Spring 2014

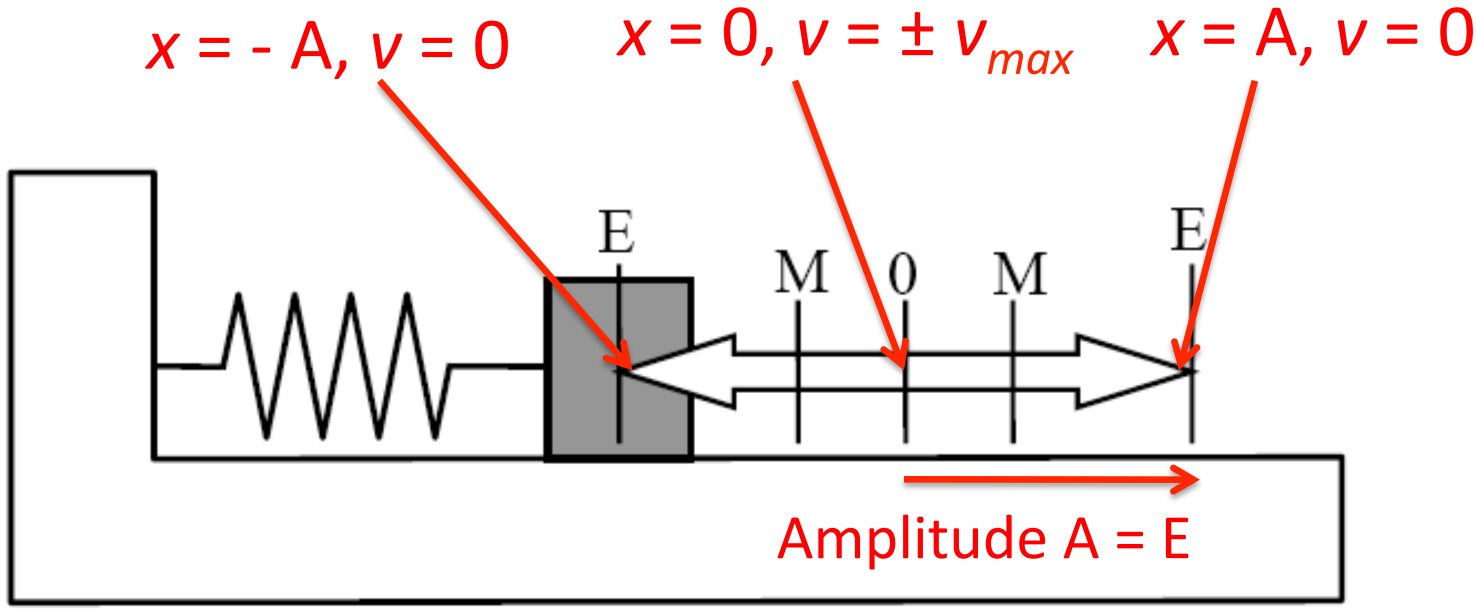
PHYS-2010

Lecture 43

Announcements

- Finish Giancoli **Chapter 10** and start **Chapter 11.**
- Extra Office Hour by TA Rosemary Wulf this Sunday, May 4, 1:30-4:00 pm in DUANE G1B30.
- Prof. Pollock's regular office hour this Friday (1-2 pm in the Help Room).
- **Final Exam:**
  - Monday, May 5, 1:30-4:00, in G1B30.
  - Cumulative, but with more focus on last chapters.
  - Practice exam posted on D2L.

Special Points of Horizontal Spring and Mass Oscillation

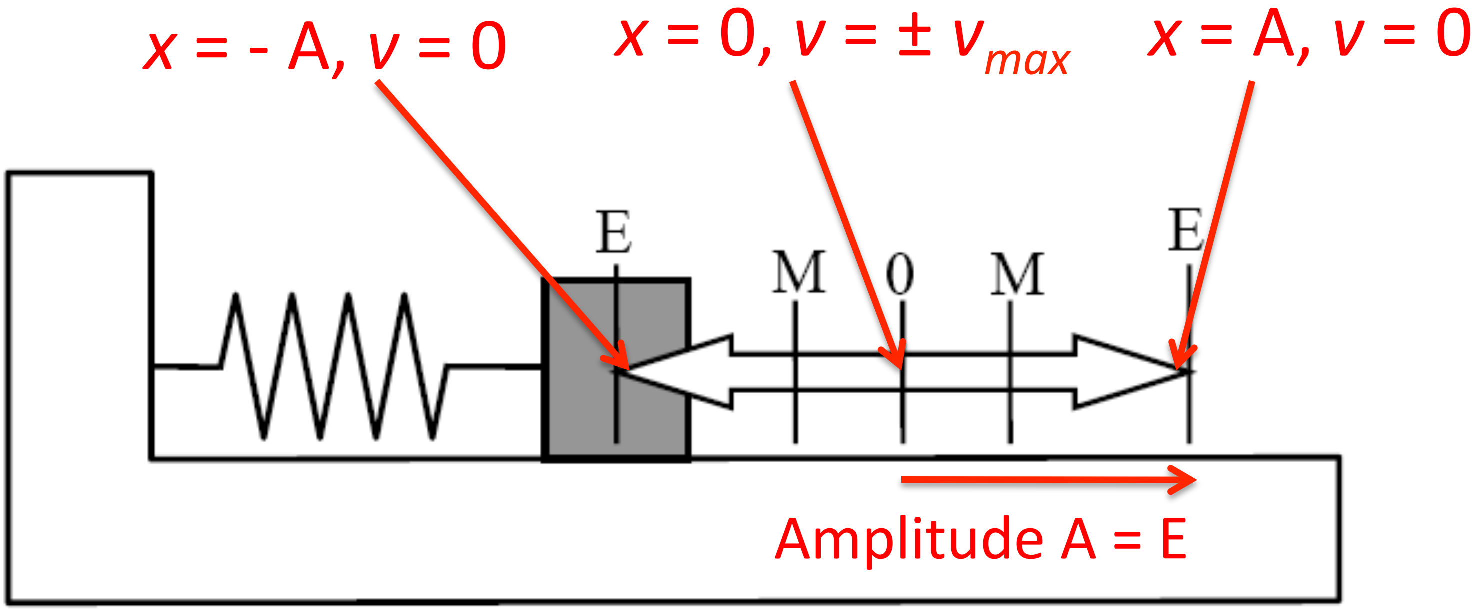


$$E_{tot} = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

At turning points $x = \pm A$, $v = 0$, E_{tot} = PE = $\frac{1}{2}kA^2$.

At equilibrium point $x = 0$, $v = \pm v_{max}$, E_{tot} = KE = $\frac{1}{2}mv_{max}^2$.

Amplitude – Max Speed Relation



Total energy is conserved!!! So we can conclude

$$E_{tot} = \tfrac{1}{2} k A^2 = \tfrac{1}{2} m v_{\max}^2$$

What are the units of this expression?

Solving for $v_{\max}$ in terms of A gives the result $v_{\max} = \sqrt{\frac{k}{m}} A.$

Clicker Question

Room Frequency BA

I take two identical masses; one is attached to a stiff (large spring constant *k*) spring; the other to a floppy (small *k*) spring. Both masses are initially positioned at *x* = 0 and given the **same** initial speeds. Which spring produces the largest amplitude motion?

$$E_{tot} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2 \longrightarrow A = \sqrt{\frac{m}{k}}\,v_{\max}$$

A) The stiff spring

B) The floppy spring

C) Same!

Clicker Question

Room Frequency BA

Now the identical masses on the two different springs are pulled to the side and released from rest with the **same initial amplitude**. Which spring produces the largest maximum speed of its mass?

A) The stiff spring

B) The floppy spring

C) Same!

$$A = \sqrt{\frac{m}{k}} v_{\max}$$

$$v_{\max} = \sqrt{\frac{k}{m}} A$$

Example of Energy Analysis I

A mass *m* attached to a spring with spring constant *k* is released *from rest* at position x_0 (the spring's equilibrium position is $x = 0$). What is the speed at some intermediate position *x* between 0 and x_0?

Are we given the amplitude?

Yes! $A = x_0$ because it was released from rest.

Total energy = $\frac{1}{2}kA^2$ and it is a constant!

$$E_{tot} = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}kx_0^2$$

Solve for *v* and find

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

Example of Energy Analysis II

Note what we can do with a little algebraic manipulation…

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = A\sqrt{\frac{k}{m}}\sqrt{1 - \frac{x^2}{A^2}} = v_{max}\sqrt{1 - \frac{x^2}{A^2}}.$$

CHECK: Units are correct and at $x = 0$, $x = A$, we get the correct results!

If you are given v at $x = 0$, you know v_{max}, and you can find a formula for x given some v with magnitude less than v_{max}…

If given a *v* at a particular *x* you can *always* find A and v_{max}

Harmonic Time Dependence of SHM

To find the ***time*** dependence of SHM we use an amazing fact:

The time dependence of one space component of circular motion is *exactly* the same as the time dependence of SHM!

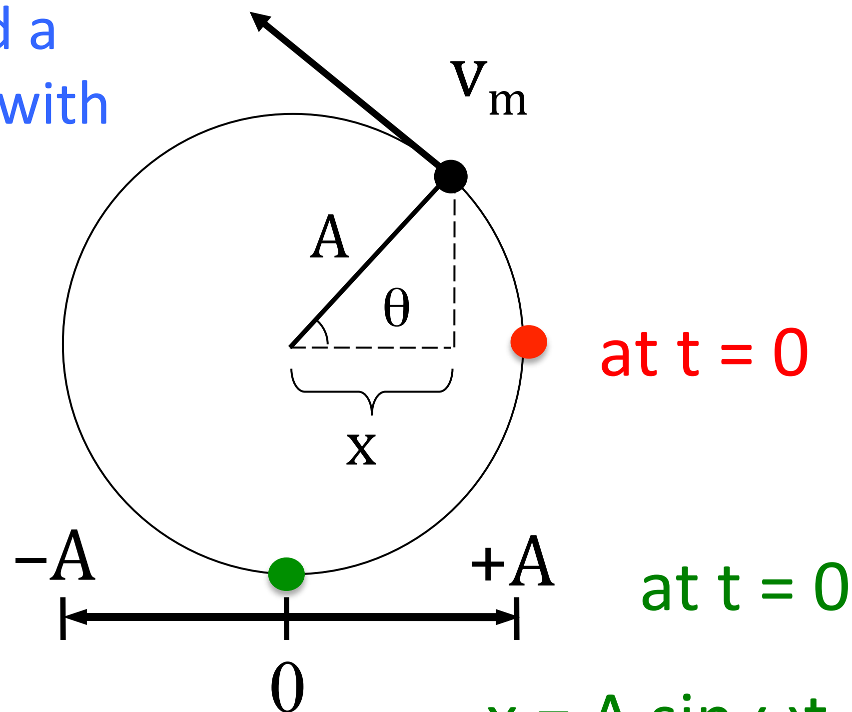
Consider a particle moving around a circle of radius *A* in the *x-y* plane, with constant speed v_m

The angular velocity $\omega = \Delta\theta / \Delta t$

For constant speed, ω is constant and so $\theta = \omega t$.

As usual $x = A \cos\theta$ so

$$x = A \cos \omega t$$



$x = A \sin \omega t$

Clicker Question

Room Frequency BA

What are the **signs** of cos(225°) and sin(225°)? Don't use a calculator!

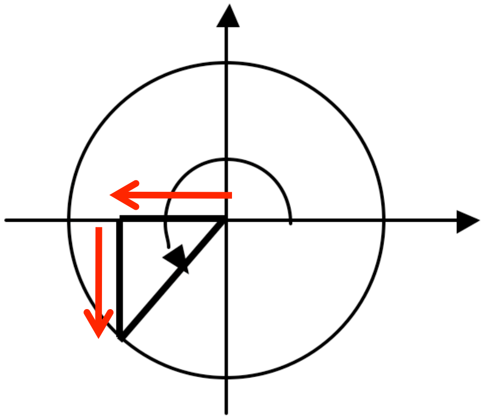
A) cos(225°) = (+) , sin(225°) = (–)

B) cos(225°) = (–) , sin(225°) = (+)

C) cos(225°) = (+) , sin(225°) = (+)

D) cos(225°) = (–) , sin(225°) = (–)

E) None of these. One of them is zero



Harmonic Time Dependence of SHM

How do we relate ω to v_m?

One cycle takes time *T*, called the *period.*

Time to go around circle is $2\pi A/v_m = T = 2\pi \text{ rad}/\omega$

Solving yields $\omega = v_m/A$

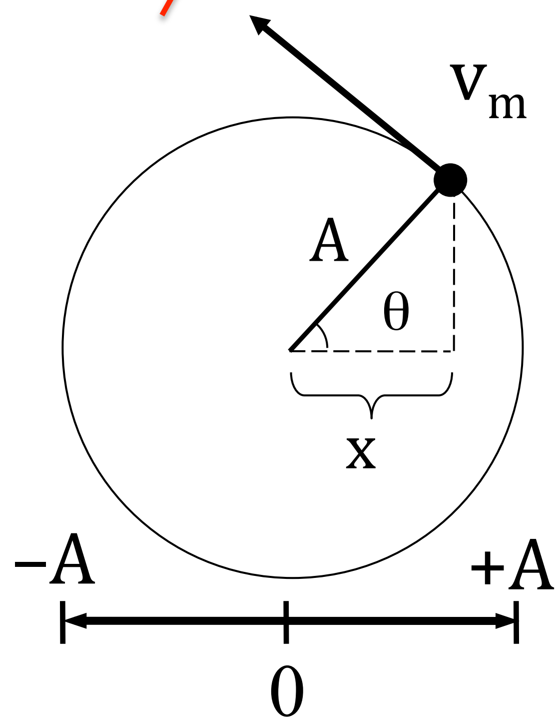
SHM has same time dependence,
so $x = A\cos[(v_m/A)t]$

But for SHM, we derived

$$\frac{v_m}{A} = \sqrt{\frac{k}{m}}$$

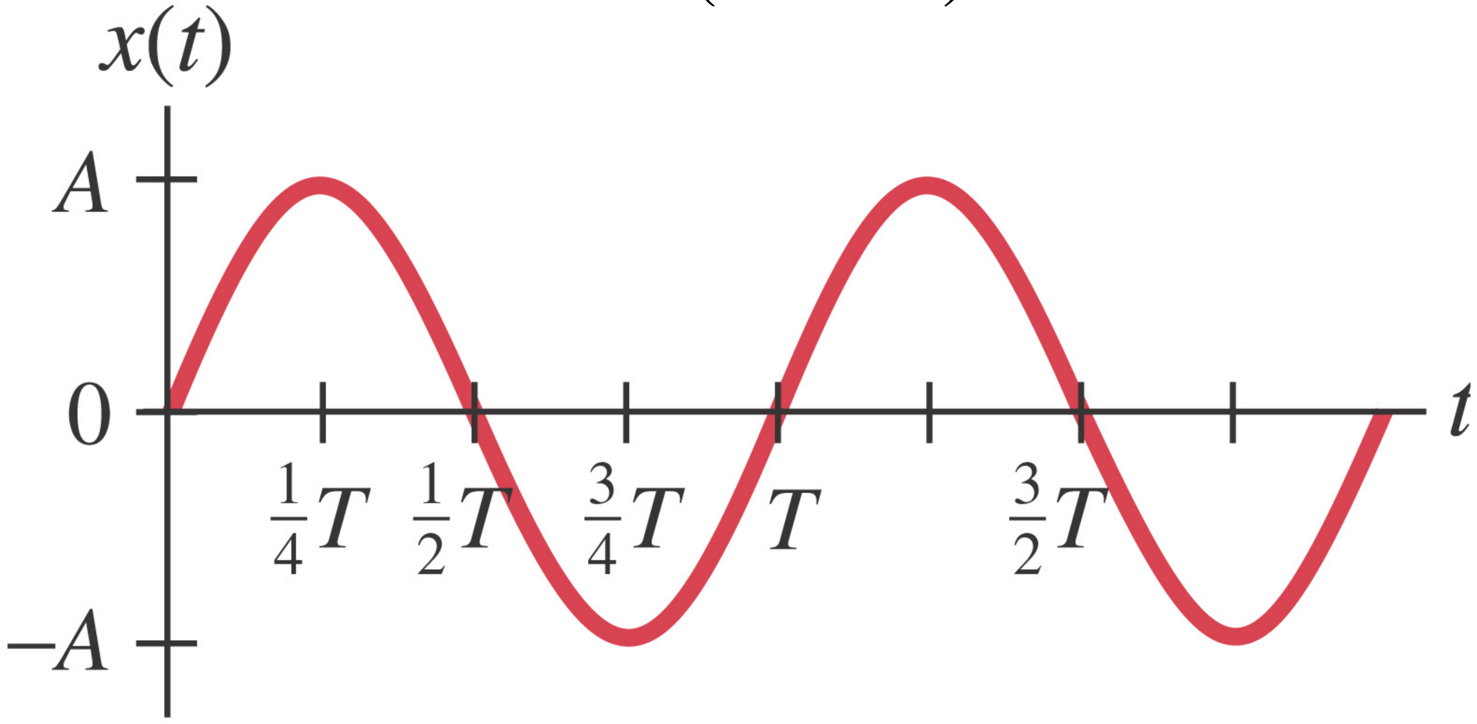
So we have

$$x = A\cos\left(\sqrt{\frac{k}{m}}t\right)$$



Simple Harmonic Oscillator with x = 0 at t = 0

$$x = A\sin\omega t$$
$$= A\sin(2\pi f\, t)$$
$$= A\sin(2\pi t / T)$$



SHM Period and Frequencies

We just found that T = 2π rad/ω = 2πA/v_m . For SHM then

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency *f* = 1/T so we find

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Independent of amplitude!

Note: even for back and forth motion we have an *angular frequency* ω!

$$\omega = \sqrt{\frac{k}{m}} \text{ in radians/sec for } k \text{ and } m \text{ in SI units}$$

Cosines and sines *must* be functions of angles; here we will always use radians

$$x = A \cos \omega t$$

Clicker Question

Room Frequency BA

A mass on a spring oscillates with a certain amplitude and a certain period *T*. If the mass is doubled, the spring constant of the spring is doubled, and the amplitude of motion is doubled, THEN the period ...

A) increases

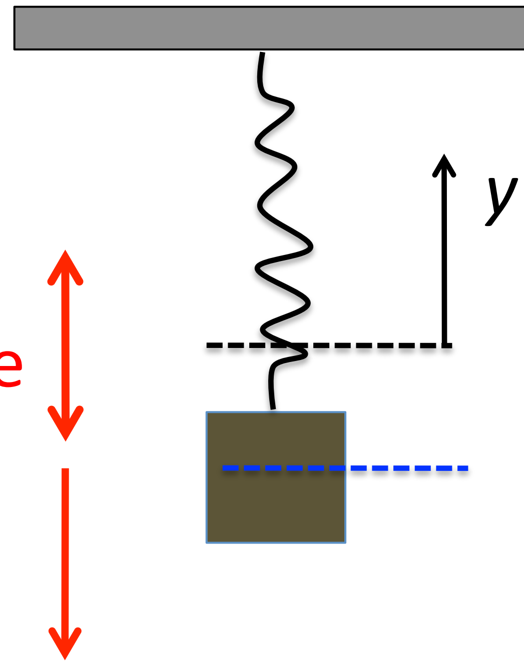
B) decreases

C) stays the same.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Ratio unchanged if *m* and *k* both doubled and the period is independent of amplitude.

Vertical Spring and Mass Oscillation



Spring equilibrium without gravity $y=0$

Spring equilibrium with gravity $y = y_E$

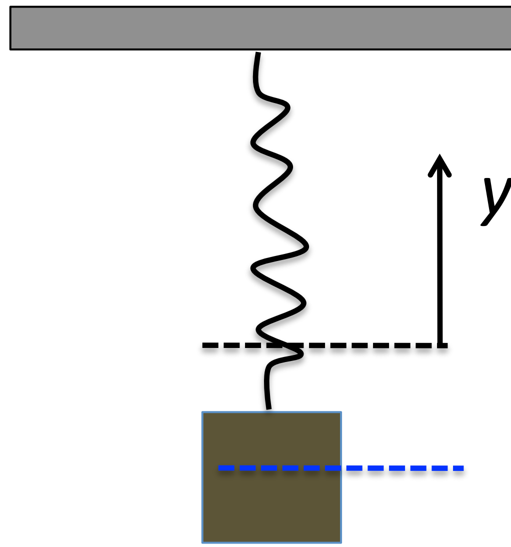
New spring equilibrium length where $-mg - ky_E = 0$

$$\boldsymbol{y_E = -mg/k}$$

Oscillation frequency is NOT changed:

$$\omega = \sqrt{\frac{k}{m}} \quad \text{(in radians/sec)}$$

Vertical Spring and Mass Oscillation



Spring equilibrium without gravity $y=0$

Spring equilibrium with gravity $y = y_E$

Mechanical Energy is still conserved! Now E_{tot} has gravity PE term:

$$E_{tot} = \tfrac{1}{2}ky^2 + \tfrac{1}{2}mv^2 + mgy$$

With a little algebra you can rewrite this as

$$E_{tot} = \tfrac{1}{2}k(y - y_E)^2 + \tfrac{1}{2}mv^2 + \tfrac{1}{2}ky_E^2$$