Name: ____

Other group members: ______________________________________

Tutorial #4

PHYS 1240: Sound and Music

Thursday, July 18, 2019

Instructions: Work in groups of 3 or 4 to answer the following questions. Write your solutions on this copy of the tutorial—each person should have their own copy, but make sure you agree on everything as a group. When you're finished, keep this copy of your tutorial for reference—no need to turn it in (grades are based on participation, not accuracy).

1. The illustration on the first page is a medieval woodcut from a manuscript on music theory from 1492 (Gafurio's *Theoria musice*). Each panel shows Pythagoras experimenting with different instruments related by integer ratios. The upper left shows his initial discovery with blacksmith hammers, as we heard in class; let's see what the other panels "sound" like.

   a) Look at the bottom right panel. Each pipe is labelled by a number representing its length, which, as we will see next week, is directly proportional to its sounding frequency. First, what is the frequency ratio of the top two pipes being played (labelled "8" and "16")? What musical interval is this?

   b) Now, arrange all six pipes in order by size and find the frequency ratios between each adjacent pair of pipes. What musical intervals do each of these ratios represent?

   c) Now, look at the bottom left panel. The numbers now represent the weights of rocks pulling each string taught. We know that the frequency of a system is proportional to the square root of the stiffness (or "tightness") of the string, so which numbered string will have the highest pitch?

   d) If the frequency $f \propto \sqrt{\text{stiffness}}$, will all the intervals between the strings be integer ratios? Which pairs of strings, if any, will sound consonant? (i.e. which still have integer frequency ratios?)

   e) The upper right panel (with glasses of water and bells) is even more complicated. Just like the weighted strings, these will not always produce pure intervals, since their frequencies depend on the size and mass in complicated ways. But what we can determine is this: which numbered bell and which numbered glass of water will have the lowest pitch?

2. The circle of fifths and the Pythagorean comma:
If you start at any note and go up by intervals of a perfect fifth, you will eventually hit all 12 notes of the chromatic scale and return to your original note. For example, starting at C, you go up a fifth to G, up another fifth to D, up another fifth to A, and so on, until you've reached C again, 7 octaves higher! Since we know what ratio a perfectly consonant fifth should have, we can use the circle of fifths to determine the frequencies of all the pitches. However, something strange happens if you do this. Let's go through the math:

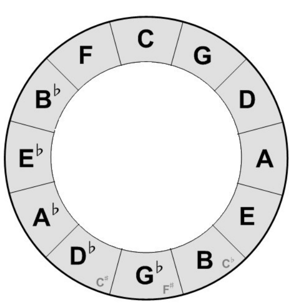
   a) Go up by 12 perfect fifths and find the frequency ratio of the ending note to the starting note. That is, calculate $(3/2)^{12}$.

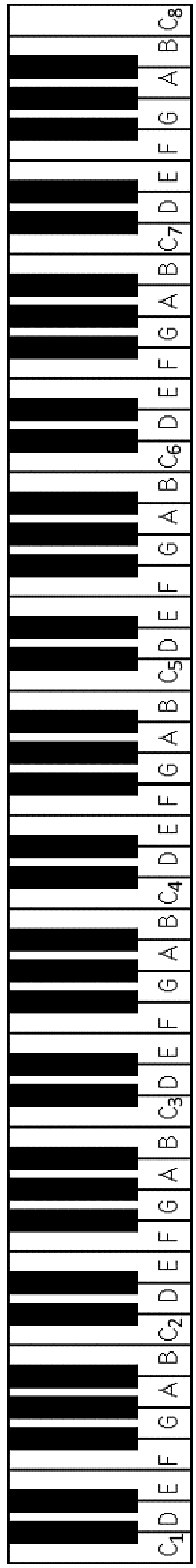
   b) Verify that we need to go down 7 octaves to return to the starting note by marking the location of the 12 fifths on the keyboard on the next page. Start at C_1 at the bottom, and mark every fifth up (G_1, D_2, etc.) until you reach C again. Then, verify that this new C is 7 octaves higher than C_1.

   c) Now, find the frequency ratio of going down 7 octaves, to return to the initial note. That is, calculate $(1/2)^7$.

   d) Multiply these two numbers together. What do you get? What should you get?

   e) We can use this same procedure of go up by fifths (3/2) and dropping down the octave when necessary (1/2) to find the frequencies of all the notes in the chromatic scale. This approach to assigning frequencies has an imperfection. Explain the problem you discovered here. If the starting frequency is middle C (262 Hz), how big is this imperfection in terms of what beats we could hear? This is the interval known as the *Pythagorean comma*.




C1 D E F G A B
C2 D E F G A B
C3 D E F G A B
C4 D E F G A B
C5 D E F G A B
C6 D E F G A B
C7 D E F G A B
C8