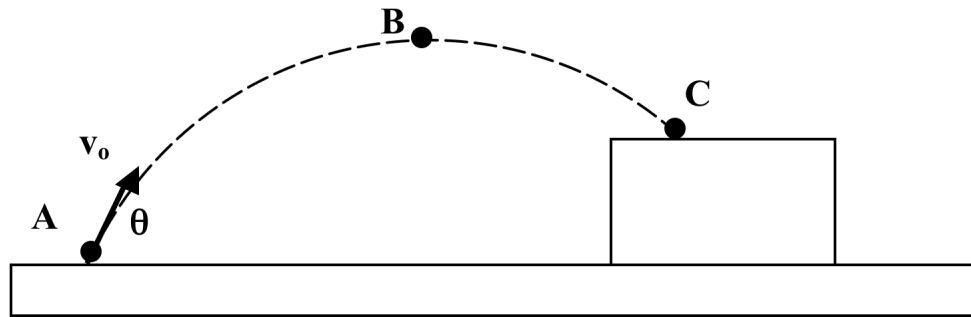


**Spring 2014**

**PHYS-2010**

**Lecture 12**

A pellet is fired from a slingshot with initial speed  $v_0$  at an angle  $\theta$ . The projectile lands on a building. Where is the speed of the projectile a **minimum**?

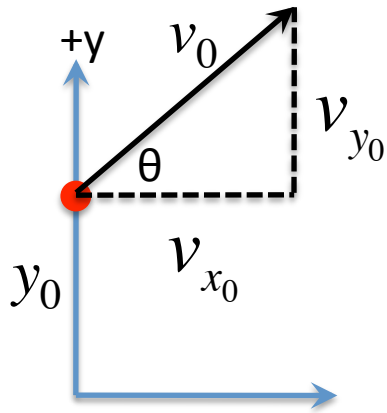


- A) Immediately after the pellet is launched.
- B) At the apex of the trajectory.**
- C) Just before the pellet hits the building.

# Announcements

- Finish reading Giancoli Chapter 3.
- **CAPA # 4** is due Tuesday at 11 pm.
- **Written homework # 3 due on Friday** this week.
- This Week: Lab # 2 with a prelab (print out and complete ahead of time, submit to your TA at the start of lab).
- Midterm exam solutions and scores are posted on D2L.

# Projectile Motion in 2D



**Horizontal  
x-direction**

$$a_x = 0$$

$$x_0$$

$$v_{0_x} = v_0 \cos \theta$$

**Vertical  
y-direction**

$$a_y = -g$$

$$y_0$$

$$v_{0_y} = v_0 \sin \theta$$

Special  
equations for  
projectiles for  
this reference  
frame

$$v_x = v_{0_x} = v_0 \cos \theta$$

$$x = x_0 + v_{0_x} t$$

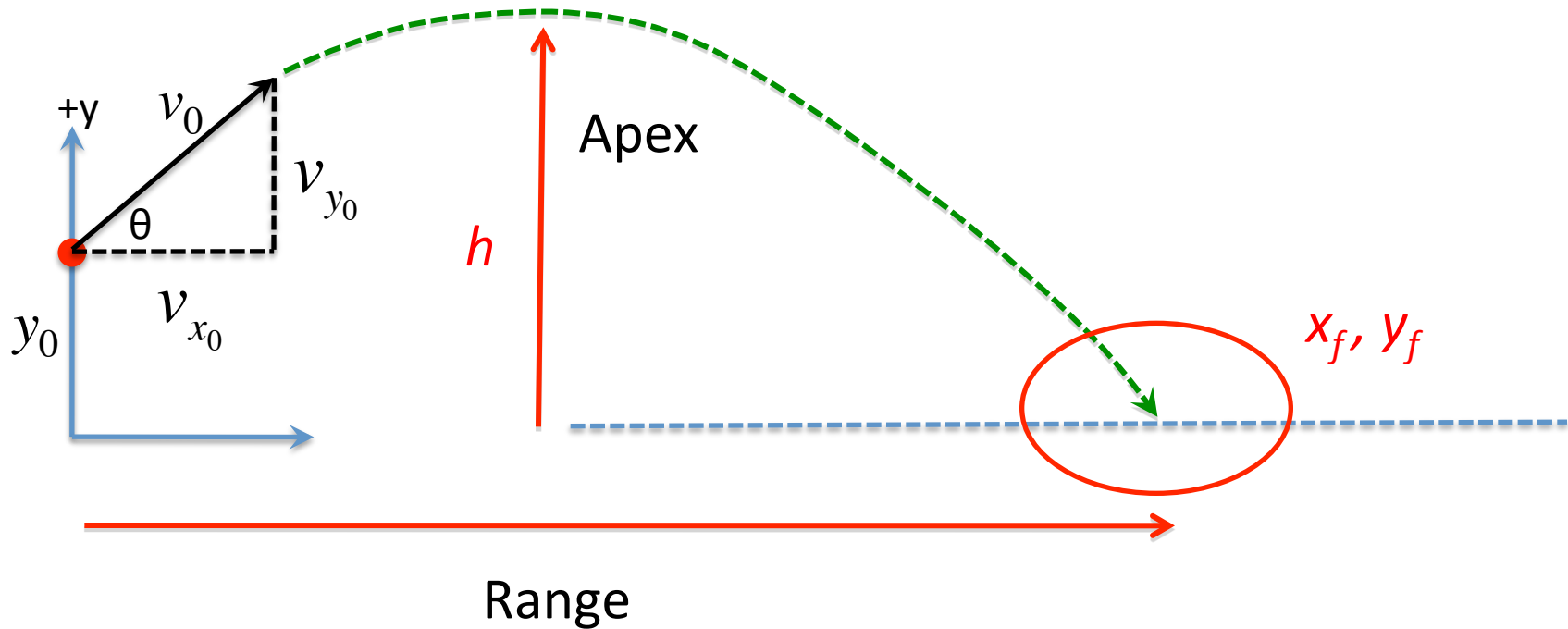
$$v_x^2 = v_{0_x}^2$$

$$v_y = v_{0_y} - gt = v_0 \sin \theta - gt$$

$$y = y_0 + v_{0_y} t - \frac{1}{2} gt^2$$

$$v_y^2 = v_{0_y}^2 - 2g(y - y_0)$$

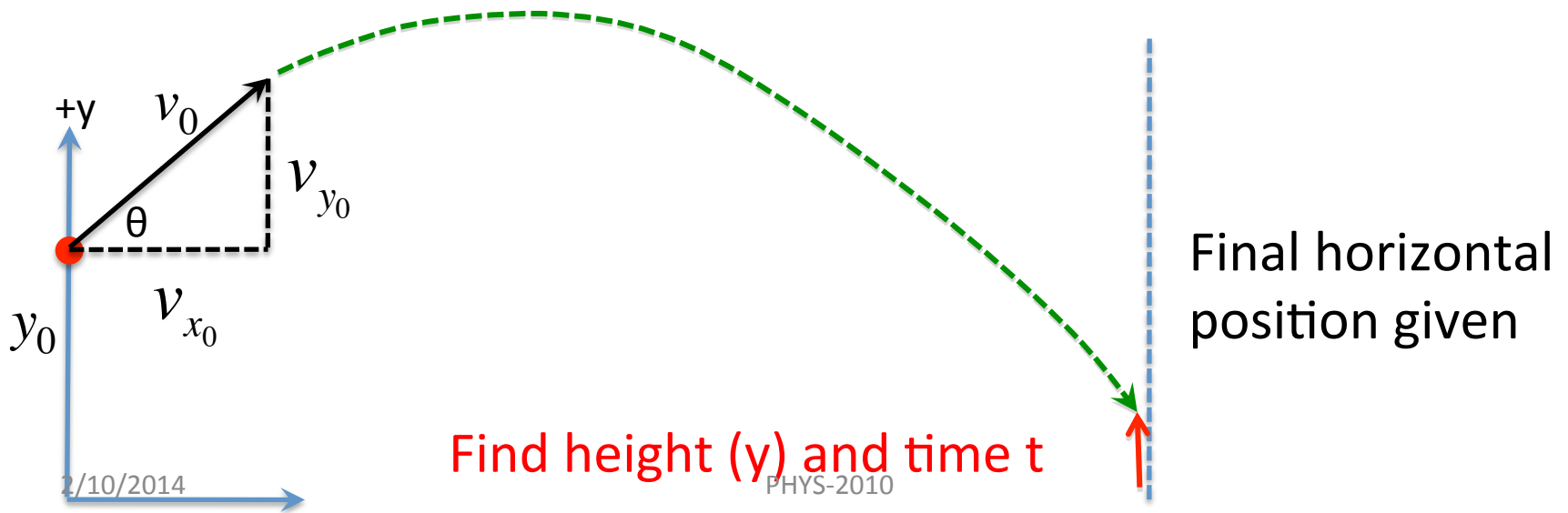
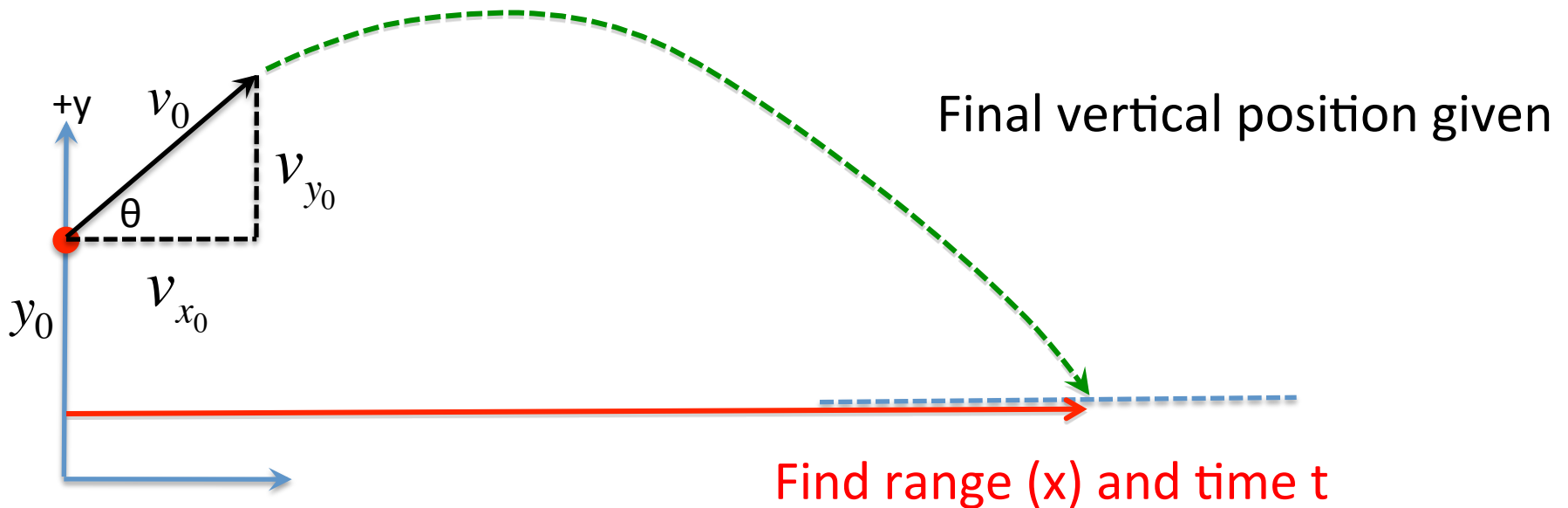
# Projectile Motion: Range



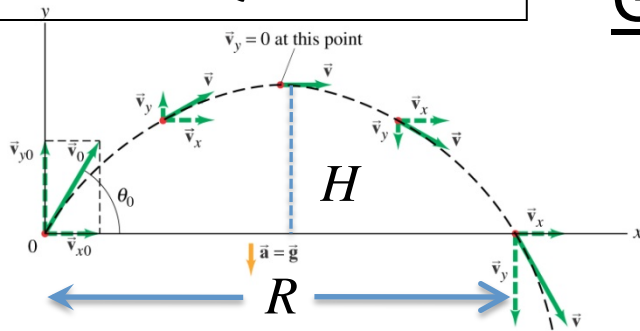
How far did the projectile go?  
How long did it take to get there?

Answers depend on  $v_0$ ,  $\theta$ ,  $x_0$ ,  $y_0$ ,  $x_f$ ,  $y_f$

# What condition ended the motion?



# Clicker Question



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$$a_y = -g \quad a_x = 0$$

$$y_0 = 0 \quad x_0 = 0$$

$$v_{0y} = v_0 \sin \theta \quad v_{0x} = v_0 \cos \theta$$

Given the initial velocity vector, we know height and time to apex:

$$t_{ap} = \frac{v_{y0}}{g}$$

$$H = \frac{v_{y0}^2}{2g}$$

## Ground to Ground Projectile:

### The Range

$$(1) \quad v_y(t) = v_{y0} - gt$$

$$(4) \quad v_x(t) = v_{x0}$$

$$(2) \quad y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$(5) \quad x(t) = v_{x0}t$$

$$(3) \quad v_y^2(y) = v_{y0}^2 - 2g\Delta y$$

$$(6) \quad v_x^2(x) = v_{x0}^2$$

What is the **range R** --- total horizontal distance traveled ?

$$A) R = x_0 v_{y0}$$

$$B) R = \frac{2g}{x_0}$$

$$C) R = \frac{2v_{x0}v_{y0}}{g}$$

$$D) R = \frac{v_{x0}v_{y0}}{g}$$

# Football Punter Physics



For a specific play, the punter wants to kick the ball as far down the field as possible (i.e. **maximum range**).

What is the optimal angle to kick the ball at, *assuming the same initial speed (regardless of the angle)?*

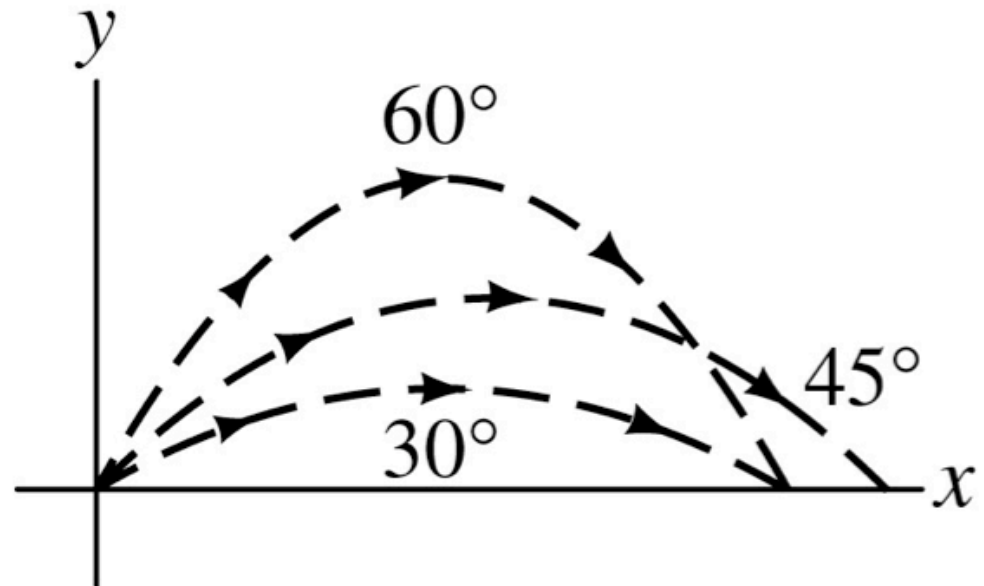
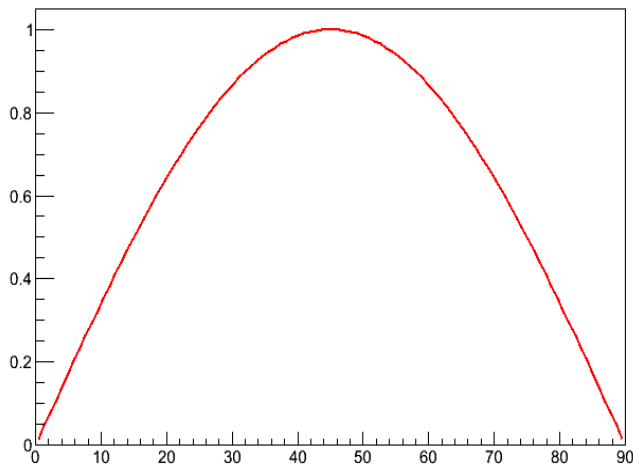
Football obeys the laws of physics.  
Constant acceleration case (ignoring air resistance).



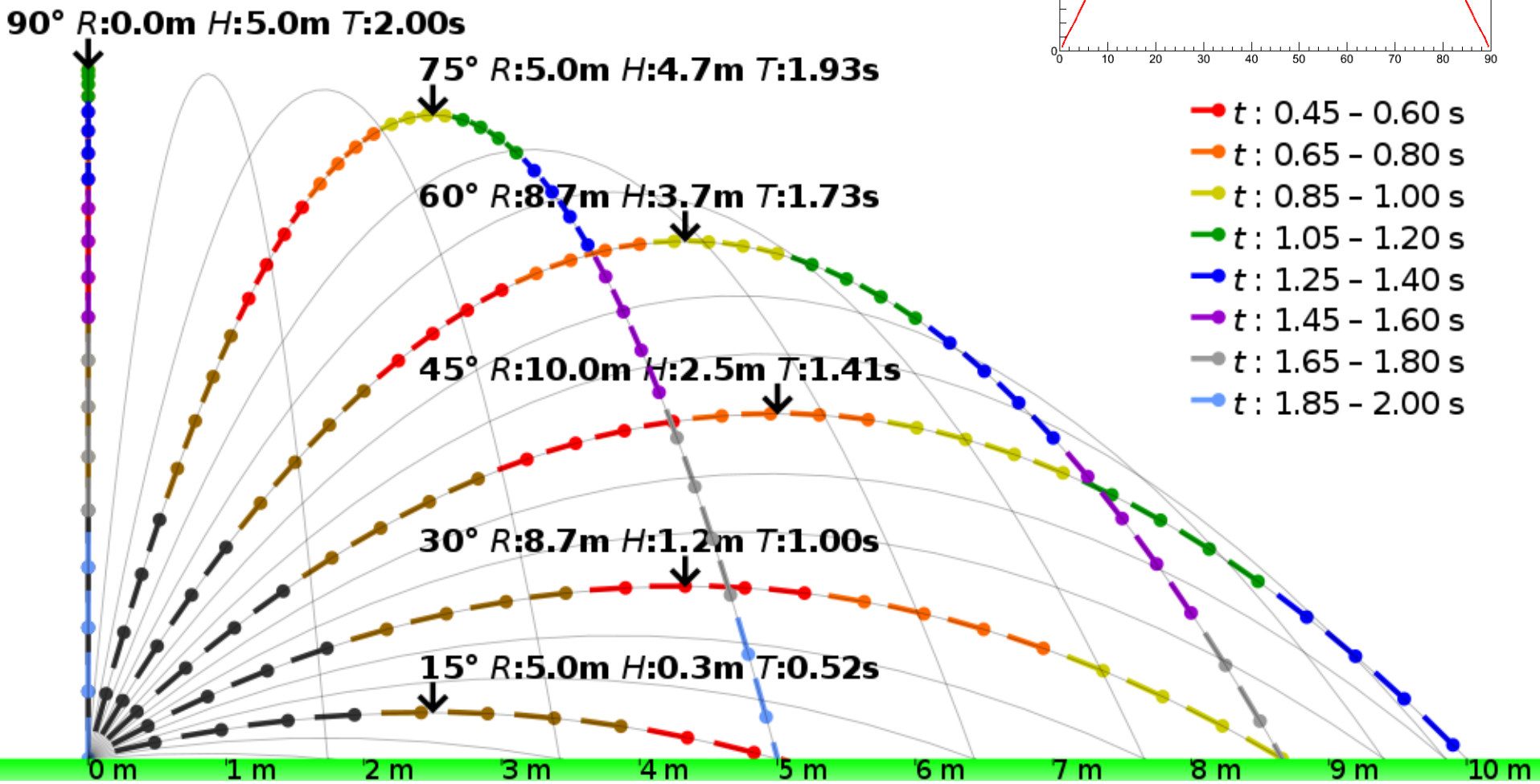
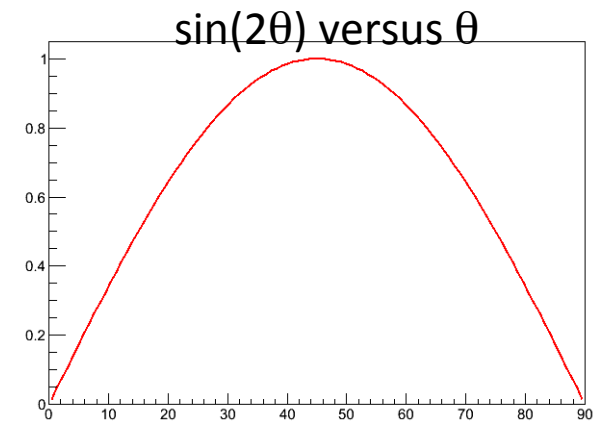
$$R = v_{x0}(2t_{ap}) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g} = \frac{v_0^2 \sin 2\theta}{g}$$

Generally, there are two initial angles ( $\theta$ ,  $90^\circ - \theta$ ) that will give the same range  $R$ .

$\sin(2\theta)$  versus  $\theta$



$$Range = \frac{|\vec{v}_0|^2 \sin 2\theta}{g}$$



$$t(\textit{Flight Time}) = 2 (|\vec{v}_0| \sin \theta) / g$$

What angle  $\theta$  gives the longest flight time?

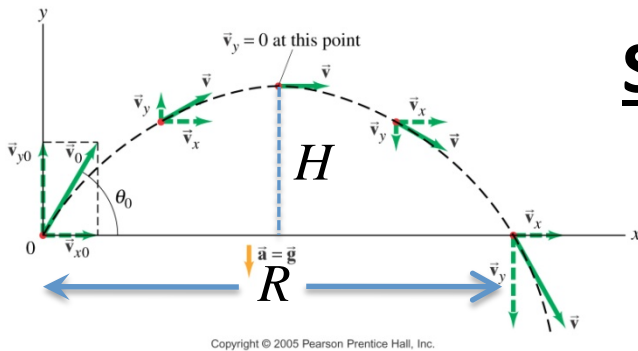
$$\theta = 90 \text{ degrees}$$

$$x(\textit{Range}) = \frac{|\vec{v}_0|^2 \sin 2\theta}{g}$$

What angle  $\theta$  gives the longest distance range?

$$\theta = 45 \text{ degrees}$$

# Summary: Ground to Ground Projectile



$$0 = x_0 = y_0$$

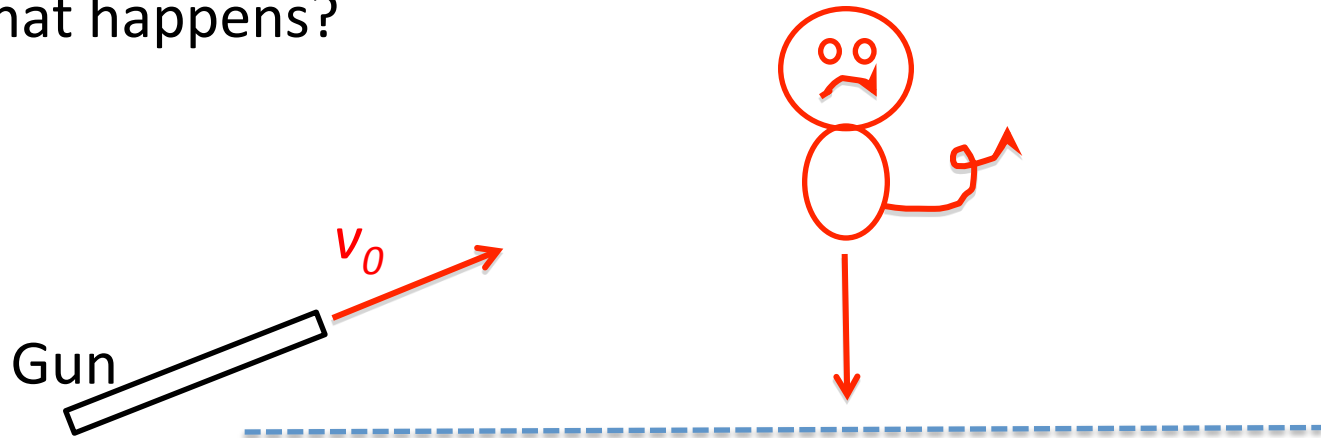
We derived three formulas about the behavior of a projectile moving from ground level to ground level:

$$t_{\max} = 2 \frac{v_{y0}}{g} = 2 \frac{v_0 \sin \theta}{g} \quad \text{Time of flight (time to apex is half that)}$$

$$H = \frac{v_{y0}^2}{2g} = \frac{v_0^2 \sin^2 \theta}{2g} \quad \text{Height of apex}$$

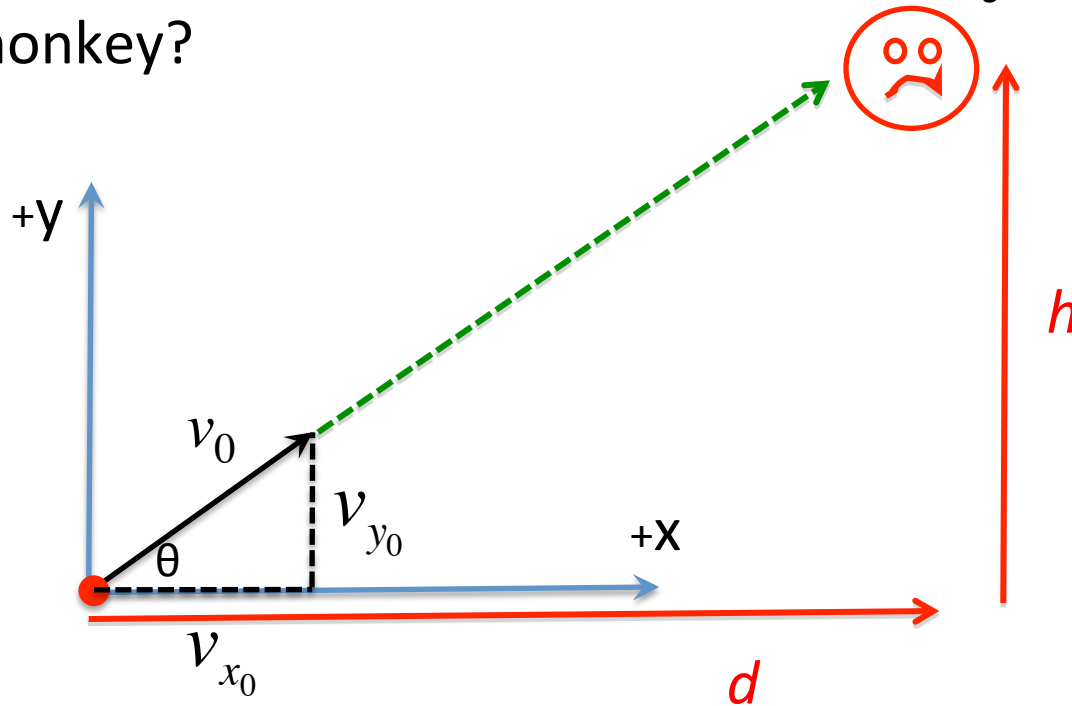
$$R = \frac{2v_{x0}v_{y0}}{g} = \frac{v_0^2 \sin 2\theta}{g} \quad \text{Range – horizontal distance traveled.}$$

A tranquillizer gun is accurately aimed at a rabid monkey hanging from the branch of a tree. The instant the gun is fired, the monkey releases the branch and starts freely falling. The monkey is well within the range of the gun. The initial speed of the tranquillizing dart is  $v_0$ . What happens?



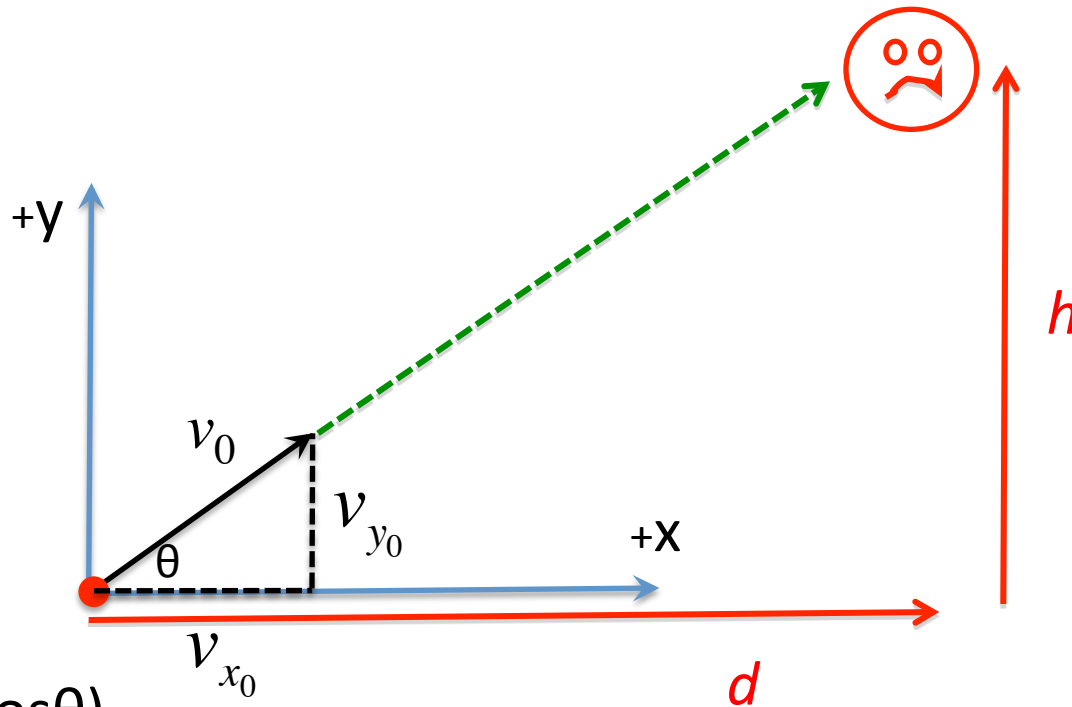
- A) The dart finds its target, regardless of the value of  $v_0$ . (Assuming  $v_0$  still large enough to reach the air below the monkey.)
- B) The dart hits the monkey only if  $v_0$  is large enough.
- C) The dart misses.

In the absence of gravity, at what height  $y_d$  is the dart after traveling horizontally a distance  $d$ , assuming  $v_0$  is pointed at the monkey?



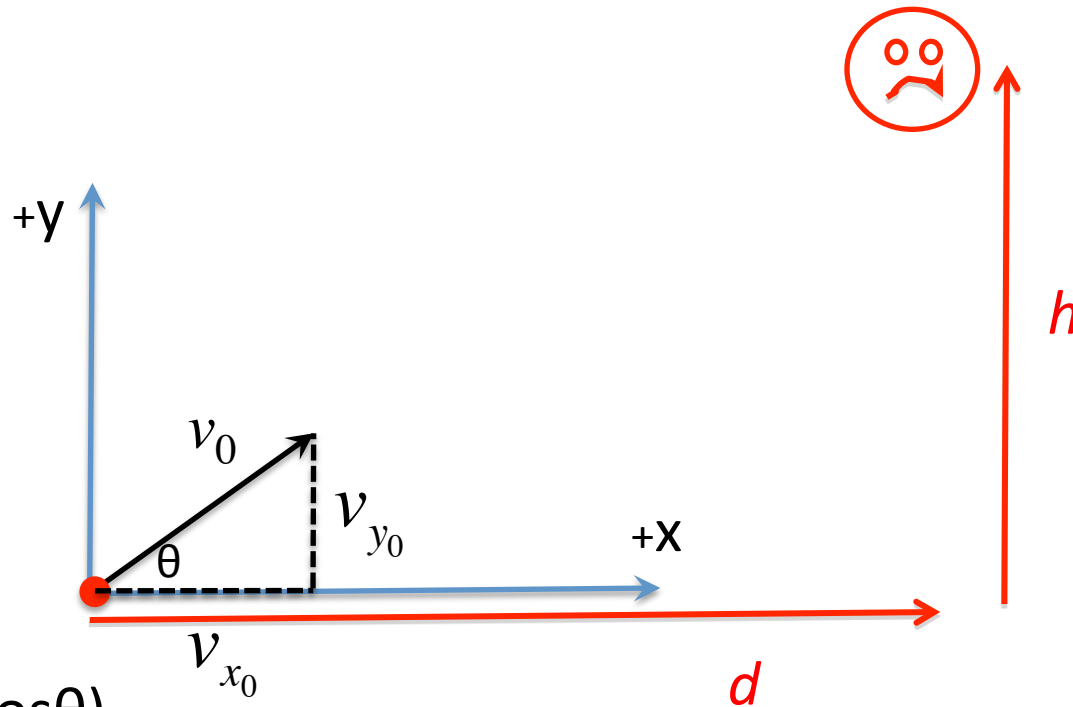
- A)  $d \cdot \tan \theta$
- B)  $h$
- C)  $\sqrt{d^2 + h^2} \cdot \sin \theta$
- D) All of the above**
- E) None of the above

*In the absence of gravity,* how long does it take the dart to travel the horizontal distance  $d$ ?



- A)  $d/(v_0 \cdot \cos\theta)$
- B)  $h/(v_0 \cdot \sin\theta)$
- C)  $\text{Sqrt}(d^2+h^2)/v_0$
- D) All of the above
- E) None of the above

With gravity, how long does it take the dart to travel the horizontal distance  $d$ ?



A)  $d/(v_0 \cdot \cos\theta)$

B)  $h/(v_0 \cdot \sin\theta)$

C)  $\text{Sqrt}(d^2+h^2)/v$

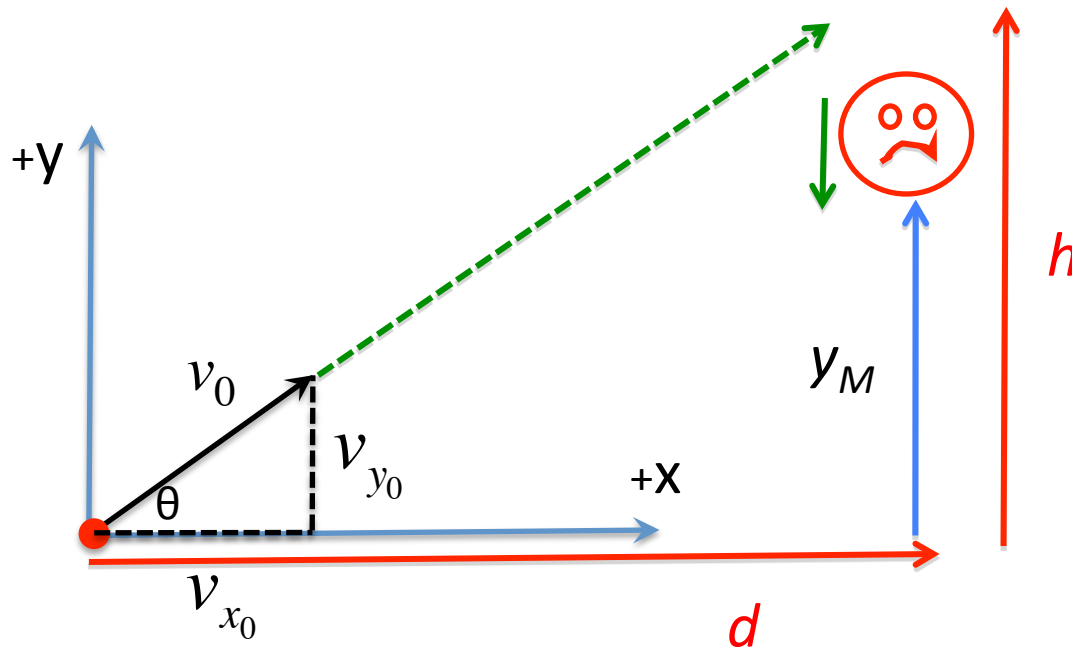
D) All of the above

E) None of the above

With gravity, the vertical velocity is changing! Horizontal velocity is constant.



# VERTICAL MOTION OF DART AND MONKEY



$$t_d = d / (v_0 \cos \theta) =$$

$$= h / (v_0 \sin \theta)$$

**DART**

$$y_d = y_{0d} + v_{0d} t_d - \frac{1}{2} g t_d^2$$

$$y_d = 0 + v_0 \sin \theta * t_d - \frac{1}{2} g t_d^2$$

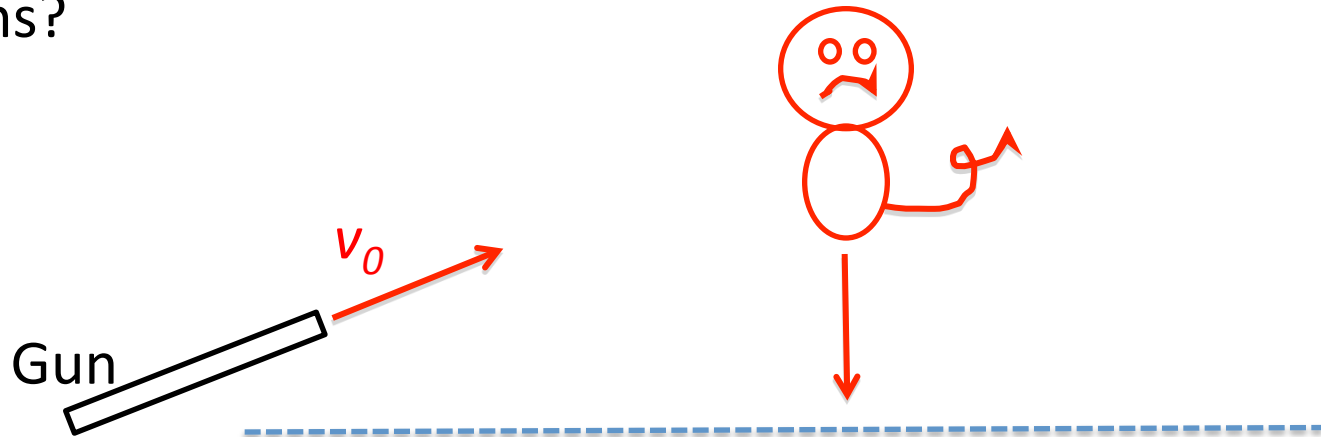
$$y_d - y_M = v_0 \sin \theta * t_d - h = 0$$

**MONKEY**

$$y_M = y_{0M} + v_{0M} t_d - \frac{1}{2} g t_d^2$$

$$y_M = h - \frac{1}{2} g t_d^2$$

A tranquillizer gun is accurately aimed at a rabid monkey hanging from the branch of a tree. The instant the gun is fired, the monkey releases the branch and starts falling. The monkey is well within the range of the gun. The initial speed of the tranquillizing dart is  $v_0$ . What happens?



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