

Electric flux, and Gauss' law

Finding the Electric field due to a bunch of charges is KEY! Once you know \mathbf{E} , you know the force on any charge you put down - you can predict (or control) motion of electric charges! We're talking manipulation of anything from DNA to electrons in circuits... But as you've seen, it's a pain to start from Coulomb's law and add all those darn vectors.

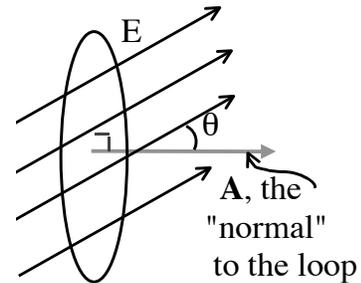
Fortunately, there is a remarkable law, called **Gauss'** law, which is a universal law of nature that describes electricity. It is more general than Coulomb's law, but includes Coulomb's law as a special case. It is always true... and sometimes VERY useful to figure out \mathbf{E} fields! But to make sense of it, we really need a new concept, Electric Flux (Called Φ). So first a "flux interlude":

Imagine an \mathbf{E} field whose field lines "cut through" or "pierce" a loop. Define θ as the angle between \mathbf{E} and the "normal" or "perpendicular" direction to the loop.

We will now define a new quantity, the **electric flux through the loop**, as

$$\text{Flux, or } \Phi = E_{\perp} A = E A \cos\theta$$

E_{\perp} is the component of \mathbf{E} perpendicular to the loop:
 $E_{\perp} = E \cos\theta$.



For convenience, people will often characterize the area of a small patch (like the loop above) as a *vector* instead of just a number.

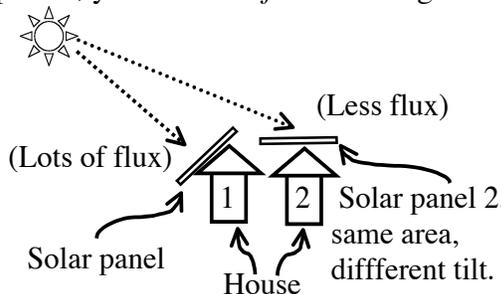
The magnitude of the area vector is just... the area! (What else?)

But the *direction* of the area vector is the normal to the loop.

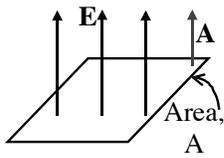
That way, we can write $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$.

(Can you see that this just gives the formula we had above?)

Flux is a useful concept, used for *other* quantities besides \mathbf{E} , too. E.g. if you have solar panels, you want the *flux* of sunlight through the panel to be large. House #2 has poorly

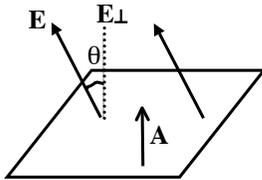
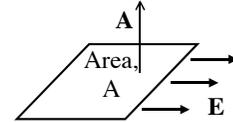


designed panels. Although the AREA of the panels is the *exact same*, and the sunshine brightness is the *exact same*, panel 2 is less useful: fewer light rays "pierce" the panel, there is less FLUX through that panel.

Examples of calculating flux:

Here (picture to the left) $\Phi = E A$, because E is parallel to the area vector. ($\theta=0$)
 (If E pointed the opposite direction, you'd have a "negative" flux!)
 Note: I've drawn an area " A " vector for you: remember, that's defined as perpendicular to the surface in question.

Here (picture to the right), $\Phi = 0$, because E is perpendicular to the area. ($\theta=90^\circ$.) No flux: the E field lines don't "pierce" this loop, they "skim" past it...



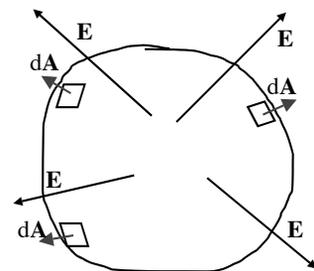
Here, (picture to the left), $\Phi = E A \cos\theta$. The flux is reduced a bit because it's not perfectly perpendicular. Again, the A vector is (by definition) perpendicular to our surface. (So θ is *also* the angle between E and A , and $\Phi = \vec{E} \cdot \vec{A} = E A \cos\theta$)

What if E is not constant? Or, if the area curves? You can still find the flux, you just break your surface up into little "patches", and add up (integrate!) all the small fluxes through each little patch.

For Gauss' law (that's what we're after, remember!) we're going to have to evaluate the electric flux through a **CLOSED SURFACE**: like the surface of a balloon, or a soda can, or a cube... or just some imaginary surface in space. (*Closed* means there's an inside and an outside. So the patches shown above are not closed, but a cube is, or the funny ball shown below...)

Of course, *any* surface can be thought of as the sum of a bunch of little patches! Just imagine taking *any* surface and drawing a little "grid" on it, completely covering it. Each tiny patch will have tiny area (dA).

So, the flux through a *closed surface* is the **SUM** (really, **INTEGRAL**) of the flux through all the patches covering the surface. (The convention is always that dA points everywhere *out* of a closed surface)



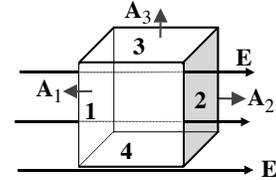
We write this flux formally as $\Phi = \oint \vec{E} \cdot d\vec{A}$

(The weird circle around the integral is to remind you that we're talking about an integral, or sum, all the way around some *closed* surface!)

In a crude sense, flux counts how many "field lines" pierce *out* a surface!

Easy example #1: \mathbf{E} is uniform (constant) from left to right, and our surface is a cube with face area A .

What's the total flux through the (closed) cube shown?



We need to add up the flux through *all 6 faces*. Now, the top and bottom faces, (3 and 4), have an area vector which is perpendicular to \mathbf{E} . So the flux through them is zero. (\mathbf{E} "skims" them, doesn't "poke through" them).

Convince yourself! The exact same argument is true for the front and back faces (not numbered in the picture). Again, each has zero flux.

What about face 2? Here, flux is $\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ (through wall 2) = $\vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA$

What about face 1? This time, the flux is $-EA$.

The minus sign comes because area vectors always point OUT. That means \mathbf{A} (face 1) points left, but \mathbf{E} points right, and the dot product is thus $EA \cos(180) = -EA$.

The total flux through entire surface is thus

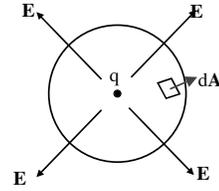
$$\text{Flux}(1) + \text{Flux}(2) + \dots + \text{Flux}(6) = -EA + EA + 0 + 0 + 0 + 0 = 0.$$

There is NO net flux through this surface!

That's correct: the number of lines "poking in" on the left is equal to the number "poking out" on the right, the total is thus zero.

Notice: If the \mathbf{E} field and/or the surface are simple enough, you don't really have to DO any integral - just think of the SUM of fluxes through all parts of the surface, and add 'em up.

Easy example #2: You have a point charge q at the center of a perfect sphere of radius R . What is the flux through the sphere?



Here's the key thing you have to realize. Since \mathbf{E} is everywhere "radially outward", and the little area vectors $d\mathbf{A}$ at any patch on a sphere are also "radially outward", then on ANY little patch, \mathbf{E} and $d\mathbf{A}$ are parallel! That means $\mathbf{E} \cdot d\mathbf{A} = E dA$ (the cosine in the dot product ALWAYS gives one, at every point everywhere around the sphere!) From Coulomb's law, we know that *at the surface of this sphere*, $E = kQ/R^2$ (It's the same in magnitude everywhere around the sphere!)

So we want to compute $\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$, which is thus just $\oint \frac{kq}{R^2} dA$.

But now notice that those are all *constants*, and constant come out of integrals.

$$\text{So } \Phi = \frac{kq}{R^2} \oint dA.$$

Now think what that funny integral *means*. It means "sum up dA for all patches everywhere around the sphere".

But that's just the total area of a sphere, $4\pi R^2$! So the R^2 cancels, and Flux = $4\pi kq$. It's a constant (*independent of radius.*)

It's positive (if q is +): there is a net flux pointing OUT of the sphere! Remember, flux is basically "counting flux lines that poke out".

If we moved q a little to the side, the flux would be bigger on that side, but smaller on the other. But still the total *number* of flux lines poking out would be the same! The total flux would be the same no matter *where* q was, as long as it is somewhere *inside* the sphere.

If you added a second charge, q_2 , it would add *more* flux = $4\pi kq_2$.

So the total flux would just be $4\pi k(q + q_2)$. (No matter where precisely those charges were inside the sphere.) This leads us to the big idea of this chapter. If you have a surface with charges inside it, the *total flux* through that surface will just be $4\pi k * (q \text{ total, inside})$

Since we defined $k = \frac{1}{4\pi\epsilon_0}$ back in Chapter 22, we can write this as

$$\Phi = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

This equation is called "Gauss' law". We just showed it for spherical surfaces.

Gauss' law his actually comes from experiment, and is true no matter WHAT the shape of the closed surface is! Ut involves measurable quantities, and as far as we know, there are NO EXCEPTIONS to it.

Again: if you have a bunch of charges enclosed by any surface (a *closed* surface, which has an inside an an outside) then:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad \text{Gauss' law, simple but cryptic! Let's review what it means:}$$

The left side is the electric flux through our surface.

$q(\text{enclosed})$ = the total amount of charge inside.

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N m}^2)$ is a constant of nature (defined in Ch. 22).

You can pick a surface, any surface you want. It can be a real physical surface (like a balloon) or an IMAGINARY surface in space. Doesn't matter.

Inside that surface will be some charges. They can be distributed arbitrarily. (Could be a mix of + and -) Together, they add up to a total charge = q_{enclosed} . Those charges will produce an electric field (all throughout the universe...!) and those **E** field lines will "pierce" your surface... We've just seen that you can always (in principle) compute the FLUX of E field through any surface...: that's the LEFT SIDE of Gauss' law: total electric flux through our surface. It tells you "how many flux lines poke OUT the surface" (really, # out - # in)

The right side is simple as can be: proportional to total charge.

Gauss' law is amazing! *It's going to take a while to sink in what it really says.*

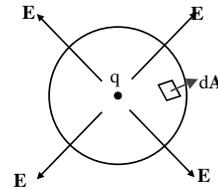
We'll start by looking at our simple examples (where we find both left and right sides). We'll see how Gauss' law *gives us* Coulomb's law. We'll see some useful *consequences* when we consider metals in the real world. We'll look at some situations where Gauss' law can be used to figure out E fields from useful (but rather complicated) charge distributions where "integrating" Coulomb's law to find E, like in the last chapter, would be truly nightmarish.

Gauss' law is one of four "master equations" we'll learn this semester. These equations are together called "Maxwell's Equations" (Maxwell tied them together). With them, *all classical electromagnetic phenomena* can be described, predicted, and explained!

Remember the cube example? The total flux was zero. (Number of lines poking out = number of lines poking in). Is that consistent with Gauss' law? Yes it is - the cube was empty! E field lines came in and went back out. (If there were any charges in there, E field lines would start or end on the charges: if they pass on through, there's no charge in there!)

Suppose we *only* knew Gauss's law. I claim we can DEDUCE Coulomb's law from it! Here's how: Place a charge q at the origin. I'd like to figure out the E field at some arbitrary point, a distance R away. (Coulomb tells me the answer, but let's work it out from Gauss' law).

I do need to make an additional argument, based on **symmetry**. A point charge is totally symmetric: no DIRECTION in space can be any different than any other. North, South, East, West... the E field must be the same in different directions. So by SYMMETRY, I can argue that whatever \mathbf{E} is, it must i) point radially away from the charge, and ii) must depend ONLY on the distance away from the charge. We call this a *spherically symmetric* field. I could say $\mathbf{E} = E(r) \hat{r}$ (That means the the E field might depend on radius, but at a given radius, it has a definite value and points outwards)



So let's draw an IMAGINARY sphere, centered on the charge, with radius R . Let's see what Gauss' law says.

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$. The right side is obviously just q/ϵ_0 . What about the left?

If we don't KNOW \mathbf{E} yet, how can we do the integral? Well, the point is, I don't know what \mathbf{E} is, but I DID argue that $\mathbf{E} = E(r) \hat{r}$, i.e. it's the SAME at all points on the sphere, and always points out (i.e. parallel to $d\mathbf{A}$). That means everywhere on our surface, $\mathbf{E} \cdot d\mathbf{A} = E(R) dA$. Since $E(R)$ is just a constant, pull it out of the integral: $E(R) \oint dA = q/\epsilon_0$. Once again, that funny integral is just the area of the sphere, $4\pi R^2$, which means $E(R) 4\pi R^2 = q/\epsilon_0$, or, solving, $E = \frac{q}{4\pi\epsilon_0 R^2}$. That's Coulomb's law!

Let me summarize: I could "do" the integral in Gauss' formula even without knowing what exactly $E(R)$ was, by cleverly CHOOSING a nice Gaussian surface and invoking symmetry. And then I came back with Coulomb's law! So Coulomb's law can be thought of as a consequence of Gauss' law, which is the deeper and more fundamental of the two...

Let's try another example. Suppose you have a spherical ball of charge (NOT a point). What's the \mathbf{E} field *outside* this ball? Coulomb's law doesn't tell us the answer (it's only valid for point charges). But Gauss' law does: follow EXACTLY the same steps as the previous example, and conclude that the \mathbf{E} field is the same as if all the charge was located at a point at its center!

What if we had a spherical *shell* of charge Q (not a sphere: a thin shell, like the skin of a balloon). What's \mathbf{E} *outside* that? The exact same argument says it's the same as if all the charge was located at a point at the center of the shell.

Let's do another one: suppose I have this spherical shell of charge Q , spread evenly.

What's \mathbf{E} *inside* the shell, a distance R from the origin?

Since the charge is spherically symmetric, my old symmetry argument still holds: \mathbf{E} can only depend on R (not on angle), and it must point radially.

(Might be radially out, or in, that I can't tell, but it can't point at some funny other angle: that would be non-symmetric.)

It's the same game as before!

Draw an imaginary spherical surface with radius R . Note: that's a *new* surface, it's NOT the same as the shell that has the charge on it. It's *inside* that one!

Now apply Gauss' law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0}$.

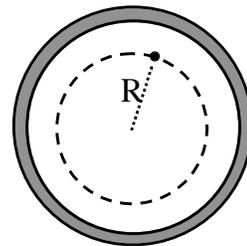
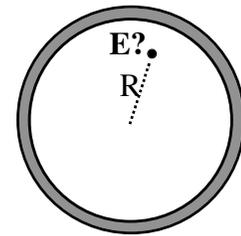
We're going to be integrating over our new imaginary sphere!

We've seen what to do with the left side: $\mathbf{E} \cdot d\mathbf{A} = E(R) dA$, the $E(R)$ comes out of the integral, and we're left with $E(R) 4\pi R^2 = q_{\text{enclosed}} / \epsilon_0$

But what is $q(\text{enclosed})$? Think about it : the charge Q of the shell is *outside* my imaginary surface. $q(\text{enclosed})$ is zero, there's no charge *INSIDE* the dashed line. That means $E(R) = 0$.

There is NO \mathbf{E} field ANYWHERE inside that shell. Not even right up close to all the charges just inside the shell!

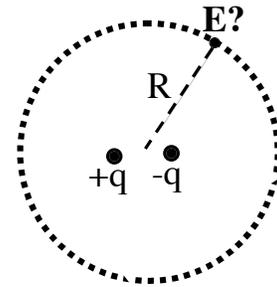
This has useful physical consequences: we'll come back to it later. (It's why you're safe in your car if lightning hits it and dumps charge all around you. If you're *INSIDE*, $E=0$!) But we need to come back and see how this is still true even with a "non-spherical" car, and "non-uniform" charge distribution!



What if we lose symmetry? E.g, suppose you have a dipole near the origin.
What's \mathbf{E} a distance R away from the origin?

It *is* still true that $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0} = 0$, but now we can't

make the symmetry argument, and I no longer can figure out \mathbf{E} in any easy way!



I certainly can NOT conclude that $\mathbf{E}=0$, just because the integral is zero. There's no reason to assume \mathbf{E} is the *same* everyone on the surface I drew, so if \mathbf{E} is not a constant, you can't pull it out of the integral.

We're stuck - I can't simplify the integral. I know that the total flux through that sphere is zero - the #of lines poking out = # poking in. But that does NOT tell us that those #'s are each separately zero!

We'd have to go back to more direct (and painful) ways: summing up $d\mathbf{E}$ from each charge.

Bottom line: Gauss' law is great, especially when there's symmetry. But, it doesn't ALWAYS tell you what \mathbf{E} is at any given point in space. (So it's always true, but not *always* helpful for problem solving!)

Another example: Suppose you have a large (effectively infinite) *flat sheet* of uniformly smeared out charge, with uniform charge density $\sigma = \text{charge/area}$.

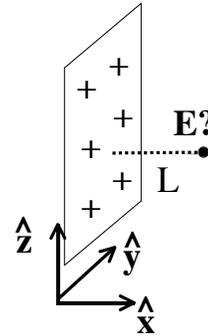
What is the \mathbf{E} field a distance L away from the sheet?

We have symmetry again, but it's not *spherical*.

What I'll argue is that if the sheet is infinite, then \mathbf{E} cannot depend on z or y . (It *might* depend on x , we'll have to see..)

Furthermore, \mathbf{E} can't point "up" or "down" because of symmetry.

In fact, the *only* way it could point is away from the sheet (in the x direction).



Symmetry arguments are subtle, and maybe unfamiliar. But they're a *really* important idea in lots of applications. Think about the argument I just made. Ask yourself why \mathbf{E} can't point any way except "directly away"!

So we've argued that $\mathbf{E} = E(x) \hat{i}$. It's time to apply Gauss' law.

But remember, you *first* have to pick a surface over which you will integrate.

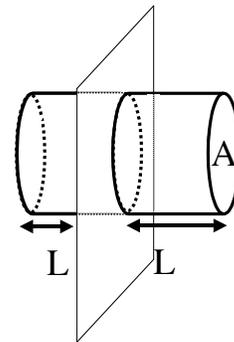
What surface should we pick? We want one that takes advantage of the symmetry. Picking a big sphere is a *bad* idea, because this problem doesn't have spherical symmetry.

(The \mathbf{E} field would not point parallel to $d\mathbf{A}$ on a sphere, and wouldn't necessarily be the same everywhere.)

Here we need a little creativity!

I'm going to propose the infamous "beer can" surface, also known as a "Gaussian pillbox".

It's an imaginary cylinder, endcap area A , centered around the charged sheet:



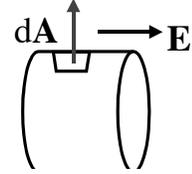
Now consider Gauss' law applied to that pillbox.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

That integral is a *surface* integral: we need to figure out $\mathbf{E} \cdot d\mathbf{A}$ all around the pillbox.

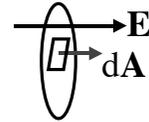
I propose thinking of the pillbox as having 3 distinct parts: the left endcap, the right endcap, and the rest of the cylinder. To find the total flux, we'll *add* the flux through all three of those parts of the surface.

Consider first the "cylinder part". It has little $d\mathbf{A}$ vectors spiking out, all of which are perpendicular to the x direction! (Do you see that?) So if \mathbf{E} points purely in the x direction, it's "skimming" the can, not poking it.

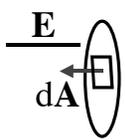


The flux through that *entire cylindrical* surface is zero!

What about the right end cap? There, \mathbf{E} points right, and so does $d\mathbf{A}$. \mathbf{E} is totally uniform over that whole end, so integrating $\mathbf{E} \cdot d\mathbf{A}$ just gives you $E A$, where A is the area of the endcap, and $E = E(L)$ is the value of the \mathbf{E} field out there a distance L away from the sheet.



What about the left end cap? There \mathbf{E} points *left*, and *so does* $d\mathbf{A}$!



You need to think about both of those.

\mathbf{E} points "away" from the sheet: on the left side, away = left.

We also agreed that $d\mathbf{A}$ always points OUT of a closed surface, (which means *left* on the left endcap)

That means $\mathbf{E} \cdot d\mathbf{A} = +EA$ again!

The sign is important: if \mathbf{E} and \mathbf{A} are parallel, the dot product is positive, even if both point left. There IS a net flux out of the can, \mathbf{E} fields poke "out" of BOTH endcaps!

Note also that I'm using symmetry to claim that $E(-L) = E(L)$ I'm calling them both just " E ".

Bottom line: The left side of Gauss' law is the flux:

$$\oint \vec{E} \cdot d\vec{A} = 0 + EA + EA = 2EA.$$

How about the right side, $q(\text{enclosed})$?

Our pillbox has surrounded a circular patch of the sheet, with area A , so the charge enclosed must just be σA .

Let's put it all together:

$2 E(L) A = \sigma A / \epsilon_0$. The area cancels. (That's good, because A was an artificially introduced area of our imaginary surface!)

And $E(L) = \sigma / 2 \epsilon_0$. Hey, check it out: there's no dependence on " L "!

E is the same at ANY distance from the sheet!

$$\mathbf{E} = \sigma / 2 \epsilon_0 \mathbf{i} \quad \text{if } x > 0$$

$$\mathbf{E} = -\sigma / 2 \epsilon_0 \mathbf{i} \quad \text{if } x < 0$$

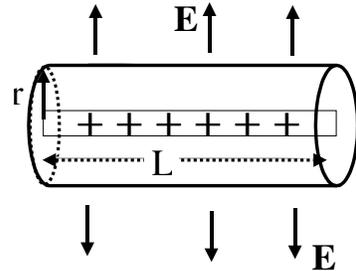
One more example: Suppose you have a very long (infinite) LINE of charge, with linear charge density λ (Coulombs/meter).

What is the electric field at some distance r away from this long line?

By now perhaps you see the method. First use *symmetry* to convince yourself that \mathbf{E} can only depend on " r ", the distance away from the line.

Also, \mathbf{E} must point straight away from the line at all points.

Then choose a sensible imaginary Gaussian surface: another "pillbox" (beer can!) shape seems promising



Then, write down Gauss' law for the pillbox surface:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

The left side requires integrating over the two endcaps plus the "can" itself. This time, it is the endcaps that both give zero contribution (because \mathbf{E} doesn't "pierce" them, it's parallel to them

For the rest: at ANY little patch of area, $\mathbf{E} \cdot d\mathbf{A}$ is just $E dA$ (because \mathbf{E} points straight out from the line at the middle, and so does $d\mathbf{A}$.) Convince yourself! Once again, E is the same value, $E(r)$ everywhere on the surface (by symmetry) so we can pull it out of the integral, and we have $E(r) \oint dA$. But that funny integral just means "add up the little area squares on the cylindrical surface", and we know that: the area of a cylinder is $2 \pi r L$.

The other side of Gauss' law requires $q(\text{enclosed})$, which is just λL (do you see that?)

Putting it together, $E(r) 2 \pi r L = \lambda L / \epsilon_0$.

The " L "s cancel (Good thing! Because L was the length of the imaginary surface, not anything physically real!) and we've got

$$E = \frac{\lambda}{2 \pi r \epsilon_0}$$

This E field *DOES* depend on r , the distance from the line. But it doesn't drop off like r^2 (like Coulomb's law), instead it drops off less rapidly.

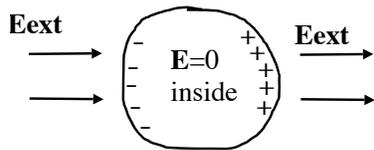
E fields and metals: Electrostatic equilibrium

I said \mathbf{E} is defined everywhere in space. What is the electric field \mathbf{E} *inside* a chunk of metal? I claim, in steady state, "electrostatic equilibrium", the answer must be zero! Why? Because metals are filled with free electrons that can move around (they conduct) - so if \mathbf{E} was NOT zero at some point inside, then the force on an electron there would be nonzero - the electron would start to move. They would continue to move, building up a "counter" \mathbf{E} field, a canceling field, until finally everything settles down, nothing moves, $\mathbf{E}=0$ throughout the metal (so $\mathbf{F}=0$) No more motion!

(Steady state MEANS everything has settled down, no charges are moving, so the feel no net force on them any more)

It's a bit of a subtle argument - you have to think about it. But it's a very important idea: \mathbf{E} WILL ALWAYS BE EXACTLY 0 everywhere *inside* a metal (conductor), in steady state.

If you *try* to add an external \mathbf{E} field, the charges (conduction electrons) inside the chunk of metal will *quickly* move around, until $\mathbf{E}=0$ everywhere inside.



Gauss' law applied to an imaginary surface *anywhere INSIDE* the conductor will give $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enclosed}}}{\epsilon_0}$

Since $\mathbf{E}=0$ everywhere inside, the left side always vanishes.

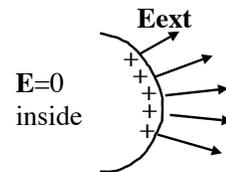
Thus, $q(\text{enclosed})$ vanishes for *any* surface inside the chunk of conductor, which tells you there are no net charges anywhere in there.

The separated charges must live right on the *surface* of the conductor!

(External field lines end on one surface, and start on the other: starting or stopping on surface charges, but never going into the metal.)

The same argument tells you that, in steady state, if there's a nonzero \mathbf{E} field outside of a chunk of metal, it will always be perpendicular to the surface right at the edge.

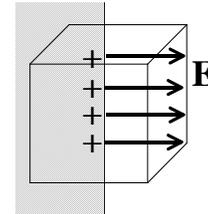
If it wasn't perpendicular, there would be a component of \mathbf{E} parallel to the surface, and electrons right at the surface would then flow (surface currents), until that piece of the \mathbf{E} field got cancelled out.



There are tons of practical consequences of the above simple statements. Just one example: if you make a metal box and put it in a region of large E field, the electrons in the metal quickly (essentially instantly) rearrange on the surface to make $E=0$ everywhere **INSIDE** the box. If you then hollow out the box, it makes no difference, $E=0$ inside, still. If there's a lightning storm (large, potentially fatal E fields all around you), a relatively safe place to be is inside a metal car, because the E field **INSIDE** the car (box) is zero. (It would be better if the car was *entirely* metal) If you're in a fancy fiberglass-body car, too bad - fiberglass doesn't conduct, it's not a metal, so the above arguments fail to hold. At least you'll stay dry..

If you look at the surface of a conductor, we've argued above that there *might* be a "surface charge density" σ . But *inside*, $E=0$ and there won't be any other net charges in there.

So let's look right close to the edge of a conductor, any conductor, and apply Gauss' law to a small imaginary *cubical* surface, half of which is inside and half outside.



Since we argued that E is perpendicular to the surface, there will *only* be flux through **ONE** side of the cube, (the right side in the picture) and that will simply be EA . So Gauss' law says

$$EA = q(\text{enc}) / \epsilon_0 = \sigma A / \epsilon_0$$

Thus $E = \sigma / \epsilon_0$ just outside of a conductor in steady state.

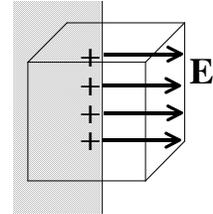
That's always true! The charge density will "adjust itself" to make it true.

It's a very general statement, no approximations involved.

Recall that we found a couple of pages ago that $E = \sigma / 2 \epsilon_0$ just outside of a *sheet* of charges. There's a factor of two difference from the formula we just found for \mathbf{E} outside a conductor.

We need to think about this a bit, it's a puzzle! Because it looks from the picture on the right like what we have is ...,well, it looks like a sheet of charge with density σ !?

So why do we get a different answer (by 2) than we got before?



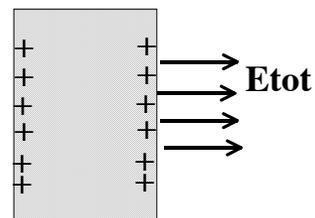
The answer is subtle and worth thinking about: The infinite sheet problem was a very special case. We had to assume we had that sheet, and *nothing else*, no other charges anywhere in the universe (!) Otherwise, the \mathbf{E} field from the other charges would superpose, add in, and we'd get a different answer.

So the formula $\sigma / 2 \epsilon_0$ is JUST what you get from a single sheet and NOTHING ELSE.

But for the case of the conductor, we're not making any such assumption. Not only CAN there be other charges around... there MUST be, in order to ensure that $\mathbf{E}=0$ inside the conductor!! (If the ONLY charges were the ones shown above, then \mathbf{E} would NOT be zero inside, it would be $\sigma / 2 \epsilon_0$ to the left)

So there must be other charges around, and their field will add to the $\sigma / 2 \epsilon_0$ from the sheet alone. The magic of Gauss' law is that we know what this sum is going to turn out to be, even without hunting around and looking for those other charges. They have to be there, they *will* be there, and they'll make the total field turn out to be σ / ϵ_0 right outside the conductor.

E.g., perhaps the "larger" picture looks more like this: Perhaps we have σ on the right side, and σ on the left side of the metal. The resulting \mathbf{E} field over on the right is really the superposition of the \mathbf{E} fields caused by the *two* sheets,



$\mathbf{E}_{tot} = \mathbf{E}_1 + \mathbf{E}_2$. But both \mathbf{E}_1 and \mathbf{E}_2 are $\sigma / 2 \epsilon_0$, arising from a sheet of charge. The total is σ / ϵ_0 , just as we said it has to be: right outside a conductor.

Inside, the fields from the two sheets will cancel, giving $\mathbf{E}_{tot}=0$, again is it has to be.

The field outside a conductor arises from *ALL* charges everywhere. The formula $E = \sigma / \epsilon_0$ outside a conductor just tells me what the total \mathbf{E} field outside a conductor is, taking into account all charges *everywhere*. It's really pretty cool!