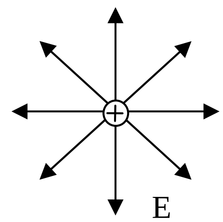
Magnetism: a new force!

So far, we've learned about two forces: gravity and the electric field force.

$\vec{E} = \frac{\vec{F}_E}{q}$, $\vec{F}_E = q\vec{E}$ ← Definition of E-field



- E-fields are created by charges: $|\vec{E}| = \frac{kQ}{r^2}$
- E-field exerts a force on other charges: $\vec{F}_E = q\vec{E}$.

The gravitational force is similar:

- Gravitational fields are created by mass: $|\vec{g}| = \frac{GM}{r^2}$.
- The gravitational field exerts a force on other masses. $\vec{F}_{grav} = m\vec{g}$.

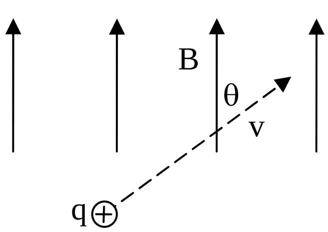
There is a different kind of field, called a **magnetic field** or **B**-field.

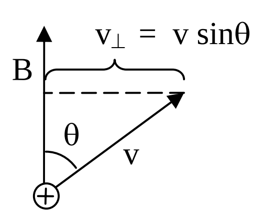
- B-fields are created by *moving* charges (currents).
- B-fields exert forces on moving charges.

Remember: I is to **B** as q is to **E**. We will see in the next chapter exactly how B-fields are made by currents. For now, we assume that we have a B-field and we want to know how the B-field exerts a force on a moving charge (a current).

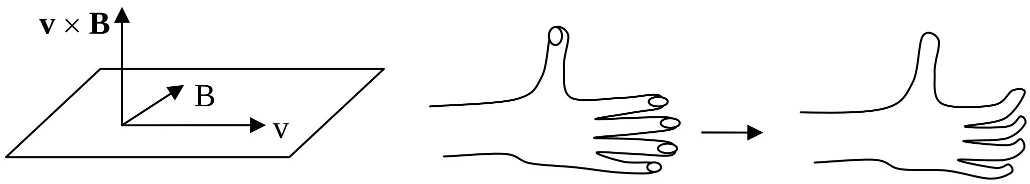
The force F_B from a B-field on a moving charge depends on the velocity of the charge in a peculiar way: a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$, feels a magnetic force given by $\boxed{\vec{F}_B = q\vec{v}\times\vec{B}}$ (q times *cross-product* of **v** and **B:** see appendix for review of the cross-product). This equation is the definition of **B**, analogous to the $\vec{F}_E = q\vec{E}$, which defines **E**.

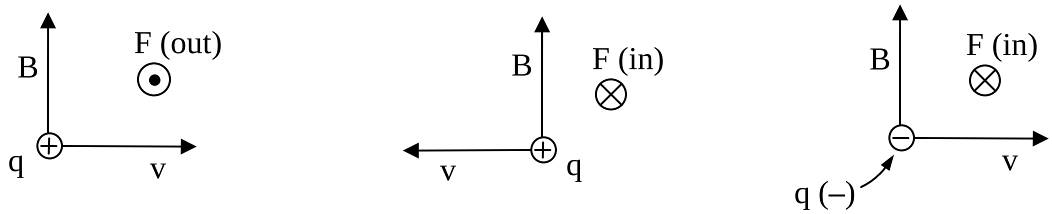
The magnitude of the magnetic force is $|\vec{F}_B| = F_B = |q| v B \sin\theta = |q| v_\perp B$





The direction of the force $\vec{F}_B$ is *perpendicular* to the plane formed by $\vec{v}$ and $\vec{B}$. The direction of $\vec{v}\times\vec{B}$ is determined by the "Right-hand rule". Use your <u>right</u> hand. Point your fingers in the direction of $\vec{v}$, curl your fingers toward $\vec{B}$. (Orient hand so that fingers curl thru angle < 180^o). Your thumb then points in the direction of $\vec{v}\times\vec{B}$, which is the direction of $\vec{F}_B$ **<u>if</u>** the charge q is positive. If q is (–), $\vec{F}_B$ is other way





Unlike gravity or the electric force, the magnetic force is a velocity-dependent force.

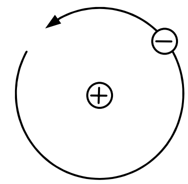
- If $\vec{v}\perp\vec{B}$, then $\sin\theta = 1 \Rightarrow$ $F_B = |q|\,v\,B$
- If $\vec{v}\,||\,\vec{B}$, then $\sin\theta = 0 \Rightarrow$ $F_B = 0$
- If v = 0, then $F_B = 0$ (<u>unlike</u> gravity or E-field force)

Units of B: $[B] = [B] = \dfrac{[F]}{[q][v]} = \dfrac{N}{C\cdot\frac{m}{s}} = 1$ tesla (T)

Older, non-SI, unit of B: 1 gauss = 10^{-4} T , 1 T = 10^4 gauss

- Earth's magnetic field ≈ 0.5 gauss = 5×10^{-5} T
- kitchen magnet : 50 – 500 gauss = 0.005 – 0.05 T
- iron core electromagnet: 2 T (max) (Strong enough to yank tools out of your hand.)
- superconducting magnet: 20 T (max)

We have said that currents make B-fields. So where's the current in a permanent magnet (like a compass needle)? An atom consists of an electron orbiting the nucleus. The electron is a moving charge, forming a



tiny current loop — an "atomic current".

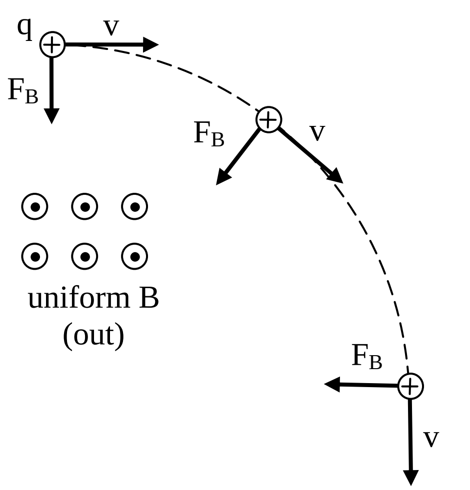
In most metals, the atomic currents of different atoms have <u>random</u> orientations, so there is no net current, no B-field. But in magnetic materials the atomic currents are aligned and they create a net current. (More on permanent magnets in the next chapter.)

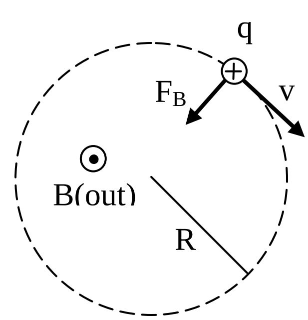
Motion of a charged particle in magnetic field

Consider a charge q moving in a uniform magnetic field **B**. Since the force $\mathbf{F_B}$ is always <u>perpendicular</u> to the velocity **v**, the force F_B does <u>no</u> <u>work</u> :

$\vec{F}_B \perp \vec{v}\ ,\ \ \vec{v} = \dfrac{d\vec{r}}{dt}\ \ \Rightarrow\ \ W_{FB} = \vec{F}_B \cdot d\vec{r} = 0$. A magnetic force cannot change the KE of a particle (recall Work-KE theorem: $W_{net} = \Delta KE$). The B-field changes the direction of the velocity **v**, but does not change the speed, so we have v = constant.

If the velocity **v** is perpendicular to the field **B**, the magnetic force bends the path of the particle in a circle.

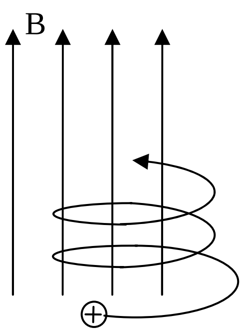




We can relate the radius R of the circular path to the magnitude of the field B and the speed v with Newton's Second Law:

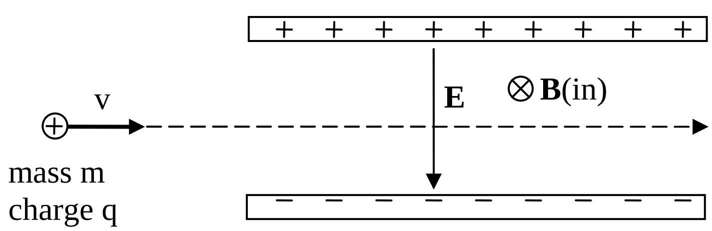
$$\left|\vec{F}_{net}\right| = m\left|\vec{a}\right| \;\Rightarrow\; q\,v\,B = m\frac{v^2}{R} \quad \text{(recall that for circular motion } \left|\vec{a}\right| = \frac{v^2}{R}\text{)}$$

Solving for R, we get $R = \dfrac{m\,v}{q\,B}$. Notice that the radius is proportional to the mass of the particle. In a *mass-spectrometer*, the mass of an unknown particle is determined from measurement of the radius (assuming charge, speed and B-field are all known).



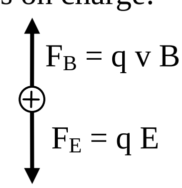
Since the magnetic force has no component along the direction of **B**, there is no acceleration in that direction, and the component of the velocity along the direction of **B** is constant. Consequently, charged particles moving in a magnetic field can form spiral trajectories, spiraling around and along the B-field lines as shown. Charged particles (protons) from the sun (solar wind) are guided along the earth's B-field to arctic regions, where they slam into the atmosphere, producing "Northern lights".

The Velocity Selector The velocity selector is a device which measures the speed v of an ion. (ion = charged atom with one or more electrons missing) . A magnet produces a uniform B-field and a capacitor produces a uniform E-field, with **E** ⊥ **B .**



uniform E down, B (in)

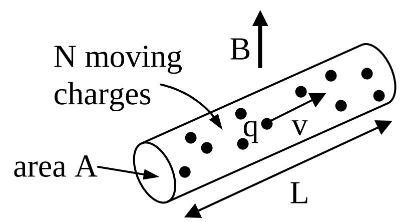
Forces on charge:



The B and E fields are adjusted until the particle goes straight through. If the path is straight, then $F_B = F_E \Rightarrow qvB = qE \Rightarrow v = E / B$.

Magnetic force on a current-carrying wire

A B-field exerts a force on a moving charge. A current-carrying wire is full of moving charges, so a B-field exerts a force on the current-carrying wire. The force on a straight wire of length L, carrying a current I, in a uniform magnetic field **B**, is given by $\boxed{\vec{F} = I\,\vec{L} \times \vec{B}}$, where we define a length vector **L**, having magnitude L = length of the wire and direction equal to the direction of the current in the wire.



Proof:

Force on a single charge = $\vec{F}_{\text{on q}} = q\,\vec{v} \times \vec{B}$.

Number of charges in wire = $N = \underbrace{n}_{\#/\text{vol}} \cdot \underbrace{A \cdot L}_{\text{volume}}$

Total force on all the charges = $\vec{F}_{\text{tot}} = N q\,\vec{v} \times \vec{B} = n A L q\,\vec{v} \times \vec{B}$

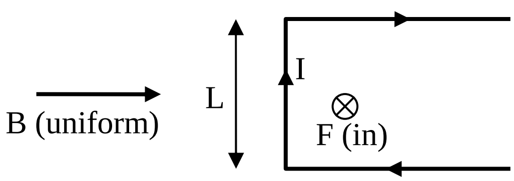
Recall from the chapter on current that $J = \dfrac{I}{A} = n q v \;\Rightarrow\; I = n q v A$ so

$\vec{F}_{\text{tot}} = n A L q\,\vec{v} \times \vec{B} = I\vec{L} \times \vec{B}$.

Alternative proof: Let t is time for charge to move distance L, so speed of moving charges =

$v = \dfrac{L}{t}$, current $I = \dfrac{N q}{t}$. Assume **B** ⊥wire, just to simplify math.

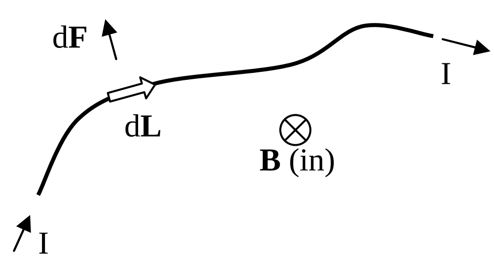
$$F_{\text{on wire}} = N \cdot F_{\text{on q}} = N q \underbrace{v}_{\frac{L}{t}} B = \underbrace{\frac{N q}{t}}_{I} L B = I L B. \quad \text{Done.}$$



F = I L B (**B** ⊥ I direction)

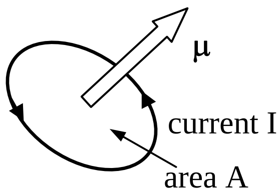
If wire is not straight or **B** is not uniform, then do that calculus thing: in your imagination, break the wire up into little segments d**L.** $d\vec{F} = I\,d\vec{L} \times \vec{B}$,

$\vec{F}_{\text{tot}} = \int d\vec{F} = I \int d\vec{L} \times \vec{B}$



Force on a Current Loop

New term: "magnetic dipole moment" or "magnetic moment" = a loop of current



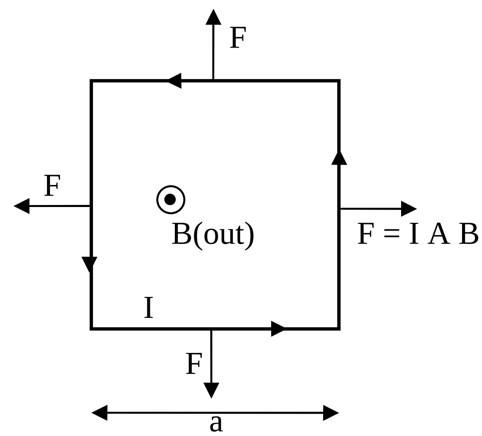
magnetic moment μ = current × area

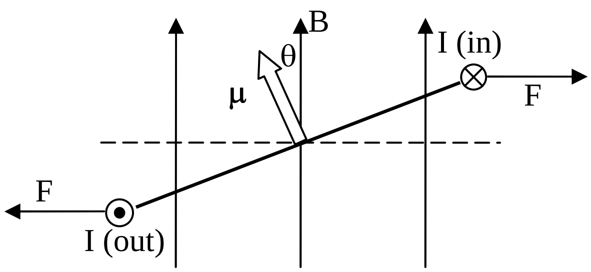
$\mu = I\,\mathbf{A}$

Direction of $\boldsymbol{\mu}$ = direction of area vector **A** + right-hand rule: curl fingers of right hand in direction of current, thumb points along $\boldsymbol{\mu}$.

(It's called a magnetic dipole moment because the B-field created by the current loop looks similar to the E-field created by an electric dipole. More on that in next chapter.)

Suppose loop is in a *uniform* **B**-field. Then $\mathbf{F}_{net} = 0$, always, since forces on opposite sides of loop will cancel.





side view of loop

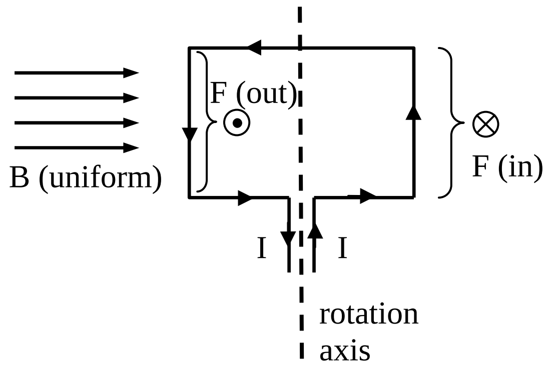
But if $\boldsymbol{\mu}$ is not parallel to **B** (loop not perpendicular to **B**) then there is a torque τ, tending to twist the loop so that $\boldsymbol{\mu}$ aligns with **B**. It's not hard to show that the torque is

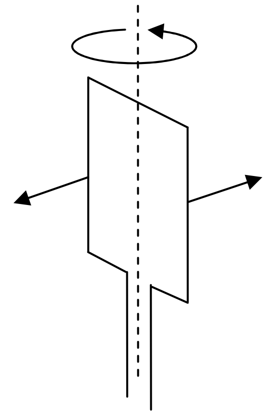
$\vec{\boldsymbol{\tau}} = I\vec{A} \times \vec{B} = \vec{\boldsymbol{\mu}} \times \vec{B}$. This equation $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{B}$ is the principle of operation of a galvanometer, which is a device that measure current: the greater the current, the greater the torque which causes a needle to rotate along a calibrated scale.

Electric Motors

An electric motor is a device which converts electrical energy into mechanical work. It consists of a rotating coil of wire carrying current I in a constant magnetic field B. (The B-field is made by a permanent magnet or by another coil of wire with current I .)

The B-field exerts forces on the coil, causing it to rotate. After the coil rotates 180^o, the current I reverses direction so that the force always causes the coil to rotate in the same sense.





So far, we have assumed the existence of **B**, and described the force on a moving charge due to that **B**. Now we will show how to make a B-field with a current.

Cross-Product Review: The *cross-product* of two vectors is a third vector $\vec{A}\times\vec{B}=\vec{C}$ defined like this: The magnitude of $\vec{A}\times\vec{B}$ is $A\,B\sin\theta$. The direction of $\vec{A}\times\vec{B}$ is the direction perpendicular to the plane defined by the vectors **A** and **B** plus right-hand-rule. (Curl fingers from first vector **A** to second vector **B**, thumb points in direction of $\vec{A}\times\vec{B}$