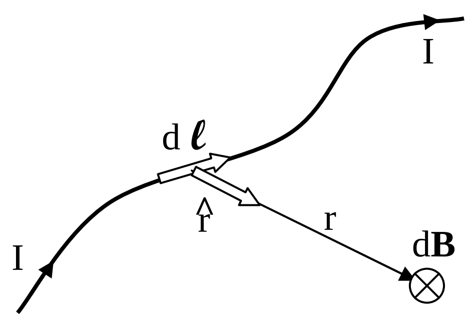
Currents make B-fields

We have seen that charges make E-fields: $d\vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dQ}{r^2}\,\hat{r}$.

Currents make B-fields according to the **Biot-Savart Law**: $\boxed{d\vec{B} = \dfrac{\mu_0 I}{4\pi}\dfrac{d\vec{\ell}\times\hat{r}}{r^2}}$



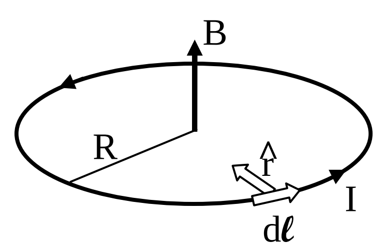
where μ_0 = constant = $4\pi\times10^{-7}$ (SI units).
d**B** is the element of B-field due to the element of current I d***ℓ*** . d***ℓ*** is an infinitesimal length of the wire, with direction given by the current. The total B-field due to the entire current is $\vec{B} = \int d\vec{B} = \dfrac{\mu_0 I}{4\pi}\int\dfrac{d\vec{\ell}\times\hat{r}}{r^2}$.

This can be a very messy integral!

This law was discovered experimentally by two French scientists (Biot and Savart) in 1820, but it can be derived from Maxwell's equations.

Example of Biot-Savart: B-field at the center of a circular loop of current I, with radius R. Here the integral turns out to be easy.

$\left|d\vec{\ell}\times\hat{r}\right| = d\ell\,|\hat{r}|\sin 90^0 = 1 \quad\Rightarrow\quad dB = \dfrac{\mu_0 I}{4\pi}\dfrac{d\ell}{R^2}$



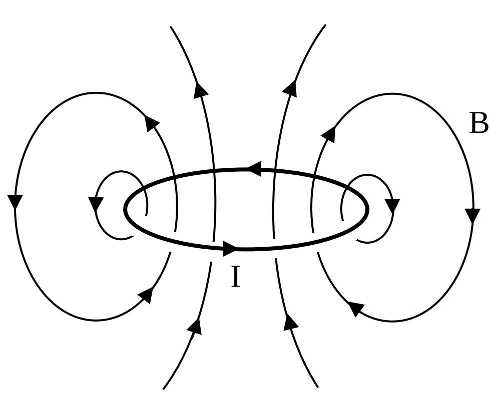
We can replace the vector integral

$\vec{B} = \int d\vec{B}$ with the scalar integral $B = \int dB$

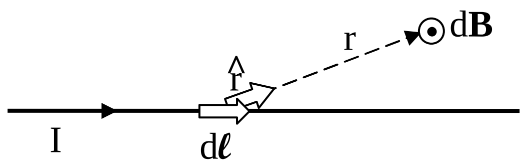
because all of the d**B**'s point in the same direction.

$$B = \int dB = \frac{\mu_0 I}{4\pi R^2}\underbrace{\int d\ell}_{2\pi R} = \frac{\mu_0 I}{2R}\,.$$

The full field at all positions near a current loop requires very messy integrations, which are usually done numerically, on a computer. The full field looks like this:

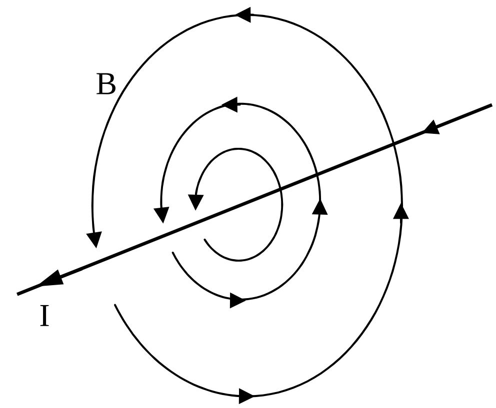


Another, more difficult example of the Biot-Savart Law: B-field due to a long straight wire with current I. The result of a messy integration is



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

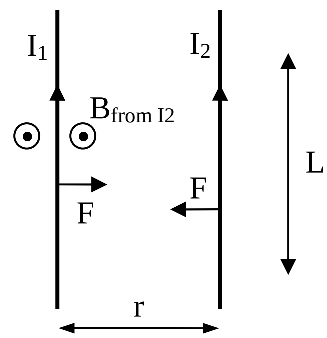
This formula can be derived from a fundamental law called Ampere's Law, which we describe below.



The B-field lines form circular loops around the wire.
To get directions right for both these examples (B due to wire loop, B due to straight wire), use "Right-hand-rule II": With right hand, curl fingers along the curly thing, your thumb points in direction of the straight thing.

Force between two current-carrying wires:

Current-carrying wires exert magnetic forces on each other. Wire2 creates a B-field at position of wire 1. Wire1 feels a force due to the B-field from wire 2: $\vec{F}_{\substack{\text{on1}\\\text{from 2}}} = I_1 \vec{L} \times \vec{B}_2$

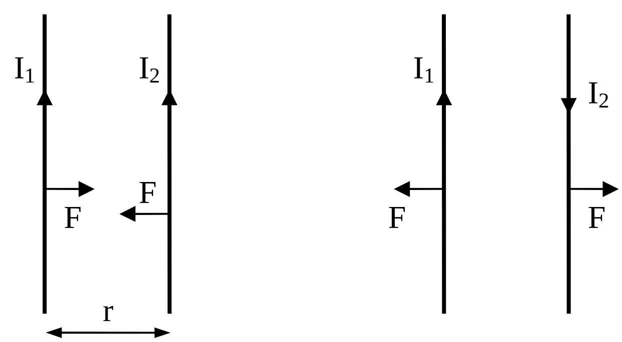


$$\Rightarrow \quad F_{\substack{\text{on 1}\\\text{from 2}}} = I_1 L B_2 = I_1 L \frac{\mu_0 I_2}{2\pi r} .$$

force per length between wires = $\dfrac{F}{L} = \mu_0 \dfrac{I_1 I_2}{2\pi r}$

- Parallel currents attract
- Anti-parallel currents repel.

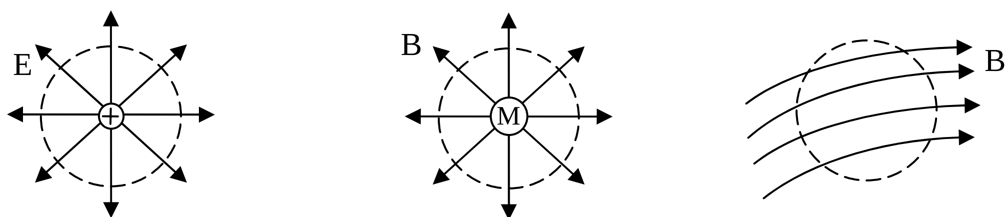
"Going my way? Let's go together.
Going the other way? Forget you!"



Gauss's Law for B-fields

B-field lines are fundamentally different from E-field lines in this way: E-field lines begin and end on charges (or go to ∞). But B-field lines always form closed loops with no beginning or end. A hypothetical particle which creates B-field lines in the way a electric charge creates E-field lines is called a *magnetic monopole*. As far as we can tell, magnetic monopoles, magnetic charges, do not exist. There is a fundamental law of physics which states that magnetic monopoles do not exist.

Recall the electric flux through a surface S is defined as $\Phi_E = \int_S \vec{E} \cdot d\vec{a}$. In the same way, we define the magnetic flux through a surface as $\Phi_B = \int_S \vec{B} \cdot d\vec{a}$. Gauss's law stated that for any closed surface, the electric flux is proportional to the enclosed electic charge:



Yes! **Impossible!** **OK !**

$\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$. But there is no such thing as "magnetic charge", so the corresponding equation for magnetic fields is

$$\boxed{\oint \vec{B} \cdot d\vec{a} = 0}$$

This equation, which has no standard name, is one of the four Maxwell Equations. It is sometimes called "Gauss's Law for B-fields".

Ampere's Law

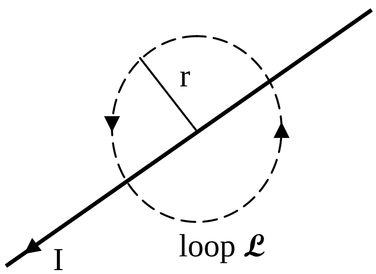
Ampere's Law gives the relation between current and B-fields:

For any closed loop $\mathcal{L}$, $\boxed{\oint_{\mathcal{L}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed}}$, where $I_{enclosed}$ is the current through the loop $\mathcal{L}$.

(It will turn out the Ampere's Law is only true for <u>constant</u> current I. If the current I is changing in time, Ampere's Law requires modification.) Ampere's Law for steady currents, like Gauss's Law, is a fundamental law of physics. It can be shown to be equivalent to Biot-Savart Law.

We can use Ampere's Law to derive the B-field of a long straight wire with current I.

B-field of a long straight wire:



$\mathcal{L}$ = imaginary circular loop of radius r

We know that **B** must be tangential to this loop; **B** is purely azimuthal; **B** can have no radial component toward or away from the wire. How do we know this? A radial component of **B** is forbidden by Gauss's Law for B-fields. Alternatively, we know from Biot-Savart that **B** is azimuthal. So, in this case, $\vec{B}\cdot d\vec{\ell} = B\,d\ell$ Also, by symmetry, the magnitude of **B** can only depend on r (distance from the wire): B = B(r).

$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{\ell} \underset{(\vec{B}\,\|\,d\vec{\ell})}{=} \oint B\,d\ell \underset{(B\text{const})}{=} B\oint d\ell = B\,2\pi r = \mu_0 I$$

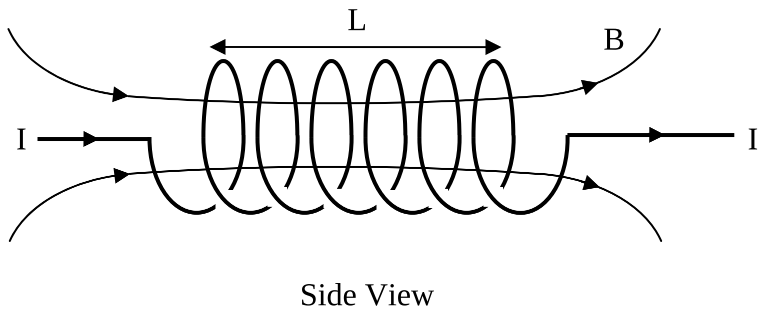
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

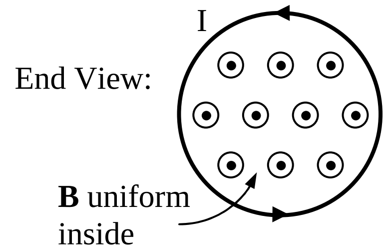
Like Gauss's Law, Ampere's Law is always true, but it is only useful for computing B if the situation has very high symmetry.

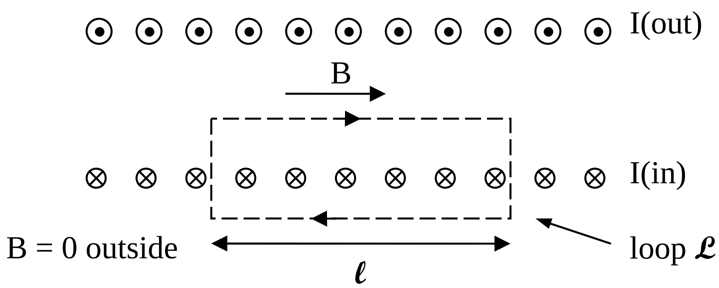
B-field due to a solenoid. solenoid = cylindrical coil of wire

It is possible to make a uniform, constant B-field with a solenoid. In the limit that the solenoid is very long, the B-field inside is uniform and the B-field outside is virtually zero.

Consider a solenoid with N turns, length L, and n = N/L = # turns/meter





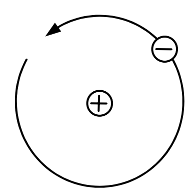


$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{\ell} \;=\; B\ell \;=\; \mu_0\, I_{\text{thru}\,\mathcal{L}} \;=\; \mu_0 \underbrace{n\,\ell}_{\substack{\#\,\text{turns}\\ \text{thru}\,\mathcal{L}}} I \qquad \Rightarrow \qquad B \;=\; \mu_0\, n\, I$$

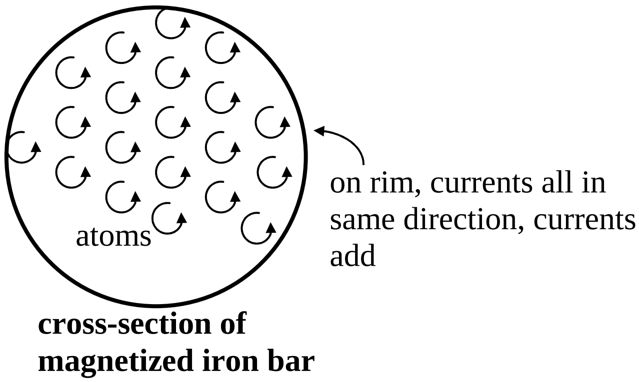
$\Rightarrow$ B-field inside solenoid is $B \;=\; \mu_0\, n\, I \;=\; \mu_0 \dfrac{N}{L} I$

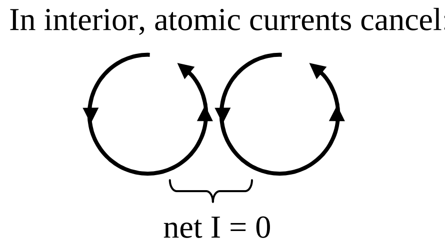
Permanent Magnets

Currents make B-fields. So where's the current in a permanent magnet (like a compass needle)? An atom consists of an electron orbiting the nucleus. The electron is a moving charge, forming a tiny current loop — an "atomic current". In most metals, the atomic currents of different atoms have <u>random</u> orientations, so there is no net current, no B-field.

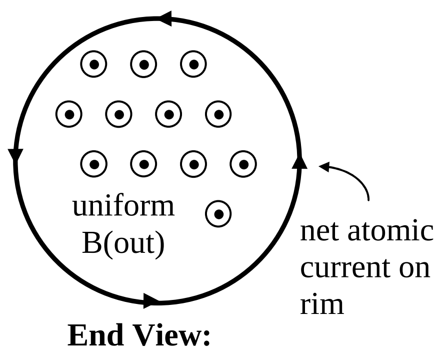


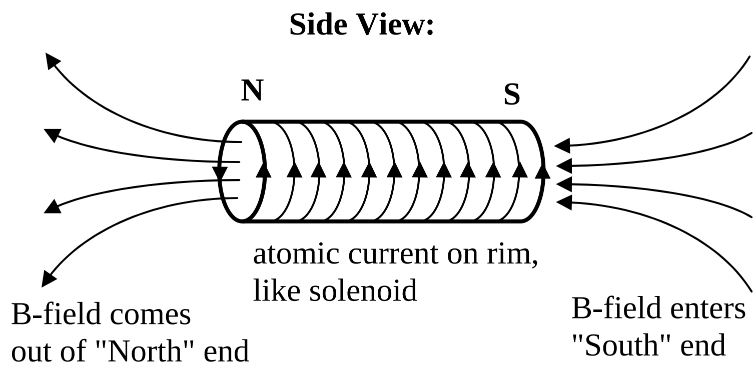
In *ferromagnetic* materials (Fe, Ni, Cr, some alloys containing these), the atomic currents can all line up to produce a large net current.





In a magnetized iron bar, all the atomic currents are aligned, resulting in a large net current around the rim of the bar. The current in the iron bar then acts like a solenoid, producing a uniform B-field inside:





Why do permanent magnets sometimes attract and sometimes repel? Because parallel currents attract and anti-parallel current repel.