A student in 1120 emailed me to ask how much extra he should expect to pay on his electric bill when he strings up a standard 1-strand box of icicle holiday lights outside his house. (total, cumulative cost)?
Try to make a real estimate, don't just guess!
Energy in Colorado costs about 10¢/kW hr.

A: Less than 1 cent
B: Between 1 cent and 10 cents
C: Between $.10 and $1.00
D: Between $1.00 and $10.00
E: More than $10.00

50-100 Watts? (Like ONE bulb). 12 hrs/day? 30 days?
~100 W * 10 hrs/day * 30 days = 30,000 W*hrs = 30 kW hrs.
⇒$3
(for 100 million Joules! Energy is cheap…)

Welcome back!!

CAPA #13 is due Friday
New online participation survey is up!
Pretest tonight (and Tut hw for tomorrow)

Reading: catch up if you're behind! E.g. 35.6 (1st 2 pp)
(We'll finish up Ch. 33 this week -all sections, including 33.6)

Last: Transformers and Induction
Today: Inductors in circuits
Next: AC circuits

Inductor = (coil of wire)

Important fact: Magnetic Flux Φ_B is proportional to the current making the Φ_B

All our equations for B-fields show that B α i

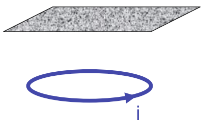
$$d\vec{B} = \frac{\mu_0 i}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2} \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0 i_{thru}$$

Biot-Savart — Ampere

$$\text{Flux } \Phi_B \propto B \propto i$$

$$\text{Flux } \Phi_B \propto i$$

Assumes leaving everything else the same.



If we double the current i, we will double the magnetic flux through any surface.

Self-Inductance (L) of a coil of wire

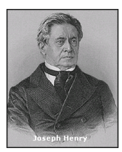
$$\Phi_B \equiv Li$$

$$L \equiv \frac{\Phi_B}{i}$$

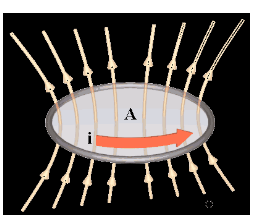
This equation defines self-inductance.

Note that since $\Phi_B \propto i$, L must be independent of the current i.

L has units [L] = [Tesla meter^2]/[Amperes]
New unit for inductance = [Henry].



An inductor is just a coil of wire.



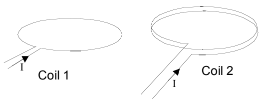
The Magnetic Flux created by the coil, through the coil itself:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

This is quite hard to calculate for a single loop. Earlier we calculated B at the center, but it varies over the area.

Inductor 1 consists of a single loop of wire. Inductor 2 is identical to 1 except it has two loops on top of each other. How do the self-inductances of the two loops compare?

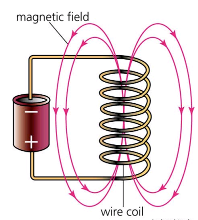
A) $L_2 = 2 L_1$ B) $L_2 > 2L_1$ C) $L_2 < 2L_1$



HINT 1: What is the B field at the center of coil 2, B2, compared to the field in the center of coil 1?
HINT 2: inductance L = Φ(total)/I

Answer: L2 > 2L1 , in fact L2 is roughly 4 L1!
Recall L =Φ/I. When N doubles => B doubles.
Φ(each loop) doubles (because *B is doubled*)
But Φ(tot) = 2 Φ(each loop), so double*double = 4 times!

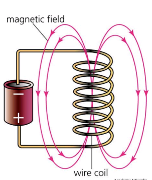
Consider a simpler case of a solenoid



$$|\vec{B}_{inside}| = \mu_0 n i = \mu_0 \frac{N}{L'} i$$

Recall that the B-field inside a solenoid is uniform!

N = number of loops
L' = length of solenoid
* Be careful with symbol L !



$$|\vec{B}_{inside}| = \mu_0 n i = \mu_0 \frac{N}{L'} i$$

$$\Phi_B = N \oint \vec{B} \cdot d\vec{A} = NBA = N(\mu_0 n i) A$$

$$L = \frac{\Phi_B}{i} = \mu_0 N n A = \mu_0 A n^2 L'$$

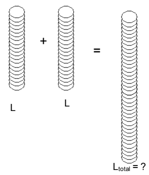
Self-Inductance

Length

Clicker Question

Two long solenoids, each of inductance L, are connected together to form a single very long solenoid of inductance L_{total}. What is L_{total}?

A) 2L
B) 4L
C) 8L
D) none of these/don't know



Answer: 2L. The inductance of a solenoid is $L = \mu_o A n^2 L'$, (n = N/L' and *L'* is the length.) In this case, we did not change n, but *L'* (length) doubled, so L doubles.

What does inductance tell us?

$$L = \frac{\Phi_B}{i}$$

$$\Phi_B = Li$$

$$\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

L is independent of time.
Depends only on geometry of inductor (like capacitance).

$$\frac{d\Phi_B}{dt} = L\frac{di}{dt}$$

Recall Faraday's Law $\varepsilon = -\frac{d\Phi_B}{dt}$

$$-\varepsilon = L\frac{di}{dt}$$

$$\varepsilon = -L\frac{di}{dt}$$

Changing the current in an inductor creates an EMF which opposes the change in the current.
Sometimes called "back EMF"

$$\varepsilon = -L\frac{di}{dt}$$

It is difficult (requires big external Voltage) to change quickly the current in an inductor.

The current in an inductor **cannot** change instantly.

If it did (or tried to), there would be an infinite back EMF. This infinite back EMF would be fighting the change!

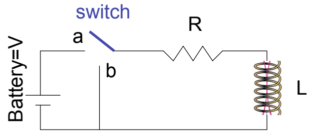
What do these inductors do in circuits?

Just recall that the EMF or Voltage across an inductor is:

$$\varepsilon = -L\frac{di}{dt}$$

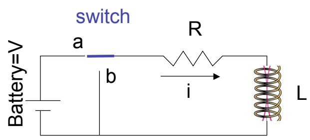
So, when we add them to circuits, we can apply the usual Kirchhoff's Voltage Law and include the inductors.

Consider a circuit with a battery, resistor and inductor (RL circuit)

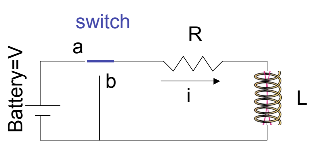


Suppose switch is in position (a) for a long time.

Consider a circuit with a battery, resistor and inductor (RL circuit)

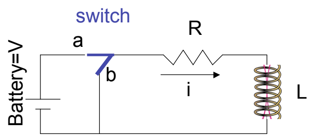


Suppose switch is in position (a) for a long time.
In steady state (after a long time), the current will no longer be changing and thus the inductor looks like a regular wire!



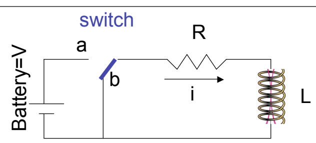
If after a long time the inductor acts like a wire:

$$\Delta V = V - iR = 0 \qquad i = \frac{V}{R}$$



At t=0, move the switch to (b).

Normally one might expect there to immediately be zero current. However, inductors don't let the current change *instantly*.



$$\Delta V = -iR - L\frac{di}{dt} = 0$$

$$\frac{di}{dt} = -\left(\frac{R}{L}\right)i$$

We need to solve this differential equation for i(t).

$$\frac{di}{dt} = -\left(\frac{R}{L}\right)i$$

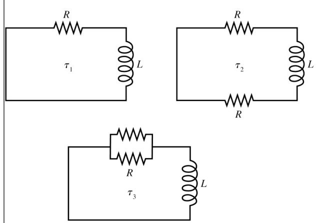
$$i(t) = i_0 e^{-\left(\frac{R}{L}\right)t} \quad \text{where} \quad i_0 = \frac{V}{R}$$

$$i(t) = i_0 e^{-t/\left(\frac{L}{R}\right)}$$

Current exponentially decays with Time Constant =τ= L/R (units of seconds).

Clicker Question

Rank in order, from largest to smallest, the time constants τ_1, τ_2 and τ_3 in the three circuits.

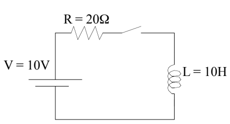


A. $\tau_1 > \tau_2 > \tau_3$
B. $\tau_2 > \tau_1 > \tau_3$
C. $\tau_2 > \tau_3 > \tau_1$
D. $\tau_3 > \tau_1 > \tau_2$
E. $\tau_3 > \tau_2 > \tau_1$

$$i(t) = i_0 e^{-t/\left(\frac{L}{R}\right)}$$

Clicker Question

The switch in the circuit below is closed at t=0.



What is the initial rate of change of current di/dt in the inductor, immediately after the switch is closed ?

A) 0 A/s B) 0.5A/s
C)1A/s D) 10A/s
E) None of these.

Hints: What is the initial current through the circuit?
Given that - what is the initial voltage across the inductor?

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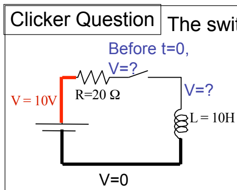

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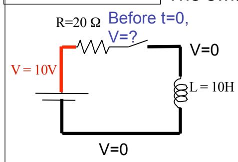

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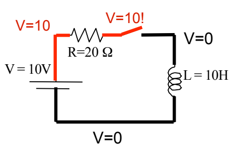

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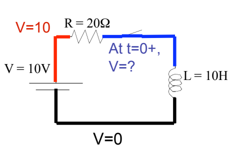

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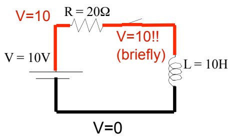

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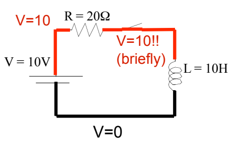

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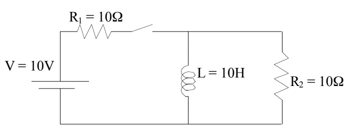
C)1A/s D) 10A/s
E) None of these.

Hints: What is the initial current through the circuit?
Given that - what is the initial voltage across the inductor?

Answer: I(initial)=0, so it must still be 0. Thus, ΔV(across R)=0
V(across inductor) = V(batt!) , but also V(inductor) = L di/dt.
So at t=0+, di/dt = V(batt)/L = 10V/10H = 1 A/s.

Clicker Question

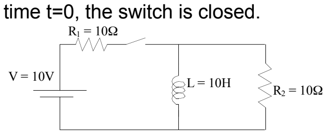
An LR circuit is shown below. Initially the switch is open. At time t=0, the switch is closed. What is the current thru the inductor L immediately after the switch is closed (time = 0+)?



A) Zero
B) 1 A
C) 0.5A
D) None of these.

Clicker Question

An LR circuit is shown below. Initially the switch is open. At time t=0, the switch is closed.



After *a long* time, what is the current from the battery?
A) 0A B) 0.5A C) 1.0A D) 2.0A
E) None of these.

What do you think of today's "all powerpoint" format?

A) Better than usual blackboard work - let's keep it!
B) Some +'s, some -'s, it's a wash for me.
C) I prefer the usual blackboard work (with powerpoint reserved for just clicker questions) Go back!
D) No opinion, the professor should decide
E) I have something different to say about this

AC Circuits

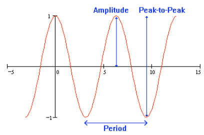
The Voltage in your wall sockets at home is AC.

AC stands for Alternating Current, but would perhaps more appropriately be called Alternating Voltage.

Alternating = Sinusoidal with time

$$V(t) = V_{peak} \sin(\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



ω is the **radial frequency** (radians / second)
f is the **frequency** in (cycles / second = Hertz)
T is the **period** in (seconds), i.e. time for one cycle