Energy and Work

Energy is difficult to define because it comes in many different forms. It is hard to find a single definition which covers all the forms.

Some types of energy:

kinetic energy (KE) = energy of motion
thermal energy = energy of "atomic jiggling"
potential energy(PE) = stored energy of position/configuration
various kinds of PE:

- gravitational
- electrostatic
- elastic (actually a form of electrostatic PE)
- chemical (another form of electrostatic PE)
- nuclear

radiant energy = energy of light
mass energy (Einstein's Relativity Theory says mass is a form of energy.)

Almost all forms of energy on earth can be traced back to the Sun.:

Example: Lift a book (gravitational PE) ← chemical PE in muscles ← chemical PE in food ← cows ← grass ← sun (through photosynthesis) !

Some textbooks say that energy is the ability to do work (not a bad definition, but rather abstract). A key idea that we will use over and over again is this: **Whenever work is being done, energy is being changed from one form to another or being transferred from one body to another. The work done equals the amount of energy transformed.**

We’ll use the symbol W for work and the symbol U for energy. (We will define work, later.) The English sentence “**The work done equals the amount of energy transformed”** we can write as

$$W = \Delta U$$

This is called “The First Law of Thermodynamics”.

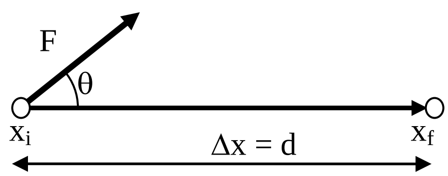
Aside: Actually the First Law of Thermodynamics is this:
“heat added plus work done equals change in energy” or $Q + W = \Delta U$. (Q is the symbol for heat). In this chapter we won’t consider adding heat to a system (like holding a flame under it), so $Q = 0$ and we have just $W = \Delta U$

As we'll see later, energy is an extremely useful concept because energy is **conserved.** When we say energy is conserved, we mean that energy cannot be created or destroyed; you can only transform energy from one form to another, or transfer it from one body to another. The total amount of energy everywhere is fixed; all we can do is shuffle it around.

[Notice that this is not what people normally mean when they say "Conserve energy." When the power company says "Conserve energy", they really mean "Don't convert the energy stored as chemical potential energy into other forms of energy too quickly." To a scientist, the phrase "conserve energy" is meaningless, because energy is always conserved. You can't NOT conserve energy.]

To understand energy and conservation of energy, we must first define some terms: work, kinetic energy (KE), and potential energy (PE).

Definition of work done by a force: consider an object pulled by some force $\vec{F}$. While the force is applied, suppose the object moves along some axis (x-axis, say) through a displacement of magnitude $|\Delta x| = d$.



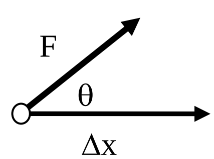
Notice that the direction of displacement is not the same as the direction of the force, in general.

Work done by a force F = $\boxed{W_F \equiv F_x \cdot d = F\cos\theta\, d = F_{||}\, d}$

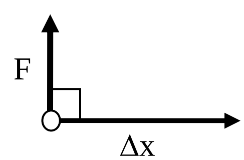
$F_{||}$ = component of force along the direction of displacement

Unit of work: $[W] = [F][d] = 1\ \text{N}\cdot\text{m} = 1\ \text{joule} = 1\ \text{J}$

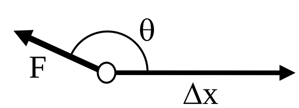
Work is not a vector, but it does have a sign (+) or (–). Work is positive, negative, or zero, depending on the angle between the force and the displacement. The formula $W_F = F\cos\theta\, d$ gives the correct sign, because $\cos\theta$ is negative when $\theta > 90$.



$\theta < 90$, W positive



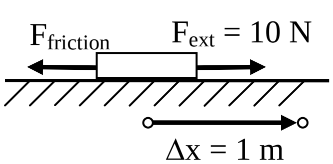
$\theta = 90$, W = 0



$\theta > 90$, W negative

(Question for later: Why do we define work this way?)

Example of work: Move book at constant velocity along a rough table with a constant horizontal force of magnitude $F_{ext} = 10$ N (10 newtons). Total displacement is $\Delta x = 1$ m.

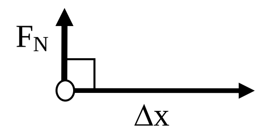


work done *by external force* =

$$W_{Fext} = +F_{ext}\cdot\Delta x = 10\ \text{N}\cdot 1\ \text{m} = 10\ \text{N}\cdot\text{m} = +10\ \text{J}$$

Since velocity = constant, $F_{net} = 0$, so $|F_{ext}| = |F_{fric}| = 10$ N
Work done *by force of friction* = $W_{Ffric} = -F_{fric} \cdot \Delta x = -10$ J (since $\cos 180^o = -1$)

Work done *by normal force* F_N is zero. $W_{FN} = 0$
(since normal force is perpendicular to displacement, $\cos 90^o = 0$.)



Work done *by the net force* is zero. Since v = constant $\Rightarrow F_{net} = 0$.
$W_{net} = 0$.

Moral of this example: Whenever you talk about the work done, you must be very careful to specify *which force* does the work.

Definition of kinetic energy (KE) of an object of mass m, moving with speed v:

$$\boxed{KE \equiv \tfrac{1}{2} m v^2}$$

KE > 0 always. An object has a big KE if it is massive and/or is moving fast. KE is energy of motion.

$$\text{Units of KE} = [KE] = kg \cdot \left(\frac{m}{s}\right)^2 = \underbrace{kg \cdot \frac{m}{s^2}}_{\text{units of force} =[m][a]} \cdot m = N \cdot m = J \text{ (joules)}$$

Units of KE = units of work = joules

Example of KE:
Bowling ball (weight mg = 17 lbs, mass m = 7.7 kg) with speed 7 m/s (typical bowling speed).
KE = 0.5 (7.7 kg) $(7 \text{ m/s})^2 \cong$ 190 J

Why do we define work and KE like we have? Because work and KE are related by the …

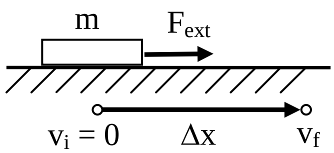
Work-Energy Principle

The Work-Energy Principle states:
The work done by the *net force on a single object* is equal to the change in kinetic energy of that object.

$$\boxed{W_{net} = W_{Fnet} = \Delta KE = KE_f - KE_i}$$

Notice that this is the work done by the total force, the net force. The Work-Energy Principle applies in the special cast that the object has a constant internal energy (meaning the object doesn't having a changing internal potential or kinetic energy – it has no internal moving parts).

"Proof" $W_{net} = \Delta KE$. Here we show that the Work-Energy Principle is true for one special case. I push a book of mass m along a frictionless table with a constant external force of magnitude $F_{ext.}$ The book starts from rest and while the force is applied, the book moves a distance Δx. System = book, and my hand is exerting an external force on the system. What is W_{net}? What is ΔKE? Are they equal?



$F_{ext} = F_{net}$ since normal force and force of gravity cancel.

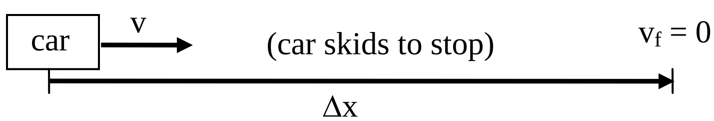
$W_{net} = W_{ext} = F_{||}\,\Delta s = +F_{ext}\,\Delta x$
What is ΔKE? KE involve v^2, so we look for a formula involving v^2. One of the 1D constant acceleration formulas (see Lecture Notes 1D-8) is
$v_f^2 = v_i^2 + 2a(x_f - x_i)$. So we have

$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2(F_{ext}/m)(\Delta x)$ (using $a = F_{net}/m = F_{ext}/m$)

$\Delta KE = KE_f - KE_i = KE_f - 0 = (1/2)mv_f^2 = (1/2)m \cdot 2(F_{ext}/m)(\Delta x) = +F_{ext}\,\Delta x$.
Voila! We have just shown that both W_{net} and ΔKE are equal to $+F_{ext}\,\Delta x$, so $\mathbf{W_{net} = \Delta KE}$.

Energy was transferred from me into the KE of the book. The amount of energy transferred was $+F_{ext}\,\Delta x$. We have shown that the Work-Energy Principle is true in this one case, but it turns out to be always true whenever there is no heat flow and no change in internal energy of the object.

The work-energy principle applies even if there is friction involved.
Example: A car of mass m is moving with speed v. The driver applies the brakes and the car skids to a stop. What was the magnitude of the work done by the friction force on the tires?



At first glance, it seems that we don't have enough info to answer the question. We don't know the coefficient of kinetic friction μ_K and we don't know how far (Δx) the car skidded. So how are we to compute the work done $|W_{fric}| = F_{fric} \cdot \Delta x = \mu_K N \cdot \Delta x$?

Easy if we use the work-energy principle: $|W_{net}| = |W_{fric}| = |\Delta KE| = (1/2)\, m\, v^2$.

Another question: If the initial speed v of the car is doubled, how much further does the car skid? (twice as far?, 3 times as far?)

Answer: Begin with $|W_{fric}| = |\Delta KE| = (1/2)\, m\, v^2$

The work done is $|W_{fric}| = F_{fric} \cdot \Delta x = \mu_K N \cdot \Delta x = \mu_K mg \cdot \Delta x$. So we have …

$(1/2)\, m\, v^2 = \mu_K mg \cdot \Delta x$, (m's cancel) $\Delta x = \dfrac{v^2}{2\,\mu_K\, g}$. Since $\Delta x \propto v^2$, if the v is doubled, the car skids 4 times as far.

Definition of potential energy PE. The change in PE is the amount of work done on a system by an external force, when the KE does not change and no heat flows.

$$\boxed{\Delta PE = W_{ext}\text{ , when } \Delta KE = 0 \text{ and no heat}}$$

Units of PE = units of work = joules. KE, PE, and work all have the same units, the unit of energy = joule.

Potential energy is a kind of *stored energy* associated with position or configuration. PE comes in several varieties. We can use our definition of PE to derive a formula for the **gravitational PE**. If an arrangement of masses has gravitational PE, there exists a *potential* to change that PE into KE.

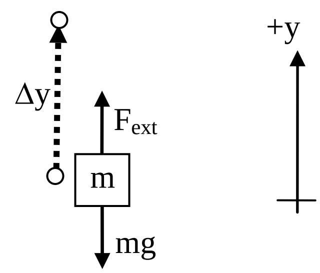
Change in PE of a mass of mass m when it changes height by Δy = $\boxed{\Delta PE_{grav} \equiv m\,g\,\Delta y}$

Only *changes* in PE_{grav} and *changes* in y have physical significance. We are free to set the zero of height and the zero of PE wherever convenient. If we agree that that PE = 0 when y = 0, then we can write $PE_{grav} \equiv m\,g\,y$

Derivation of $\Delta PE_{grav} = m\,g\,\Delta y$. Suppose I slowly and steadily lift an object of mass m = 1 kg a distance $\Delta y = y_f - y_i$ = 1 m above a table. What is the *change* in the gravitational potential energy? According to the definition of PE, $\Delta PE = W_{external} = F_{ext}\,\Delta y$. Since v = constant and acceleration a = 0, then F_{net} = 0, and the external force of my hand has magnitude F_{ext} = mg. So

$$\Delta PE_{grav} = F_{ext}\,\Delta y = m\,g\,\Delta y =$$

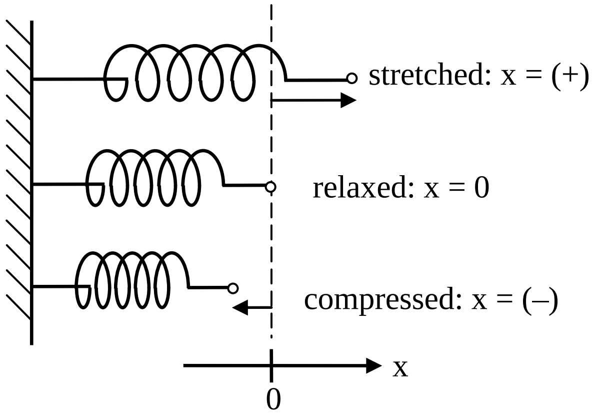
$$(1\text{ kg})(9.8\text{ m/s}^2)(+1\text{ m}) = +9.8\text{ J}$$



Gravitational PE is associated with the system of (mass m + earth + gravitational attraction between mass m and earth). The PE is not "in the mass" or "in the earth"; it is in the mass-earth system.

Springs We want to derive an expression for the elastic potential energy contained in a spring, so we have to take a little detour and talk about springs.

Most springs obey "Hooke's Law" which says that the force exerted by a spring is proportional to the displacement from the equilibrium (relaxed) position.



x is the amount that the spring is stretched (+) or compressed (–)

$| F_{spring} | \propto | x |$

double $| x |$ $\Rightarrow$ double $| F_{spring} |$

"Hooke's Law" : $\boxed{\mathbf{F_{spring} = - k\, x}}$

k = spring constant = measure of stiffness, big k $\Leftrightarrow$ stiff spring, small k $\Leftrightarrow$ floppy spring

units of k = [k] = [F]/[x] = N/m (newtons per meter)

Why the (–) sign in Hooke's Law? It's a reminder that direction of spring force is opposite direction of displacement. When displacement is to the right (x is +), the spring pulls back to the left (F is –); when x is (–), F is (+).

Why "Hooke's Law" in quotes? Because it is not really a law. It is only approximately true for most springs as long as the extension is not too great. If a spring is stretched past its "elastic limit", it will not obey Hooke's Law.

When a spring is stretched or compressed, the spring contains **elastic potential energy**:

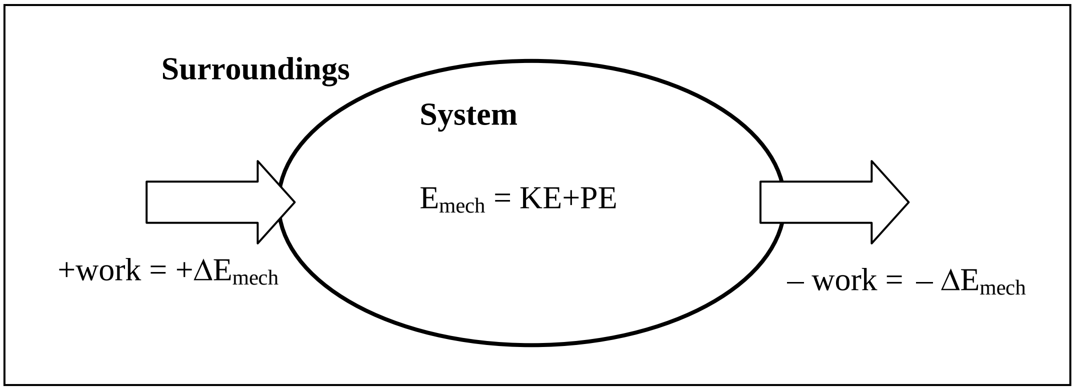
$$\boxed{PE_{elastic} = \tfrac{1}{2} k\, x^2}$$

This formula comes from our definition of PE: $\Delta PE = W_{ext}$ If you slowly compress (or stretch) a spring by an amount x, the force you exert varies from zero (when x = 0) to $F_{max} = k \cdot x$. The *average* external force you exert is $(1/2)k \cdot x$, and the work you do is W_{ext} = force × distance = $F_{average}$ × distance = $(1/2)k \cdot x \cdot x = (1/2)kx^2$.

Another definition: **total mechanical energy of a system** = $E_{mechanical} \equiv KE + PE$

Think of work as "energy input". If positive work is done by an external force on the system, the system energy increases. If negative work is done by an external force, then the system energy decreases. If no work is done on the system by the surroundings (and no heat is added), then the energy of the system is constant.

work done on system = energy change of system



dissipation = conversion of mechanical energy into thermal energy (because of frictional forces)

<u>IF</u> NO DISSIPATION (meaning no friction – more on friction later), then….

1) IF a system is *isolated* from outside forces, then one can prove that

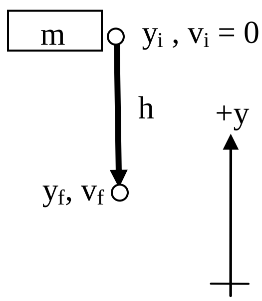
$$\boxed{E_{mechanical} \equiv KE + PE = \text{constant} \quad \text{(isolated system, no dissipation)}}$$

KE can change into PE, and PE can change into KE, but the total (KE + PE) is constant. This is a statement of the conservation of energy.

2) If the system is *not isolated* from outside forces and there is no heat transferred, then the work done on a system by the external force is equal to the change in total mechanical energy (KE+PE) of the system. When *positive* work is done by the external force on the system, the energy of the system increases. (Energy is transferred into the system from the surroundings.) When *negative* work is done by the external force, then the mechanical energy of the system decreases (energy is transferred from the system to the outside).

$$\boxed{W_{external\ force} = \Delta E_{mechanical} = \Delta KE + \Delta PE \quad \text{(no heat transfer, no dissipation)}}$$

Example of KE + PE = constant for isolated system. A book of mass m falls from rest a distance h, but has not yet hit the floor. Here, the system = (book + earth + gravity). What is the change in PE? What is the change in KE? What is the change in (KE + PE)?



$\Delta PE = mg\,\Delta y = -mgh$ (from definition of PE)

$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0 - 2g(-h) = +2gh$ (from constant *a* formula)

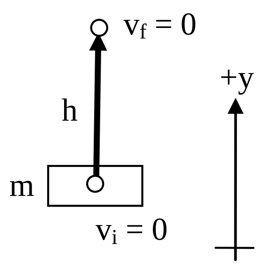
$\Delta KE = KE_f - KE_i = KE_f = (1/2)mv_f^2 = (1/2)m(2gh) = +mgh$

$\Delta E_{mechanical} = \Delta KE + \Delta PE = +mgh - mgh = 0$

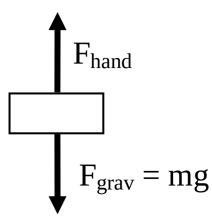
As the book falls, PE is converted into KE, but the total KE+PE remains constant. Once the book hits the floor, the KE will get converted into thermal energy (book, ground, air will heat up) – an example of the process we call dissipation.

Commentary: For the particular case of a falling book, we have shown that (KE + PE) = constant for an isolated system with no internal friction (no dissipation). This turns out to be true <u>always</u>.

Example of $W_{ext} = \Delta KE + \Delta PE$. I steadily lift a book a distance *h*, starting and ending at rest. System = book+earth, and my hand is exerting an external force on the system. What is ΔPE? What is W_{ext}?



Forces on book:



KE = 0 at beginning and end, so $\Delta KE = 0$.

$\Delta PE = mg\,\Delta y = +mgh$

Lift book with v = constant $\Rightarrow F_{net} = 0 \Rightarrow$
$F_{hand} = F_{grav} = mg$

$W_{ext} = W_{hand} = F_{||}\, d = +mgh$

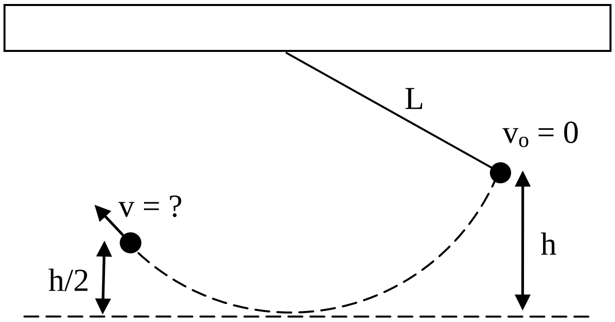
(work by hand positive because F_{hand} in same direction as displacement.)

Notice that $W_{ext} = \Delta KE + \Delta PE$ in this case.

My hand did positive work (+mgh), so system's mechanical energy increased (by +mgh). Energy flowed out of me through complex bio-chemical processes (I'm burning up the eggs I ate that morning) and went into the gravitational potential energy of the book+earth system.

Commentary: We have just shown that $W_{ext} = \Delta(KE + PE)$ in two particular cases where there is no heat transfer and no dissipation (no friction). Using Newton's laws, it is possible to prove that $W_{ext} = \Delta(KE + PE)$ is <u>always</u> true (when no dissipation or heat transfer occurs).

Example of Conservation of Energy (no friction). A pendulum consists of a mass *m* attached to a massless string of length *L*. The pendulum is released from rest a height *h* above its lowest point. What is the speed of the pendulum mass when it is at height *h*/2 from the lowest point? Assume no dissipation (no friction).



In all Conservation of Energy problems, begin by writing **(initial energy) = (final energy)** :

$E_i = E_f \Rightarrow KE_i + PE_i = KE_f + PE_f$

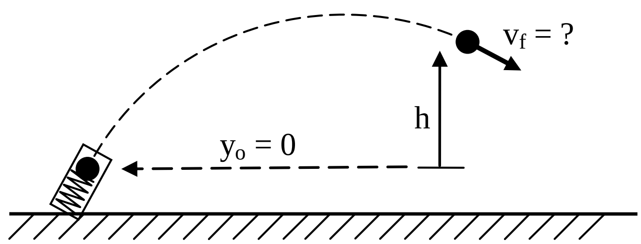
$\Rightarrow 0 + mgh = (1/2)\,mv^2 + mg(h/2)$

(cancel m's and multiply through by 2) $\Rightarrow$

$2gh = v^2 + gh \Rightarrow v^2 = gh \Rightarrow \boxed{v = \sqrt{g\,h}}$

Notice: Using Conservation of Energy, we didn't need to know anything about the details of the forces involved and we didn't need to use $\mathbf{F}_{net} = m\mathbf{a}$. The Conservation of Energy strategy allows us to relate conditions at the beginning to conditions at the end; we don't need to know anything about the details of what goes on in between.

Another example of Conservation of Energy (no friction): A spring-loaded gun fires a dart at an angle θ from the horizontal. The dart gun has a spring with spring constant *k* that compresses a distance *x*. Assume no air resistance. What is the speed of the dart when it is at a height *h* above the initial position?



$E_i = E_f \Rightarrow KE_i + PE_{grav,i} + PE_{elas,i} = KE_f + PE_{grav,f} + PE_{elas,f}$

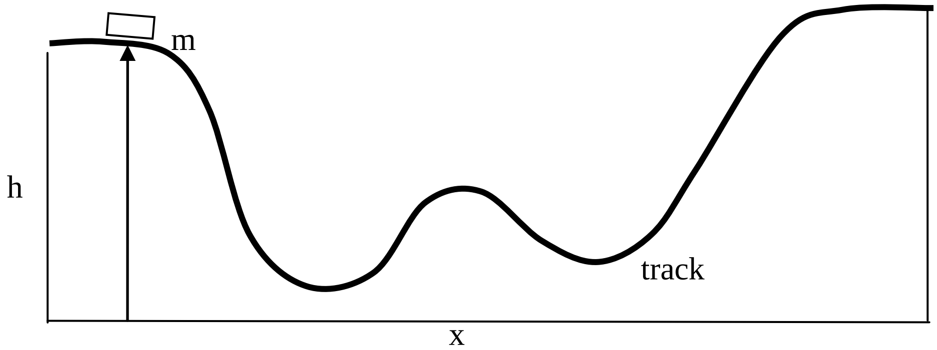
$$0 + 0 + (1/2)kx^2 = (1/2)mv^2 + mgh + 0$$

$$x = \sqrt{\frac{m\,v^2 + 2\,m\,g\,h}{k}}$$

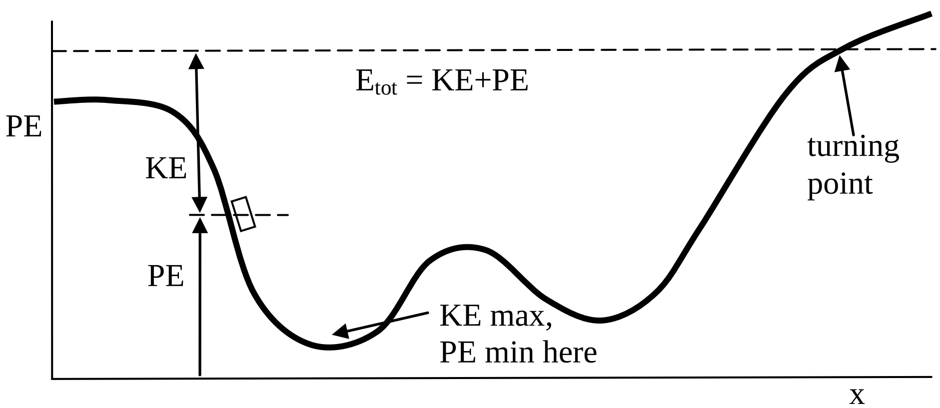
Notice that the angle θ never entered into the solution.

KE+PE=total energy graphs

Suppose a roller coaster of mass m rolls along a track shaped like so:



The shape of the track is a graph of height h vs. horizontal position x. Since the gravitational potential energy of the coaster is PE = mgh, where mg is a constant, a graph of PE vs. x looks the same as the graph of h vs. x, but the vertical axis measuring energy (joules) rather than height (meters). Assuming no friction, the total mechanical energy E_{tot} = KE+PE of the roller coaster remains constant as it rolls along the track. We can represent this constant energy with a horizontal line on our graph of energy vs. x. From this "energy graph", we can read the KE and the PE of the coaster at any point.



What if there is friction and thermal energy involved?

Up till now, we have assumed that there is no sliding friction and no heat transferred in any of these problems. (Having static friction in a problem causes no difficulties, because static friction does not generate thermal energy.) How do we handle sliding friction and heat generated?

If a system is isolated from external forces so that no work is done, and if no heat is transferred, and if there is no sliding friction so that no thermal energy is generated (that's a lot of "if's"), then we can assert that

$$KE + PE = \text{constant} \quad \text{(isolated system, no sliding friction)}$$

If, however, there is sliding friction, then some of the mechanical energy (KE+PE) can be transformed into thermal energy (E_{therm}). In this case, we can say

$$KE + PE + E_{therm} = \text{constant} \quad \text{(isolated system)}$$

It turns out that the amount of thermal energy generated is the negative of the work done by friction:

$$\Delta E_{therm} = -W_{fric}$$

The work done by sliding friction is always negative, since sliding friction always exerts a force in the direction opposite the motion. Consequently, $-W_{fric}$ is a positive quantity.

Power

Power = *rate* at which work is done = rate at which energy is converted from one form to another:

$$\boxed{P \equiv \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}}$$

units of power = [P] = joules/second = J/s = watts (W)

Every second, a 100 W light bulb converts 100 joules of electric potential energy into heat and light.

The power company sells potential energy in units of *kilowatt–hours*.
$1\ \text{kW}\cdot\text{hr} = 1000\ \text{J/s} \times 3600\ \text{s} = 3.6\times10^6\ \text{J}$

Another popular unit of energy is the Calorie (spelled with a capital C). A typical candy bar has 300 Calories of stored chemical energy. There are two kinds of calories, spelled with a little "c" or a big "C":

1 calorie (cal) = "little calorie" = 4.186 J
1 Calorie = 1000 cal = "big Calorie" = "food calorie" = 1 kcal = 4186 J

The "food Calorie" is the "big Calorie" and it should be spelled with a big C. (Chemists like to use the little calorie, which is defined as the amount of heat required to raise the temp of a gram of water by 1^{o} centigrade.)

Calorie example: Dr. D, who has mass m = 68 kg, eats a 300 Cal candy bar and then climbs 10 stories (Δh = 35 meters) to his office on the 10^{th} floor of Gamow Tower. How many Calories has he burned?

Work done = $\Delta PE = mg\,\Delta h = (68\text{ kg})\,(9.8\text{ m/s}^2)\,(35\text{ m}) = 23300\text{ J} \times (1\text{Cal}/\ 4186\text{ J}) = 5.6\text{ Cal}$

A measly 5.6 Cal !?!? Well, it's not quite that bad. He was also doing a lot of ineffective work turning around in the stairwell, flailing his limbs, etc as he climbed, so the total mechanical work was more, maybe 10 Cal total. Also, the human body is not a very efficient machine: only about 25% of the food Calories burned come out of the body as mechanical work; the rest goes into heat. (Dr. D was flushed and panting after his 10-story climb.) So to produce 10 Cal of work, his body burned about 40 Cal — still not very much.

Moral: You can't burn many Calories instantly by exercising. However, by exercising regularly, you build muscles which increases your resting metabolic rate (RMR). A typical out–of-shape male has a RMR of about 70 watts, meaning 70 joules per second burned by just breathing, digesting, thinking. (70 W is about 1400 Cal/day). By exercising regularly, that RMR can be raised to 90 watts (1860 Cal/day). So by exercising regularly, you burn about an extra 500 Cal per day just from your increased resting metabolic rate. "Lose weight while you sleep!" With the increased RMB, you can eat about 1 candy bar per day more than normal and still not gain weight.

Power example: The same Dr. D can climb to the 10^{th} floor in 60 seconds (if he pushes!). What is the mechanical power he generates (due to increased PE only, not including heat generated)?

$P = \Delta PE / \Delta t = mg\,\Delta h/\ \Delta t = (68\text{ kg})\,(9.8\text{ m/s}^2)\,(35\text{ m}) / (60\text{ s}) = 390\text{ W}$

— almost enough to light four 100W light bulbs for that 1 minute.

A horse can generate a power of 1 horsepower for several hours. 1 hp = 746 W. So Dr. D can generate about ½ hp for 1 minute (and then he is needs to take a nap). Horses are pretty powerful!

Currently (2006) the power company sells energy at a rate of $0.08 per kilowatt·hour. One kW·hr is enough to light ten 100 W bulbs for 1 hour — and you get that for 8 cents!