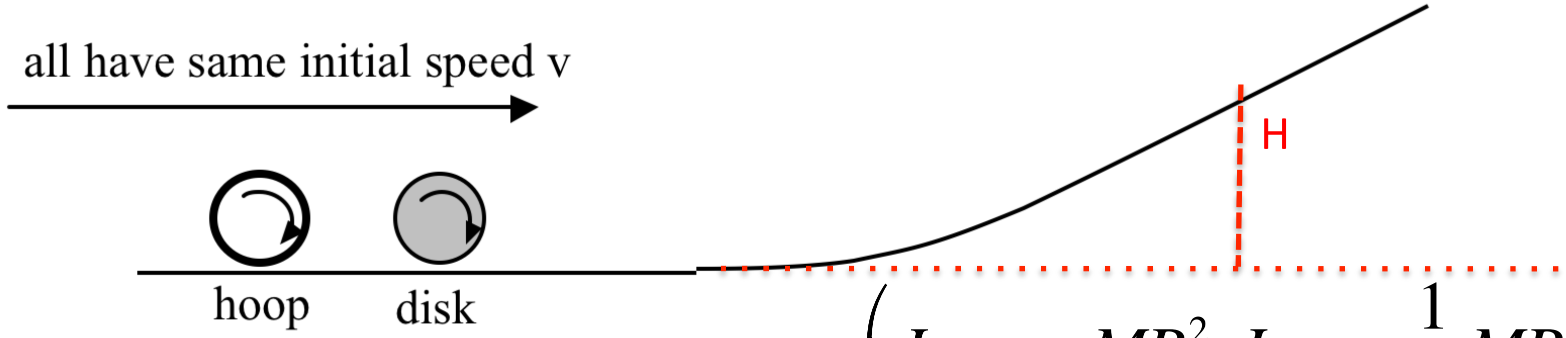
Spring 2014

PHYS-2010

Lecture 36

Clicker Question

Room Frequency BA



$$\left(I_{hoop} = MR^2, I_{disk} = \frac{1}{2} MR^2 \right)$$

A hoop and a disk of equal mass *M* and radius *R* roll towards an incline plane with the same speed. Which object has larger initial total kinetic energy?

A) Disk B) Hoop C) Same.

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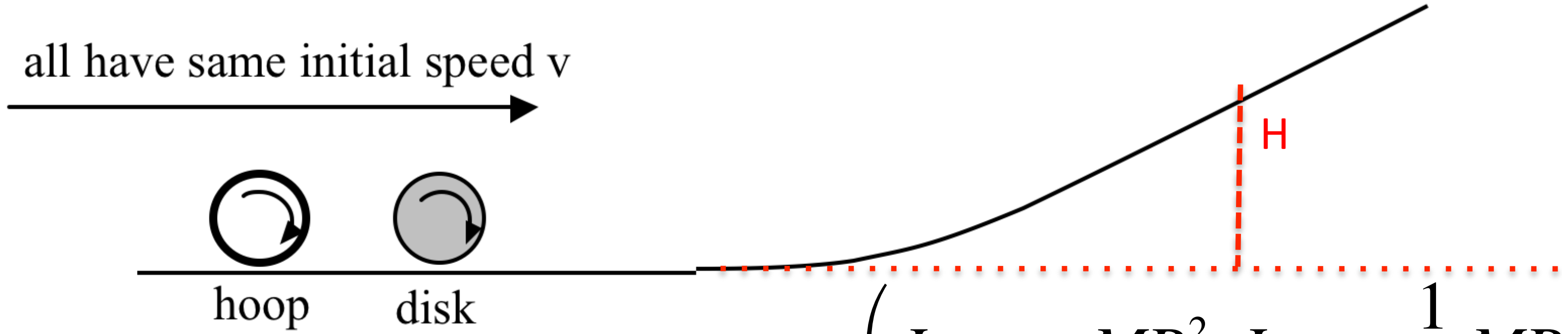

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A hoop and a disk of equal mass *M* and radius *R* roll towards an incline plane with the same speed. Which object has larger initial total kinetic energy?

A) Disk

B) Hoop

C) Same.

$$KE_{tot} = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left(1 + \frac{I}{MR^2}\right)$$

The hoop has a larger moment of inertia, and therefore a higher total kinetic energy!

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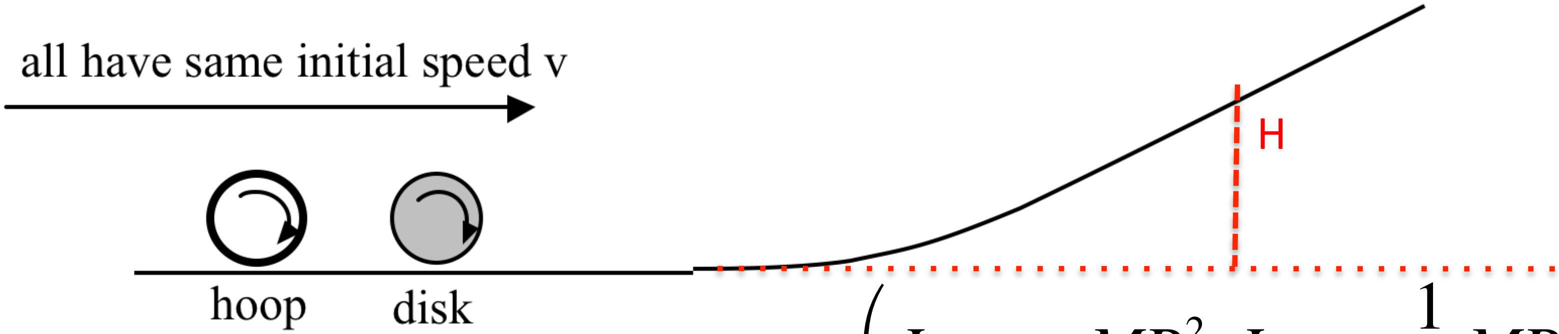


$$\left(I_{hoop} = MR^2, I_{disk} = \frac{1}{2}MR^2 \right)$$

A hoop and a disk of equal mass *M* and radius *R* roll towards an incline plane with the same speed. Which object has larger initial total kinetic energy?

A) Disk B) Hoop C) Same.

Which object will go furthest up the incline?

A) Disk B) Hoop C) Same height.

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 = MgH$$

Total mechanical energy is conserved, so higher initial kinetic energy → higher final potential energy.

Announcements

- Read Giancoli **Chapter 9.**
- **CAPA # 12** due tomorrow, Tuesday, April 15.
- **Written Homework # 9** due Friday, April 18.
- This week in Section: **Lab # 7** Rotational Motion (no prelab)
- **Midterm Exam 3** results and solutions are on D2L.
- Average (adjusted) exam score was 71, median score 70 .

Rotational Dynamics

Translation	↔	Rotation	
x	↔	θ	angle of rotation (rads)
$v = \dfrac{\Delta x}{\Delta t}$	↔	$\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{v}{r} = \dfrac{2\pi}{\tau} = 2\pi f$	angular velocity (rad/s)
$a = \dfrac{\Delta v}{\Delta t}$	↔	$\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{a_{tan}}{r}$	angular acceleration (rad/s^2)
F	↔	$\tau = r F_{\perp}$	torque (N m)
M	↔	$I = \sum m r^2$	moment of inertia (kg m^2)
$F_{net} = M a$	↔	$\tau_{net} = I \alpha$	Newton's 2nd Law
$KE_{trans} = (1/2) M v^2$	↔	$KE_{rot} = (1/2) I \omega^2$	Kinetic Energy (joules J)

$$\Delta KE + \Delta PE = \text{constant} \quad \leftrightarrow \quad \Delta KE_{trans} + \Delta KE_{rot} + \Delta PE = \text{constant}$$

(Conservation of Mechanical Energy)

$$p_{tot} = \sum_i m_i v_i = \text{constant} \leftrightarrow \quad L_{tot} = \sum_i I_i \omega_i = \text{constant}$$

Conservation of Angular Momentum

Momentum	$p = mv$	$L = I\,\omega$	Angular momentum
Relation to force	$F = \Delta p/\Delta t$	$\tau = \Delta L/\Delta t$	Relation to torque
No external force (momentum is conserved)	$\Delta p = 0$	$\Delta L = 0$	No external torque (angular momentum is conserved)

$$L_i = L_f$$

$$I_i\,\omega_i = I_f\,\omega_f$$

By changing the distribution of mass, the moment of inertia is changed.

By conservation of angular momentum, the angular velocity is therefore modified.

$$L = I\omega$$

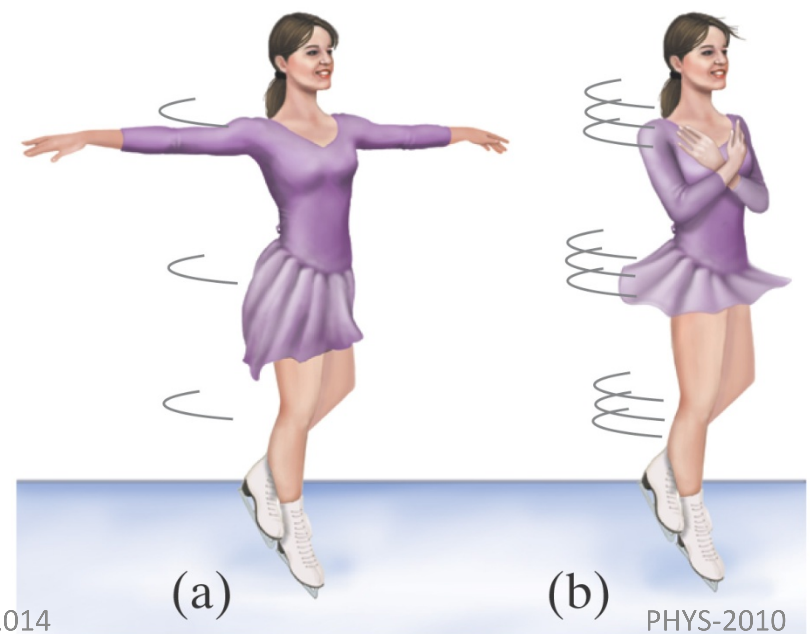
$$\tau = \frac{\Delta L}{\Delta t}$$

$$I_i\omega_i = I_f\omega_f$$

$$(\text{if } \mathrm{F}_{ext} = 0)$$

I_i large ω_i small

I_i small ω_i large



Conservation of L:

$$L_{initial} = L_{final}$$

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$$

By changing the distribution of mass, the moment of inertia is changed.

By conservation of angular momentum, the angular velocity is therefore modified.

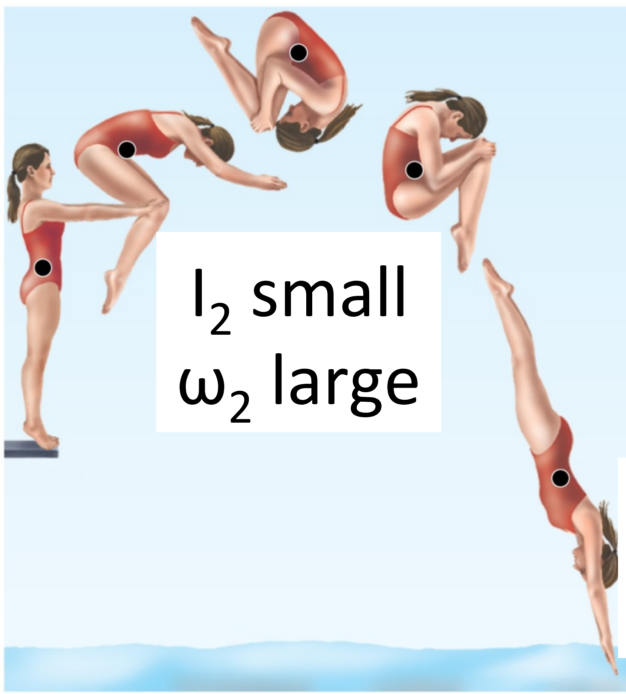
$$L = I\omega$$

$$\tau = \frac{\Delta L}{\Delta t}$$

$$I_i\omega_i = I_f\omega_f$$

$$(\text{if } F_{ext} = 0)$$

I_1 large
ω_1 small



I_3 large
ω_3 small

Conservation of L:

$$L_{initial} = L_{final}$$

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$$

Clicker Question

Room Frequency BA

A star is rotating with a period of T. Over a span of a million years, its radius decreases by a factor of 2 while the mass remains constant.

$$L = I\omega$$

$$\tau = \frac{\Delta L}{\Delta t}$$

$$I_i\omega_i = I_f\omega_f \quad (\text{if } F_{ext} = 0)$$

What is the new period of the star?
(Recall: $I_{sphere} = (2/5)\, MR^2$)

A) T/2 B) 2T C) 4T **D) T/4** E) None of these.

Conservation of Angular Momentum:

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i \qquad \omega_f = 4\omega_i$$

- Radius goes ***down*** by a factor of **2**;
- Moment of inertia I goes ***down*** by **4**;
- Angular velocity goes ***up*** by a factor of **4**;
- Period goes ***down*** by a factor of **4**.

$$T_f = \frac{1}{4} T_i$$

HOBERMAN SPHERE

HOBERMAN SPHERE

Don't try this at home!!!

Rotational Kinetic Energy and Angular Momentum

- Rotational Kinetic Energy:

$$E_{rot} = (1/2)\, I\, \omega^2$$

- Angular Momentum:

$$L = I\,\omega \quad \rightarrow \omega = L/I$$

- Thus, $\omega = L/I$ and hence

$$E_{rot} = (1/2)\, I\, \omega^2 = (1/2)\, L^2/\, I$$

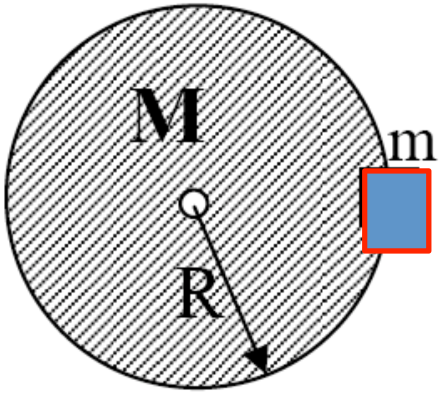
Clicker Question

Room Frequency BA

Consider a solid disk of mass **M** and radius **R** with an axis through its center.

$$I = \frac{1}{2}MR^2 + mR^2$$

An ant of mass **m** is placed on the rim of the disk.



The ant-disk system is rotating.

The ant walks toward the center of the disk.

The magnitude of the angular momentum *L* of the system:

A) increases B) decreases **[C) remains constant]**

Unless an outside torque is applied, $L = I\omega = \text{const.}$

As ant moves inward, the system's kinetic energy:

[A) increases] B) decreases C) remains constant

$E_{rot} = (1/2)\, I\, \omega^2 = (1/2)\, L^2 / I$