

Spring 2014

PHYS-2010

Lecture 32

A space ship is initially rotating on its axis with an angular velocity of $\omega_0 = 0.5$ rad/sec. The Captain creates a maneuver that produces a constant angular acceleration of $\alpha = 0.1$ rad/sec² for 10 sec.

1) What is its angular velocity at the end of this maneuver? $\omega = 1.5$ rad/sec

2) At how many rpms is it rotating after the maneuver? $f = \frac{45}{\pi}$ rpm

3) How far (in radians) did the space ship rotate while the Captain was applying the angular acceleration?

A) 1 rad

B) 5 rad

C) 10 rad

D) 45 rad

$$\omega = \omega_0 + \alpha t$$

$$= \frac{v}{r} = \frac{2\pi}{\tau} = 2\pi f$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

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$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2 = (0.5 \text{ rad/sec})(10 \text{ sec}) + \frac{1}{2} (0.1 \text{ rad/sec}^2)(10 \text{ sec})^2$$

$$= 5 \text{ rad} + 5 \text{ rad} = 10 \text{ rad}$$

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3) How far (in radians) did the space ship rotate while the Captain was applying the angular acceleration? $\theta = 10$ rad

4) How many degrees is this?

A) 180°

B) 1800°

C) $(1800/\pi)$ degrees

D) 360°

$$\omega = \omega_0 + \alpha t$$

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$$10 \text{ rad} \left(\frac{180 \text{ deg}}{\pi \text{ rad}} \right)$$

$$= \frac{1800}{\pi} \text{ deg}$$

Announcements

- Read Giancoli **Chapter 8 on Rotational Motion**.
- Next CAPA # 11 due April 8.
- Written **Homework # 8** due today at 4 pm.
- Next week: Review Recitation and **missed Lab make-up**.
 - at least 7 labs are required to pass the course;
 - contact your TA to arrange lab makeup ahead of time.
- **Study Session** by Prof. Pollock: Tue, Apr. 8, 5-6pm, G125.
- **Midterm Exam 3** on Thursday, April 10, 7:30 pm.
- Material covered on exam: Giancoli Chapters 5-7
(Gravity, Kepler's laws, Work, Energy, Power, Momentum, Collisions)

Rotational Kinematics

Translation

\leftrightarrow

Rotation

x

\leftrightarrow

θ

angle of rotation (rads)

$$v = \frac{\Delta x}{\Delta t}$$

\leftrightarrow

$$\omega = \frac{\Delta \theta}{\Delta t} = \frac{v}{r} = \frac{2\pi}{\tau} = 2\pi f$$

angular velocity (rad/s)

$$a = \frac{\Delta v}{\Delta t}$$

\leftrightarrow

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{a_{\text{tan}}}{r}$$

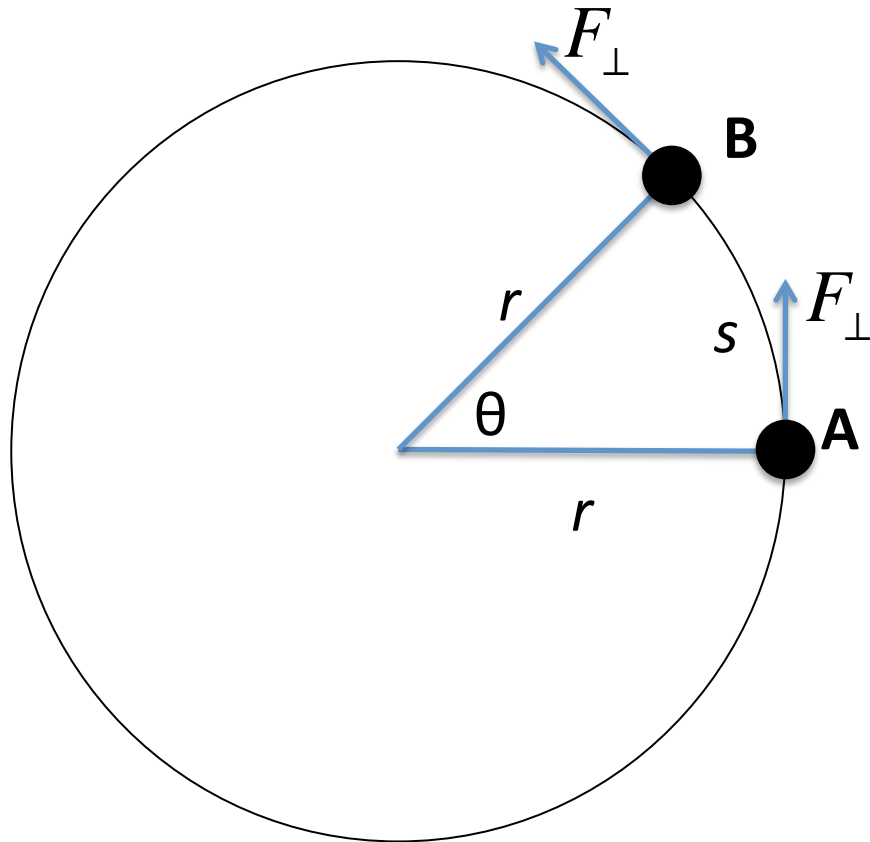
angular acceleration (rad/s²)

Rotational Dynamics

<u>Translation</u>	\leftrightarrow	<u>Rotation</u>	
x	\leftrightarrow	θ	angle of rotation (rads)
$v = \frac{\Delta x}{\Delta t}$	\leftrightarrow	$\omega = \frac{\Delta \theta}{\Delta t} = \frac{v}{r} = \frac{2\pi}{\tau} = 2\pi f$	angular velocity (rad/s)
$a = \frac{\Delta v}{\Delta t}$	\leftrightarrow	$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{a_{\text{tan}}}{r}$	angular acceleration (rad/s ²)
<hr/>			
F	\leftrightarrow	$\tau = r F_{\perp}$	torque (N m)
M	\leftrightarrow	$I = \sum m r^2$	moment of inertia (kg m ²)
$F_{\text{net}} = M a$	\leftrightarrow	$\tau_{\text{net}} = I \alpha$	Newton's 2 nd Law
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Torque

Rotational Dynamics



Consider the work done by a constant force F_{\perp} in moving an object from point A to point B on a circle.

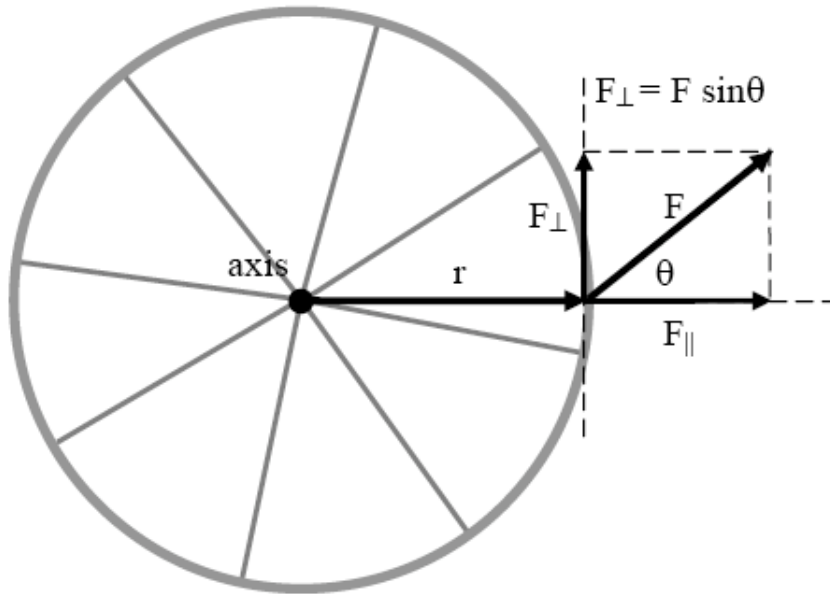
$$\begin{aligned} W &= (\text{Component of force along displacement}) \times \text{displacement} \\ &= F_{\perp} s = F_{\perp} (r\theta) = (rF_{\perp})\theta = \tau\theta \end{aligned}$$

Note: Units of torque are the same as for work/energy ($1\text{J} = 1\text{ N m}$), but these are very different quantities!

Torque

$$\tau \equiv rF_{\perp}$$

Torque



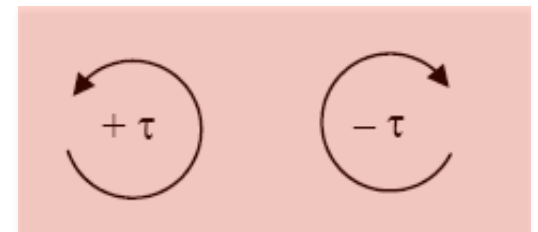
\therefore Torques are related to F_{\perp}

\therefore The amount of work done by F_{\perp} increases linearly with both r and F_{\perp} .

Torque:

$$\tau \equiv rF_{\perp}$$

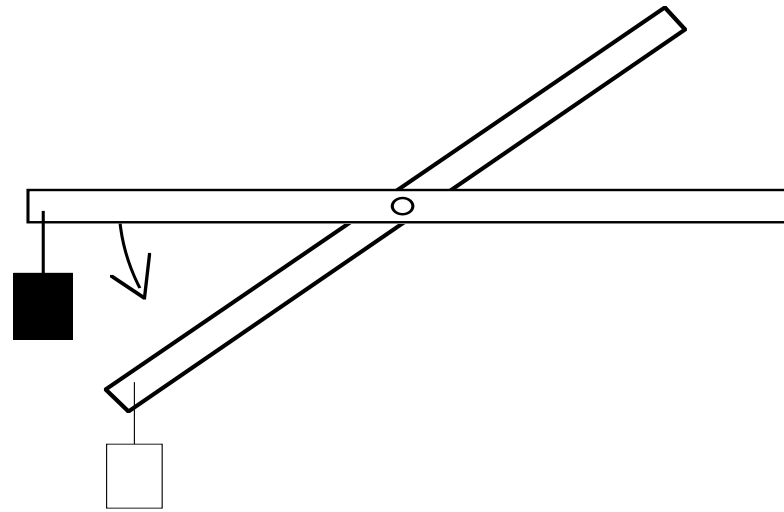
= (lever arm) x (Component of force perpendicular to lever arm)



Clicker Question

Room Frequency BA

A mass is hanging from the end of a horizontal bar which pivots about an axis through its center, but it is being held stationary. The bar is released and begins to rotate. As the bar rotates from horizontal to vertical, the magnitude of the torque on the bar..



$$\tau \equiv rF_{\perp}$$

A) increases

B) decreases

C) stays constant

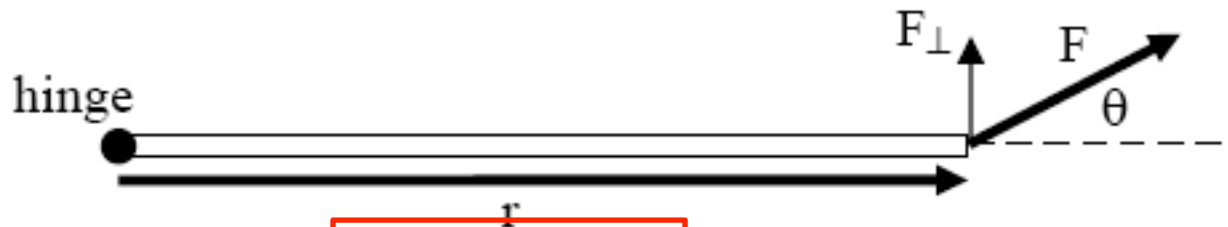
Both the lever arm and the magnitude of the force of gravity stay constant but the angle between them decreases!

Torque

$$\tau \equiv rF_{\perp}$$

You pull on a door handle a distance $r = 1\text{m}$ from the hinge with a force of magnitude 20 N at an angle $\theta = 30^{\circ}$ from the plane of the door.

What's the torque you exerted on the door? (Note: $\sin 30^{\circ} = 1/2$)



A) 8 Nm

B) 10 Nm

C) 12 Nm

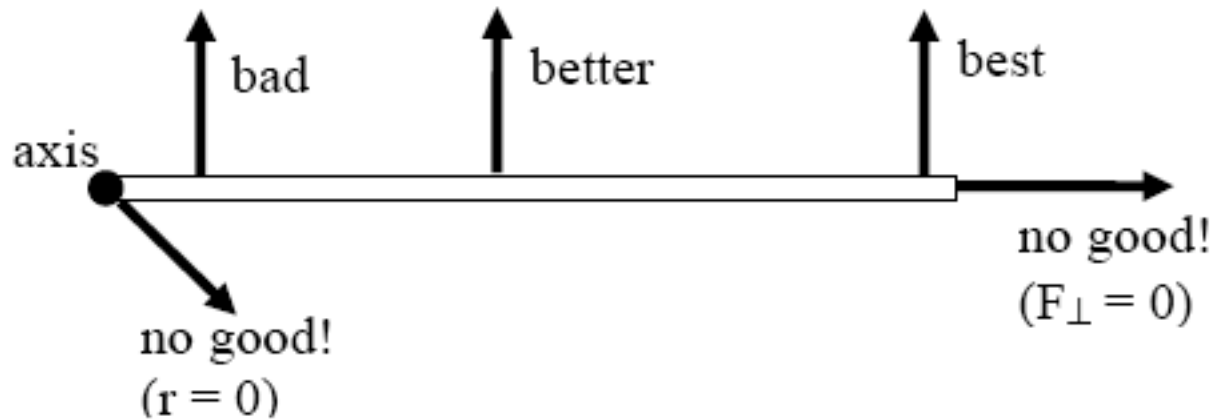
D) 20 Nm

$$\tau = rF_{\perp} = rF \sin \theta = (1\text{m})(20\text{N})(1/2) = 10\text{Nm}$$

Torque

$$\tau \equiv rF_{\perp}$$

Therefore, to easily rotate an object about an axis, you want a large lever arm r and a large perpendicular force F_{\perp} :



Torque

$$\tau \equiv rF_{\perp}$$

Therefore, to easily rotate an object about an axis, you want a large lever arm r and a large perpendicular force F_{\perp} :



(a)



(b)

Torque

$$\tau \equiv rF_{\perp}$$

Three forces labeled A, B, and C are applied to a rod which pivots on an axis through its center.

Which force causes the largest size torque?

The diagram shows a horizontal rod pivoted at its center. Three forces are applied to it:

- Force A: Applied at a distance L to the left of the pivot, at an angle of 45° below the rod. Its magnitude is F .
- Force B: Applied at a distance $L/2$ to the left of the pivot, pointing vertically downwards. Its magnitude is F .
- Force C: Applied at a distance $L/4$ to the right of the pivot, pointing vertically upwards. Its magnitude is $2F$.

The following trigonometric identity is provided:

$$\sin 45 = \cos 45 = 1 / \sqrt{2} = 0.707$$

The torques are calculated as follows:

$$|\tau_A| = LF \sin 45 = 0.707 LF$$

$$|\tau_B| = \frac{L}{2} F = 0.5 LF$$

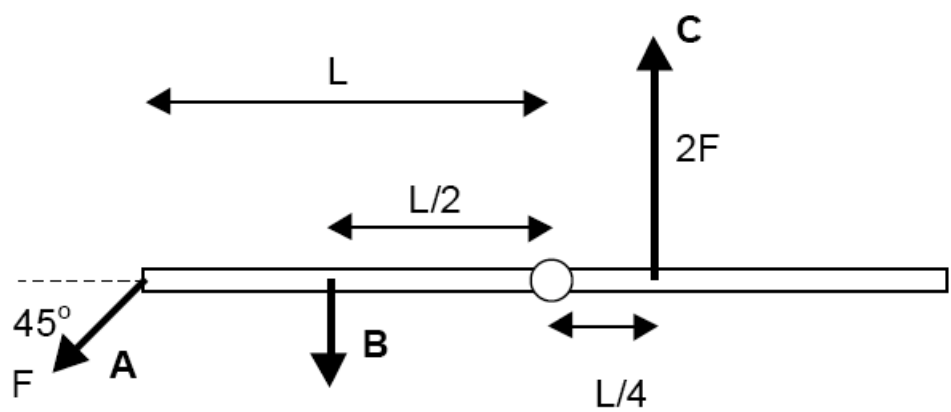
$$|\tau_C| = \frac{L}{4} (2F) = 0.5 LF$$

Torque

$$\tau \equiv rF_{\perp}$$

Three forces labeled A, B, and C are applied to a rod which pivots on an axis through its center.

What is the net torque on the rod?



$$\tau_A = LF \sin 45 = 0.707 LF$$

$$\tau_B = \frac{L}{2} F = 0.5 LF$$

$$\tau_C = \frac{L}{4} (2F) = 0.5 LF$$

$$\tau_{net} = (0.707 + 0.5 + 0.5) LF = 1.707 LF$$

- A) $0.707 LF$
- B) $-0.707 LF$
- C) $1.707 LF$
- D) $-1.707 LF$
- E) Zero

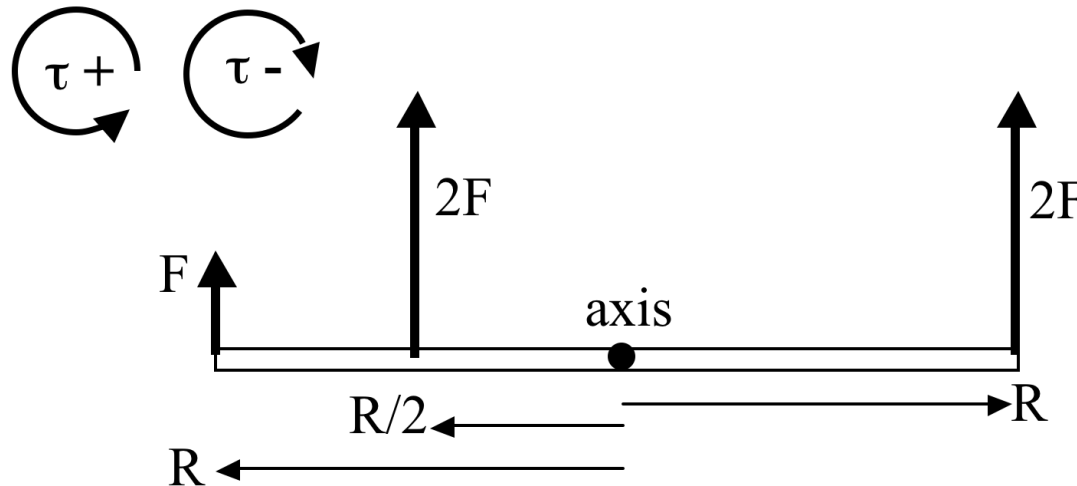
Clicker Question

Room Frequency BA

Torque

$$\tau \equiv rF_{\perp}$$

Three forces are applied to a rod which rotates about the center. What is the **net torque** about the axis?
Recall the sign convention.



A) $+RF$

B) $-RF$

C) zero

D) $+3RF$

E) $-3RF$

Total Torque:

$$\tau_{\text{tot}} = + (2F) \times R - (2F) \times (R/2) - F \times R = 0$$