1.33}\right) \ = \ 48.8^o$$

Light pipes guide light rays by total internal reflection.

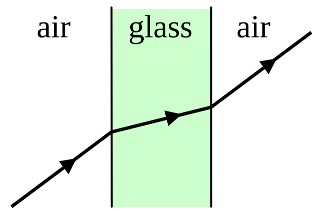


Lenses and image formation

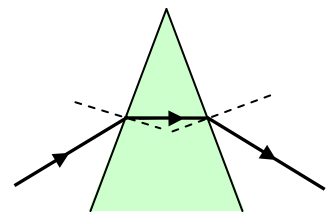
Images can be formed with lenses or mirrors. Most texts starts with a discussion of mirrors; we'll start with lenses.

Key ideas in lens design:

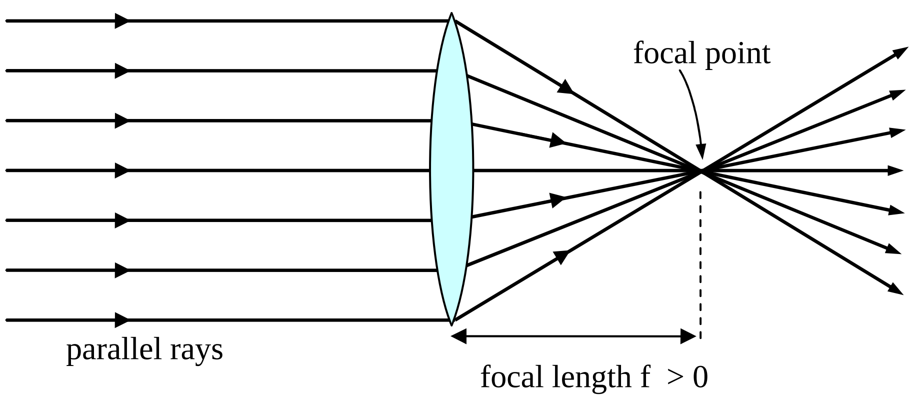
1) For a ray passing through a flat plate of glass (with parallel surfaces), the incoming ray and the outgoing ray are parallel. The refraction (ray bending) at the air/glass interface on the way in is exactly undone by the refraction at the glass/air interface on the way out.



2) For a wedge-shaped piece of glass, like a prism, the ray is bent toward the thicker end.

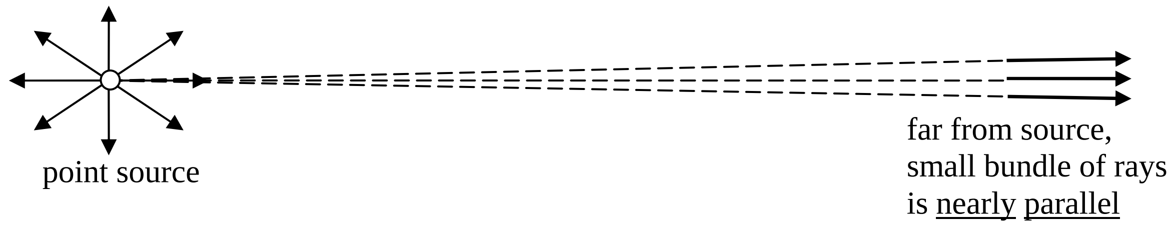


From these two ideas, we see that a **convex** lens (one that is thick in the middle and thin on the edges) tends to focus a bundle of parallel rays to a point.

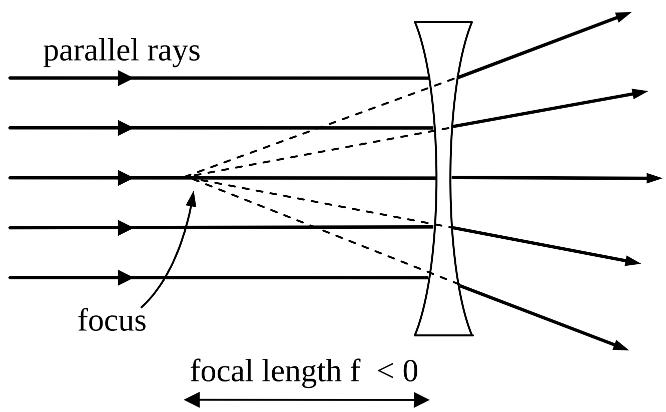


The center ray is not bent because the surfaces are parallel. The edge rays are bent toward the thicker part. A convex lens is also called a **converging lens** since the rays converge on the focus. The *focal length* f is the distance from the lens to the *focal point*, where all the parallel rays from the other side of the lens come to a focus. The focal length depends on both the index of refraction n of the glass and the shape of the lens.

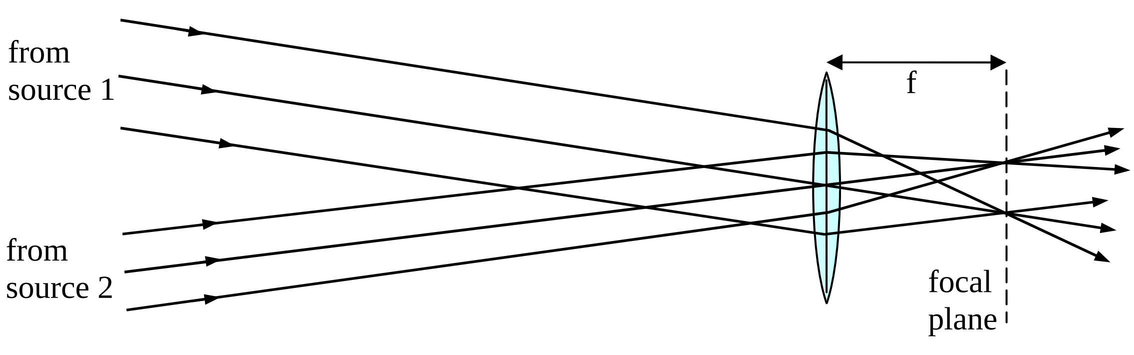
Parallel rays are produced by distant point sources ⇒ light rays from a star in the sky are parallel.



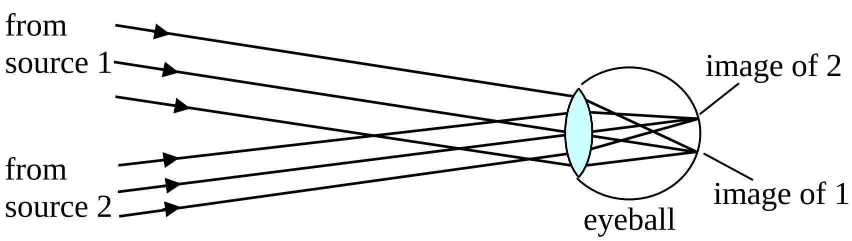
Diverging lens or **concave lens:** Thin in the middle, thick at the edges. Remember, rays bend toward the thicker end.



You can form **images** on a screen of distant objects using a converging lens.

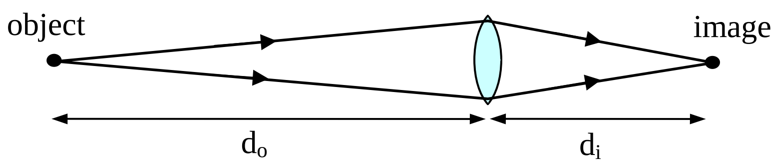


If the screen is placed at the focal plane of the lens (one focal length f away from the lens), then you will see two points of light on the screen, which are the images of point sources 1 and 2. These are **real images**. A real image, as opposed to a virtual image, is an image formed by the actual convergence of light rays at the location of the image. Your eye contains a converging lens with a screen (the retina) behind it.



A converging lens can also form an image of nearby object (if not too close).

d_o = distance from object to lens d_i = distance from image to lens



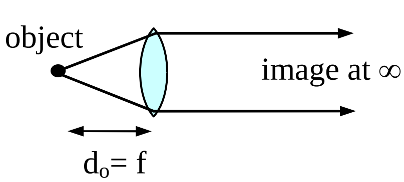
d_o, d_i, and focal length f are related by the image equation:

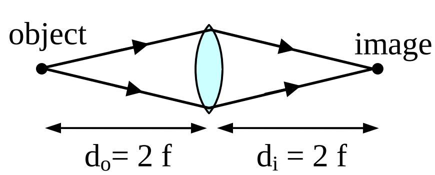
$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

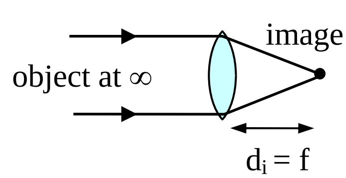
(Proof of the image equation is in the Appendix.)

Note these special cases:

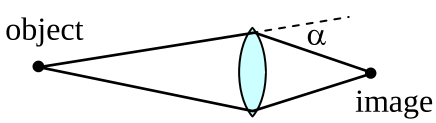
- $d_o = \infty \Rightarrow d_i = f$ [if object very far away, then image is 1 focal length from the lens]
- $d_o = f \Rightarrow d_i = \infty$
- $d_o = 2f \Rightarrow d_i = 2f$ since $\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$

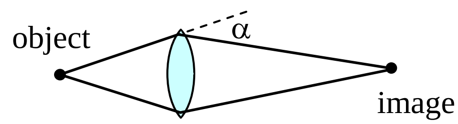




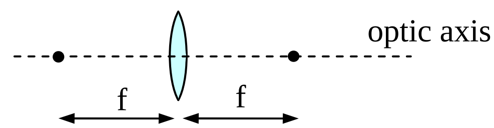


In general, as the object gets closer to lens (d_o decreases), the image gets further from the lens (d_i increases). Why? A given converging lens has only so much "bending power". A ray hitting the edge of the lens is bent the same amount regardless of the angle of the incoming ray.



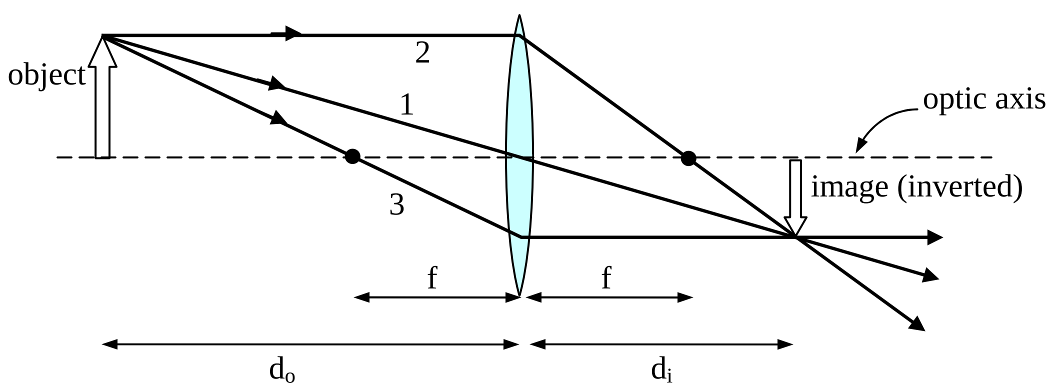


A given lens have 2 focal points, one on each side of the lens, each corresponding to parallel rays on the other side of the lens



Ray Diagrams: Use 3 special rays to get the position of image.

1. The ray through the center of the lens is undeviated (we are assuming a *thin* lens)
2. The ray parallel to the optic axis on object side passes through the focal point on image side.
3. The ray through the focal point on the object side is parallel to the optic axis on image side.

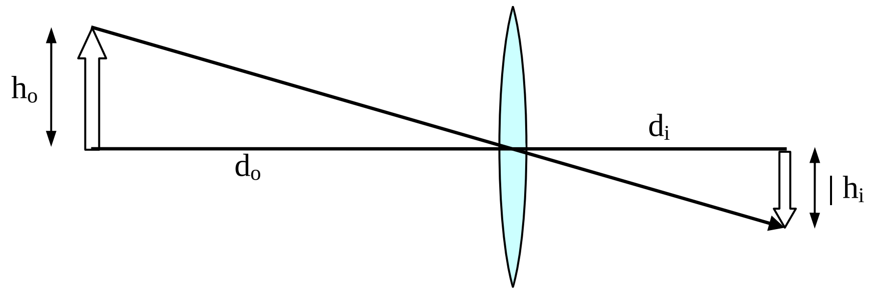


Actually, you only need two of these rays to locate the image. The 3rd ray is a check.

h_o = object height

$|\ h_i\ |$ = image height

$h_i < 0$, if image is inverted



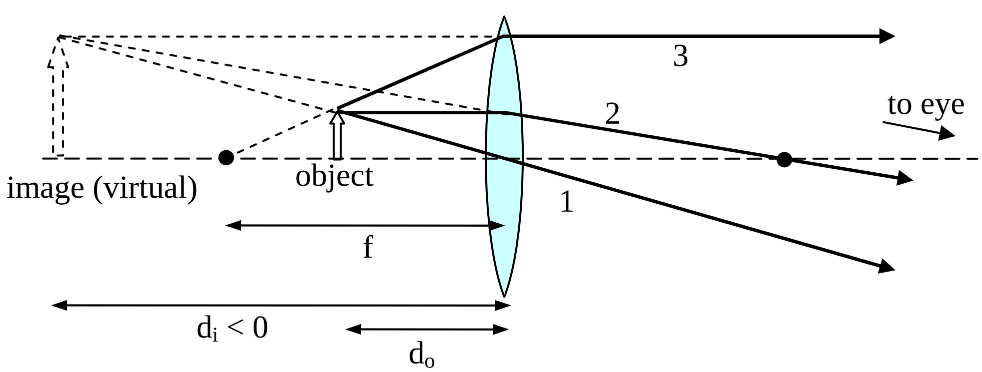
From similar triangles, we see that the *linear magnification* M is $\boxed{|M| \equiv \frac{|h_i|}{h_o} = \frac{|d_i|}{d_o}.}$

(Some textbooks define M to be negative if the image inverted. With this sign convention $M \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}$.)

Sign conventions in using image equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

- Rays go from left to right ($\rightarrow$) always.
- $f > 0$ if converging lens. $f < 0$ if diverging lens.
- $d_i > 0$ if real image on right side of lens. $d_i < 0$ if virtual image on left side of lens
- $h_i > 0$ if image is upright. $h_i < 0$ if image is inverted.

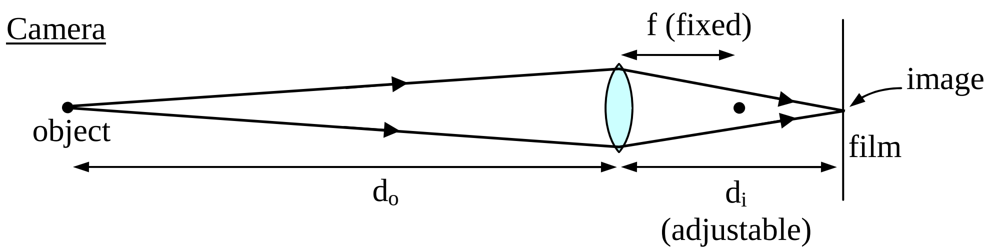
Magnifying glass: When the object is closer to the lens then the focal length, a *virtual* image is formed on the left side of the lens with $d_i < 0$, $h_i > 0$ (image virtual, upright), and magnification $M > 0$.



Camera vs. Human Eye

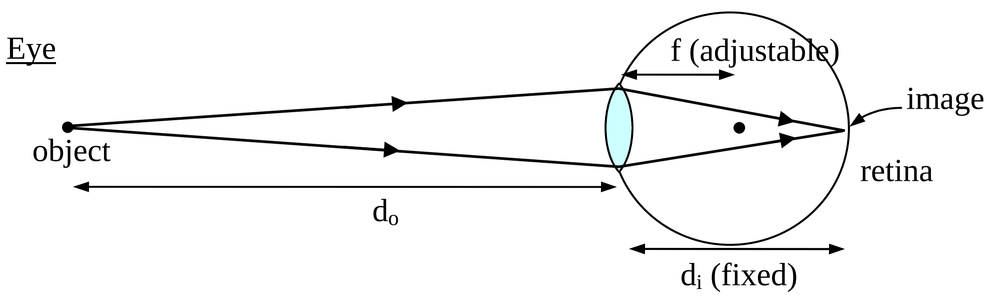
In a camera, the focal length f is fixed (unless you have a zoom lens). The lens/image distance d_i is adjusted as the object/lens distance d_o varies, in order to maintain focus. For good focus, the film must be placed at the image location, at distance d_i from the lens.

In the human eye, the "film" is the retina. The lens/image distance d_i is fixed by the size of the eye. As the object/lens distance d_o varies, the focal length f of the eye lens is adjusted to maintain focus.

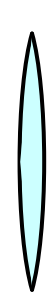


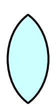
"75 mm lens" means f = 75 mm.

In the old days (a few years ago), before digital cameras, cameras used film that had to be developed by wet chemistry. "35 mm camera" meant that the width of film was 35 mm.



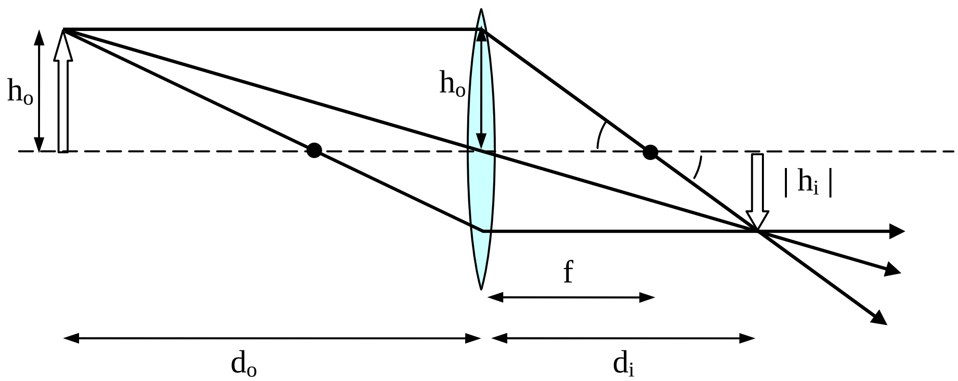
The focal length f of the eye lens is adjusted by muscles on its rim which can squeeze the lens to make it fatter in the middle.

thinner lens:  bends rays less $\Rightarrow$ longer f. "Relaxed eye" to focus on faraway objects.

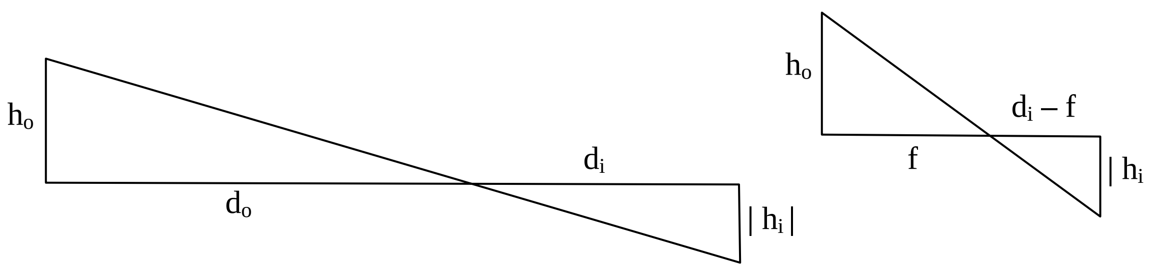
fatter lens:  bends rays more $\Rightarrow$ shorter f. "Strained eye" to focus on near objects.

Telescopes

Appendix: Proof of the image equation $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$



From the diagram above, we can draw two sets of similar triangles:



By similar triangles, we can write (1) $\frac{|h_i|}{h_o} = \frac{d_i}{d_o}$ and (2) $\frac{h_o}{f} = \frac{|h_i|}{(d_i - f)}$.

We can rewrite (2) as $\frac{(d_i - f)}{f} = \frac{|h_i|}{h_o}$. Comparing this and equation (1), we have

$\frac{(d_i - f)}{f} = \frac{d_i}{d_o}$, which is the same as $\frac{d_i}{f} - 1 = \frac{d_i}{d_o}$.

Finally, we divide through by d_i: $\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o} \Rightarrow \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. Done.