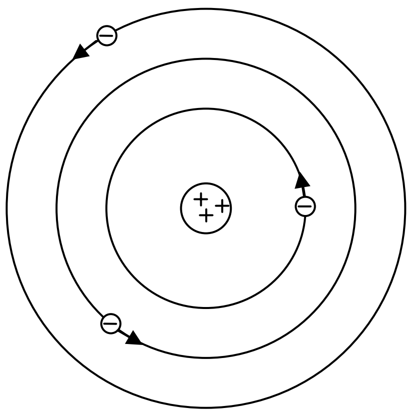
Charges, Coulomb's Law, and Electric Fields

Some experimental facts:

Experimental fact 1: Electric charge comes in <u>two</u> types, which we call (+) and (–).

An atom consists of a heavy (+) charged nucleus surrounded by light (–) electrons.
nucleus = (+) charged protons and (0) charged neutrons



$m_{proton} \approx m_{neutron} >> m_{electron}$ ($m_{proton} \approx 1800\ m_{electron}$)

$q_{proton} = q_{electron}$ ("q" is symbol for charge)

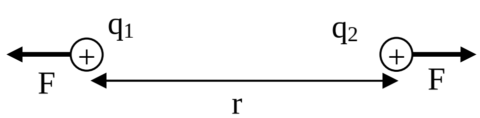
Calling protons (+) and electrons (–) is a <u>convention</u>. We could just have easily called electrons (+) and protons (–), but Ben Franklin chose the other, and we're stuck with it.

The number of protons in the nucleus of an atom is called "Z". Z determines the element: Z =1 is hydrogen, Z = 3 is lithium.

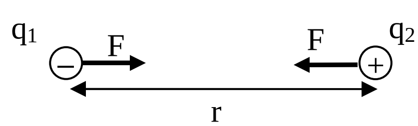
Fact 2: Unlike charges attract, and like charges repel according to **Coulomb's Law**, which says that the magnitude F of the force between two charges q_1 and q_2 separated by a distance r is given by

$$\boxed{F = k\frac{|q_1||q_2|}{r^2}}$$

where k = constant = 9.0×10^9 N m^2/C^2 .



Like-sign charges repel.



Unlike-sign charges attract.

In SI units, the unit of charge is the coulomb (C).

magnitude of charge of electron = e = $+1.602 \times 10^{-19}$ C

charge of electron = –e , charge of proton = +e (by convention, the symbol e > 0, always)

A coulomb is a <u>huge</u> amount of charge: Number N of e's in 1 C = ?

$$N \cdot e = 1C \quad \Rightarrow \quad N = \frac{1C}{e} = \frac{1C}{1.6\times10^{-19}C} = 6.3\times10^{18}$$

Fact 3: Electric charge is conserved. The net charge of an isolated system cannot change. It is impossible to create or destroy net charge. Except in nuclear or "high-energy" reactions, you can never create or destroy electrons, protons, and other charged particles – all we can do is move them around. In high energy reactions, we can create charged particles from energy (energy = mc^2), but the particles are always created or destroyed in pairs (+1 and −1) so that the net charge is conserved.

Aside: As far as we know, only 4 things in the universe are conserved:
(1) Energy (2) Linear momentum ($\mathbf{p} = m\mathbf{v}$) (3) Angular momentum (spin = $\mathbf{L} = I\omega$)
(4) Charge

[Not quite true: in high energy physics, there may be other quantities, like "baryon number" that are conserved.]

Fact 4: The charge e is the fundamental unit of charge. You never find a free particle in nature with charge = fraction of e. You only find charge = e or integer multiple of e.

Statements (1) thru (4) are experimental facts. Why are they true? Why are there 2 kinds of charge, not 3? Why $e = 1.6 \times 10^{-19}$ C, not 4.2×10^{-19} C? Why is charge conserved? We don't know! And to some extent, physicists don't care. It is the primary goal of physics to describe how nature behaves; a secondary goal is to explain why it behaves that way. (Many theorists are looking to explain why, but no luck yet…)

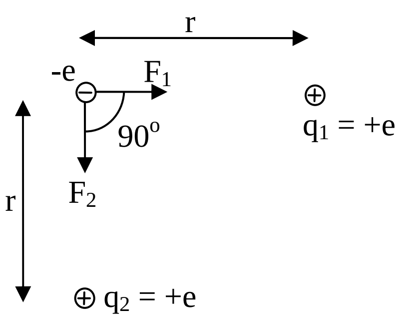
Notice that Coulomb's law is similar to Newton's Universal Law of Gravitation:

$$F_{grav} = \frac{G\ m_1 m_2}{r^2}, \qquad F_{coul} = \frac{k\ |q_1||q_2|}{r^2}$$

Similar, except that there are two kinds of charge (+ and –), but only one kind (sign) of mass. Gravity is always attractive, but electrical force can be attractive or repulsive.

Recall that force is a vector – a mathematical object that has a size (magnitude) and a direction. Forces add like vectors, not numbers.

Example: Net force on an electron due to two nearby protons, each a distance r away, 90° apart as shown.

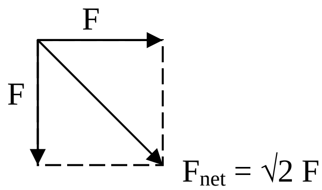


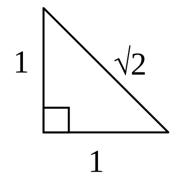
Net force = $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$

In this particular case, $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = F = \dfrac{k\,e^2}{r^2}$.

$F_{net} = \sqrt{2}\,F$ (not 2F) $\Rightarrow$ $F_{net} = \sqrt{2}\,\dfrac{k\,e^2}{r^2}$

Recall:





Here we have used the **Superposition Principle**: the net force on a charge due to other nearby charges is the **vector** sum of the individual forces:

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots$, where $\vec{F}_1 =$ net force due to charge 1, etc.

The Electric field (a new concept)

Surrounding every charge (or group of charges) is a thing, called an *electric field* $\vec{E}$ (it is a vector thing)

Definition: The electric field $\vec{E}$ at a point <u>in empty space</u> is a vector quantity which can be measured by the following procedure: place a small <u>test charge</u> q at that point, measure the force $\vec{F}$ on q due to all other charges. The electric field at that point is given by

$\boxed{\vec{E} \equiv \dfrac{\vec{F}_{\text{on q}}}{q}}$ E-field at a point is the force per charge on a test charge placed at that point.

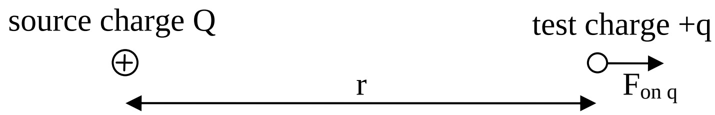
Note! The E-field exists even if there is no test charge present to measure it. Similarly, a gravitational field surrounds the earth, even if there is no "test mass" nearby to measure the pull of earth's gravity:

gravitational field $\equiv \dfrac{\vec{F}_{\text{on m}}}{m} = \dfrac{m\vec{g}}{m} = \vec{g}$, magnitude $g = \dfrac{F}{m} = \dfrac{1}{m}\dfrac{G\,M\,m}{r^2} = \dfrac{G\,M}{r^2}$

(M = earth mass, m = test mass, r = distance from m to Earth's center)

The electric field is not just an mathematical invention; it is real. We cannot (usually) see it or smell it, but we can *feel* it. In some situations, you can see an electric field: visible light is a rapidly oscillating electric field (more on that later in the semester.)

What is the E-field around a point charge Q? (Q = "source charge" = "source" of E-field, q = "test charge" or "probe charge")



$$\vec{E} = \frac{\vec{F}_{\text{on q}}}{q} = \frac{1}{q}\frac{k\,Q\,q}{r^2}\,\hat{r} = \frac{k\,Q}{r^2}\,\hat{r}$$

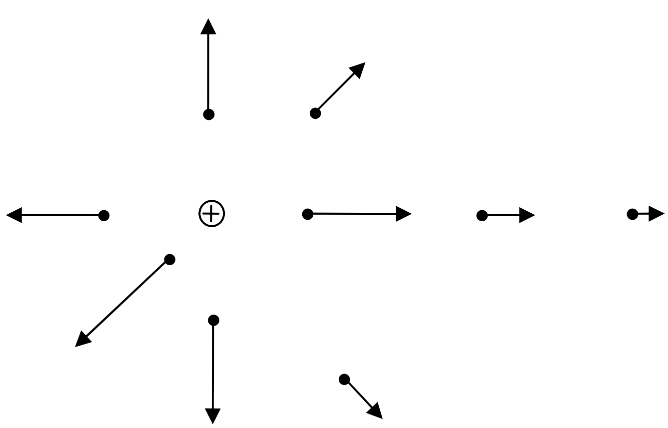
($\hat{r}$ pronounced"r-hat" is the unit vector pointing away from the origin, where Q is. r-hat has no dimensions).

Magnitude of the E–field due to a point charge Q: $\boxed{\vec{E} = k\frac{Q}{r^2}\,\hat{r}}$

If the source charge Q is positive, then the E-field points away from Q, in the direction of r-hat. If the source charge Q is negative then the E–field points toward Q in the direction opposite r-hat. This follows directly from the definition $\vec{E} = \vec{F}/q$. For instance, if both Q and q are positive then the force **F** points away from Q and so does **E**. If Q is negative and q is positive, then both **F** and **E** point toward Q.

What if the test charge q is changed from positive to negative? Then the direction of the force **F** *and* the sign of q *both* flip, which leaves the direction of **E** unchanged. The size and direction of the E-field is independent of the test charge. The test charge is just an imaginary artifice which we use to measure something which is already there.

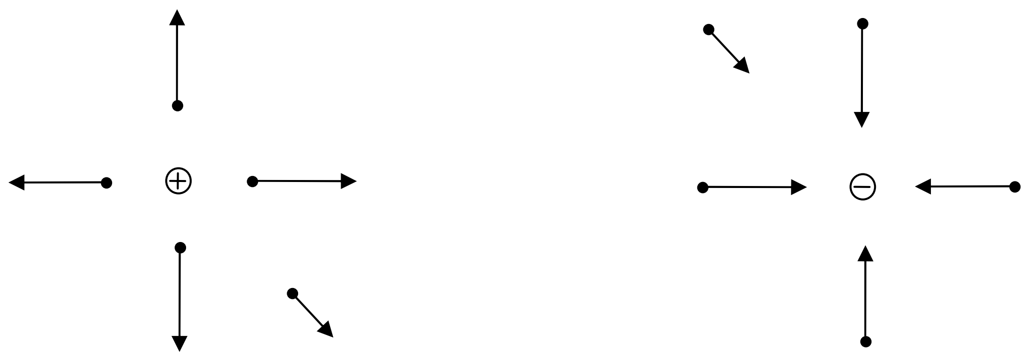
The E-field around a positive charge points always from the charge, and decreases in magnitude with distance r as $E \propto \frac{1}{r^2}$. We can represent the E-field at various points in space by drawing a little dot at those points and drawing an arrow coming out of that dot. The arrow represents the E-field at the dot point. Think of the E-field arrow as "packed into the point". The E-field arrow is not something "reaching from beginning to end of arrow". The E-field at a point in space exists at that point.



Notice that $E \to \infty$ as $r \to 0$. The electric field *diverges* near a point charge.

Again, what if the test charge q in $\vec{E} = \dfrac{\vec{F}_{\text{on q}}}{q}$ is negative? E-field still points away from positive source charge Q, since both $\vec{F}$ changes direction and q switches sign, which leaves the vector $\vec{E}$ unchanged.

The E-field points away from positive charges. It points toward negative charges.



We can think of the interaction between charges in two different ways: "Action at a distance" vs. "Fields"

"Action at a distance" : Coulomb's Law suggests that two charges exert a force on each other through empty space, instantaneously. But Coulomb's law is only valid for *stationary* charges. If charge 1 moves, it takes some time for charge 2 to sense the change.

The more modern "field-view" is: Charge 1 creates an E-field around it. Charge 2 feels that field. If Charge 1 moves, it takes some time for the surrounding E-field to change, so it takes some time for charge 2 to react.

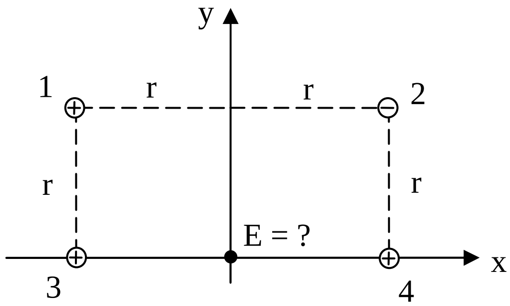
The total E-field due to a collection of charges is the vector sum of the E-fields due to the individual charges:

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \ldots = \sum_i \vec{E}_i \ , \ \text{where } E_1 = \left|\vec{E}_1\right| = k\frac{|Q_1|}{r^2} \ , \ E_2 = .. , \text{etc}$$

Why? Superposition Principle says that if we place a small test charge q near other charges Q_1, Q_2, Q_3,… , then the net force on q is

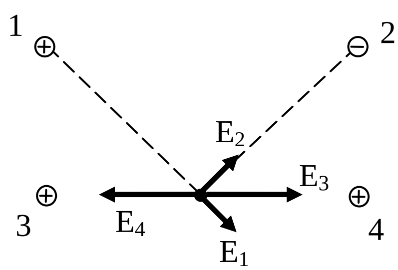
$$\vec{F}_{\text{total}} = \vec{F}_1 + \vec{F}_2 + \ldots \quad \Rightarrow \quad \frac{\vec{F}_{\text{total}}}{q} = \frac{\vec{F}_1}{q} + \frac{\vec{F}_2}{q} + \ldots \quad \Rightarrow \quad \vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \ldots$$

Example: Electric fields (qualitative) Four point charges, labeled 1 through 4, all with the same magnitude q, are placed around the origin as shown. Charge 2 is negative, the rest are positive. What is the direction of the E-field at the origin?

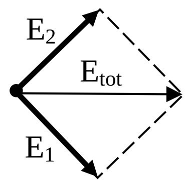


The total E-field at origin is the vector sum
$\mathbf{E}_{tot} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \mathbf{E}_4$

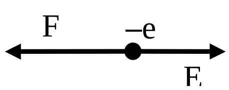
(I'll use bold type to indicate vector.)



Notice that $\mathbf{E}_3$ and $\mathbf{E}_4$ cancel since they have equal magnitude, but opposite directions. The total E-field points right.



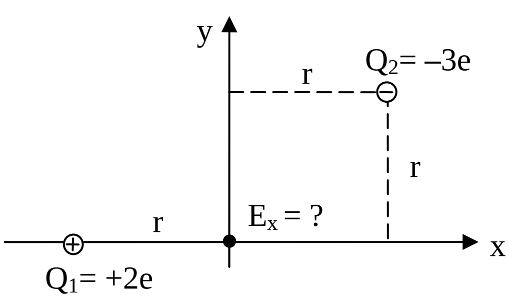
What would be the direction of the force on an electron (charge q = –e) placed at the origin?
Since $\vec{E} = \vec{F}/q$, we have $\vec{F}_{\text{on q}} = q\vec{E}$. If q is negative, the direction of the force on q is opposite the direction of the E-field. So the force is to the left.



$$\boxed{\vec{F}_{\text{on q}} = q\vec{E}}$$

In this equation, the E-field is due to all the *other* charges, not the field due to the charge q itself.

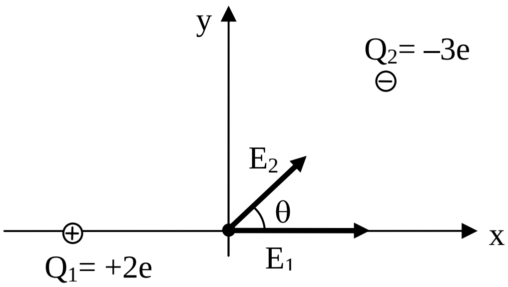
Example Electric fields (quantitative) Two charge Q_1 = +2e and Q_2 = –3e are placed as shown. What the x-component of the electric field at the origin?



$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 \Rightarrow E_{tot,x} = E_{1x} + E_{2x}$$

$$E_1 = k\frac{Q_1}{r^2} = k\frac{2e}{r^2} \quad , \quad E_2 = k\frac{|Q_2|}{\left(\sqrt{2}r\right)^2} = k\frac{3e}{2r^2}$$

(Have used the fact that the distance from the origin to Q_2 is $\sqrt{2}$ r)



$E_{1x} = E_1$, since $\mathbf{E}_1$ is along the x-axis.

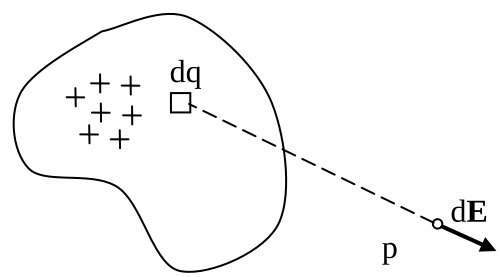
$$E_{2x} = E_2 \cos\theta = E_2 \cos 45^o \cong 0.707\, E_2$$

$$E_{tot,x} = E_{1x} + E_{2x} = k\frac{2e}{r^2} + 0.707\, k\frac{3e}{2r^2}$$

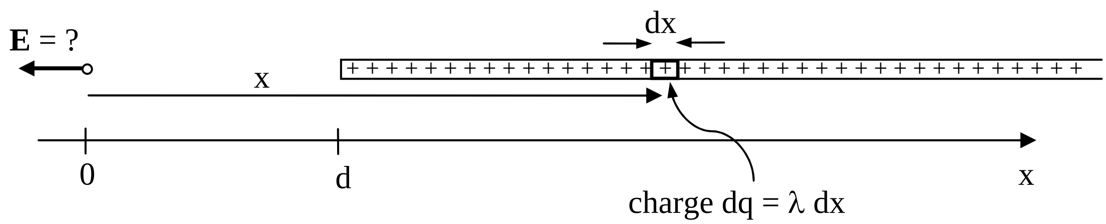
$$= [2 + 0.707(3/2)]\; k\frac{e}{r^2} = 3.06\, k\frac{e}{r^2}$$

E-field due to Continuous distribution of charge

Image a *continuous* distribution of charge with the charge spread out smoothly over the volume of some object. What is the electric field at some point p due to this volume of charge? A very small (infinitesimal) volume of the the object has an infinitesimal charge dq. "dq" means a "little bit of charge" This little bit of charge dq creates an infinitesimal electric field d**E**. The total electric field **E** at p due to all the bits of charge is $\vec{E} = \int d\vec{E}$.



Example: A semi-infinite line of charge with charge per length = λ , units [λ] = C/m. What is the E-field at a distance d from the end of the line, as shown?



dx = "little bit of x" , dq = "little bit of charge"

Coulomb's Law gives us the magnitude of the field dE due to the charge dq, a distance x away:

$$dE = \frac{k\,dq}{r^2} = \frac{k\,dq}{x^2} = \frac{k\;\lambda\;dx}{x^2}$$ = " little bit of E due to little bit of charge dq"

Since all directions here are along the x-axis, we are only interested in dE_x and can just drop the subscript x. So instead of working with the 3D integral $\vec{E} = \int d\vec{E}$, we work with the 1D integral

$$E = \int dE = \int_d^\infty \frac{k\,\lambda\,dx}{x^2} = -\frac{k\lambda}{x}\bigg|_d^\infty = 0 - \left(-\frac{k\lambda}{d}\right) = \frac{k\lambda}{d}\,.$$

Check units: $[\lambda]$ = charge/length, so $[k\,\lambda\,/\,d]$ has units of $[k]\times\dfrac{\text{charge}}{\text{length}^2} = \left[\dfrac{k\,q}{r^2}\right]$. Units check!

In 2D problems where the E-field has components along x and y, break the problem up into x- and y-components.

$$\vec{E} = \int d\vec{E} \quad \Leftrightarrow \quad \vec{E} = E_x\hat{x} + E_y\hat{y} = \int dE_x\,\hat{x} + \int dE_y\,\hat{y}$$

Conductors vs. Insulators

Most materials can be classified as conductor or insulator . A conductor is also called a metal; an insulator also called avdielectric.

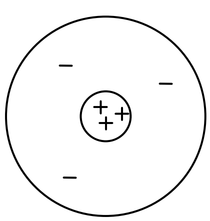
Metals (Cu, Al, Au, Ag, Fe…) conduct electricity. In metals, some of the electrons (conduction electrons) can move freely thru the metal. If there is an E-field, the conduction electrons move in response to the force F = q E, and so a flow of charge, or current, occurs.

The inner core electrons are bound strongly to their nuclei, but the outer core, conduction electrons are unbound, free to move among the nuclei. Metals usually have 1 or 2 conduction electrons per atom. (In chemist talk: “valence = 2” means “2 conduction electrons per atom”.)

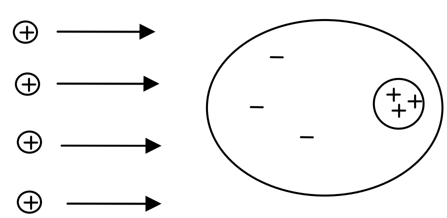
Insulators (plastic, wood, ceramic, sulfur) do not conduct electricity. In insulators, all the electrons are strongly attached to their nuclei, and do not move (much), even if there is an E-field exerting a force on them.

Metals are shiny, insulators are dull. The appearance is a consequence of the mobility of the electrons.

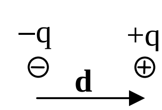
Insulators can have an induced charge due to induced dipole moments. All atoms, some molecules, have no permanent dipole moment, but acquire an *induced* moment when an external E-field is applied.



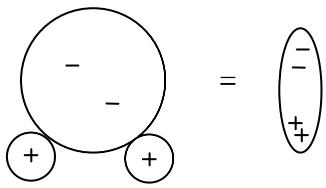
neutral atom
E = 0



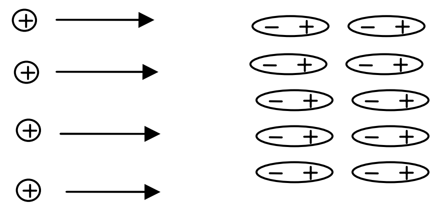
polarized atom in
external field E



Recall from chemistry, that a *dipole moment* is associated with a pair of equal and opposite charges (+q and – q) separated by a distance d. The dipole moment **p** is a vector quantity defined as $\vec{p} = q\vec{d}$, where the vector **d** points from – to +.
Some molecules, like H_2O, have a *permanent* dipole moment. In an external E-field, the moments align.



H_2O molecule = dipole moment



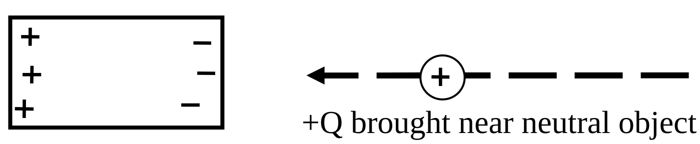
moments align in E-field

Induced Charge

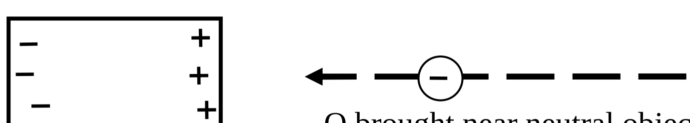
A charged object (+Q, say) brought near a neutral object *induces* a charge separation in the neutral object The equal and opposite charges on the two side of the object are called *induced* charge. Another way to describe this situation is to say that the E-field from the charge Q induces polarization charge.

Notice that an induced charge always results in a net attraction to the source of the E-field. The positively-charge +Q is attracting the negatively-charged near side of the object and repelling the positively-charged far side. But the attraction to the nearby side is greater than the repulsion from the more distant side, so the net force is attractive.

neutral object (coke can or block of wood)



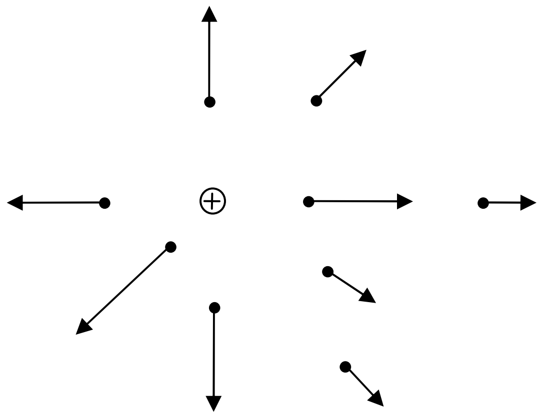
+Q brought near neutral object

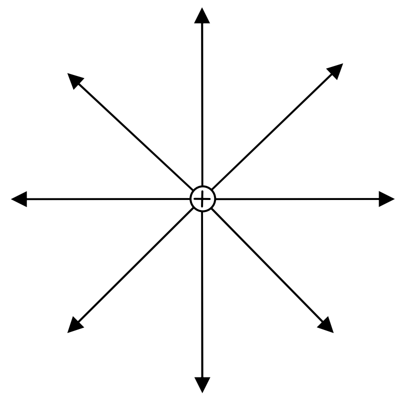


– Q brought near neutral object

Electric Field Line Diagrams

The electric field can be represented by a *field line diagram*. Instead of trying to show the E-field at a whole bunch of individual points (left diagram), we indicate the field everywhere with a field line diagram (right diagram)

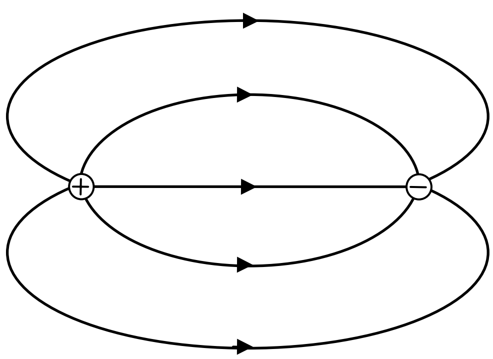


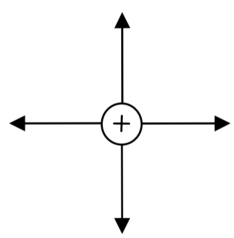


Field line diagram for (+) point charge.

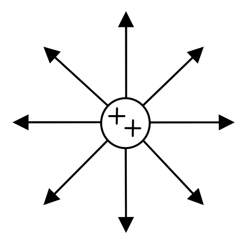
Rules for field line diagrams:

1) Field lines begin on positive charges, end on negative charges, or go off to infinity.





Charge q



Charge 2q

2) The number of field lines coming from or going to a charge is proportional to the magnitude of the charge: More field lines = bigger charge

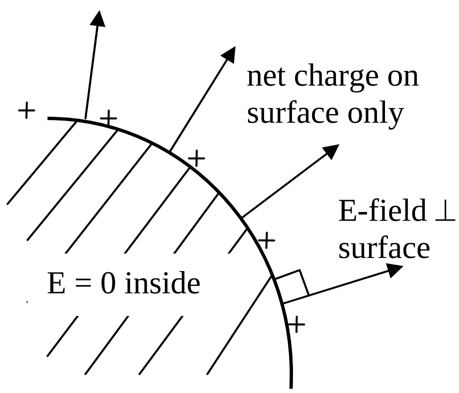
3) The direction of the E-field at a point is the direction tangent to the field line at that point.

4) The magnitude of the E-field at a point is proportional to the density of field lines at that point. To be precise, the magnitude of the E-field is proportional to the number of field per area perpendicular to the field direction. More densely packed field lines = higher magnitude E-field.

Conductors in Electrostatic Equilibrium

"Electrostatic equilibrium" means that all charges are stationary; so the net force on every charge must be zero (otherwise the charge would be accelerating).
3 interesting facts about metals (conductors) *in electrostatic equilibrium*:

- The electric field in the *interior* of a metal must be zero.
- Any net charge on the conductor resides only on the *surface* of the conductor.
- The electric field must be *perpendicular* to the surface of the conductor.

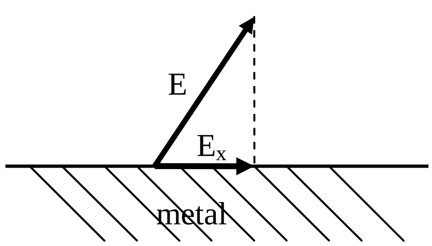


A metal in electrostatic equilibrium

The E-field must be zero in the interior, otherwise the conduction electrons in the metal would feel a force **F** = q **E** = –e **E** and would move in response. Electrons in motion would mean we are not in electrostatic equilibrium.

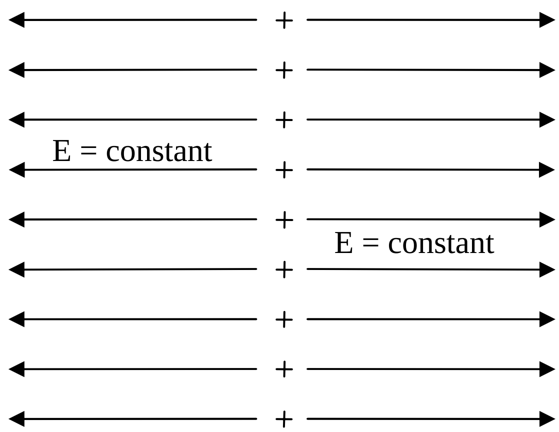
Any net charge resides only on the surface because any net charge in the interior would create an E-field in the interior which would cause the electrons to move. The electrons would keep moving until all charges have arranged themselves so that both the total E-field and net charge is zero everywhere inside the metal. In the next chapter, we will see a rigorous proof of this using Gauss's Law.

The E-field must be perpendicular to the surface (in electrostatic equilibrium), otherwise the *component* of the E-field *along* the surface would push electrons along the surface causing movement of charges (and we would not be in equilibrium).



On the surface of metal, if E_x was not zero, there would be a force $F_x = q\,E_x = -e\,E_x$ on electrons in the metal pushing them along the surface.

Another curious fact about electric fields: the E-field due to an a very large plane (sheet) of charge is constant in both direction and magnitude (as long as we are "close" to the plane). We will prove this later, using Gauss' Law.



uniform plane of charge
(seen edge-on)

One last important fact about charges and E-fields: The E-field anywhere is always due to all the charges everywhere. To get the total E-field, must always add up all the E-fields due to <u>all</u> charges everywhere: $\mathbf{E}_{tot} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots$ You cannot destroy or "block" the E-field due to a charge, but you can create a second E-field which cancels the first E-field.

