Spring 2014

PHYS-2010

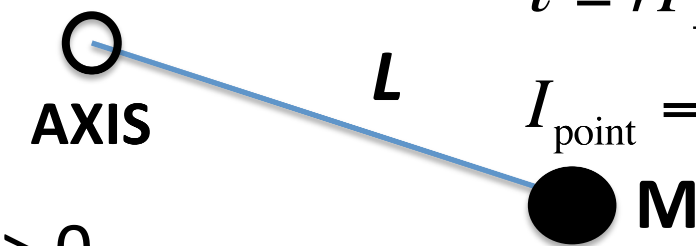
Lecture 34

Clicker Question

Room Frequency BA

A light rod of length *L* has a mass *M* attached at one end. The axis of rotation is at the other end. The rod is first held at rest horizontally and then at $t = 0$ is released from the horizontal position and starts to rotate clockwise due to the torque due to the force of gravity on *M*.

$t = 0$



$t > 0$

$$\tau \equiv rF_{\perp} = I\alpha$$

$$I_{\text{point}} = mr^2$$

How does the magnitude of the rod's angular acceleration change with time?

A) increases

B) decreases

C) stays constant

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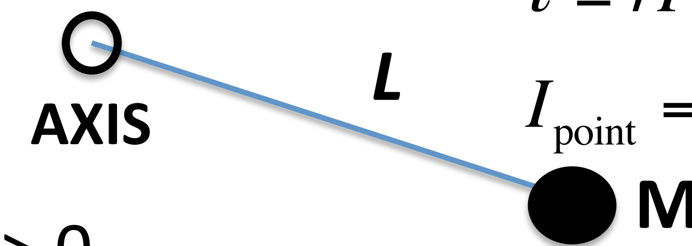

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Announcements

- Read Giancoli **Chapter 8 on Rotational Motion**.
- No written Homework due this week.
- **Midterm Exam 3** on Thursday, April 10, 7:30 pm.
- **Practice exam** is posted on D2L.
- Material covered on exam: Giancoli Chapters 5-7

(Gravity, Kepler's laws, Work, Energy, Power, Momentum, Collisions)

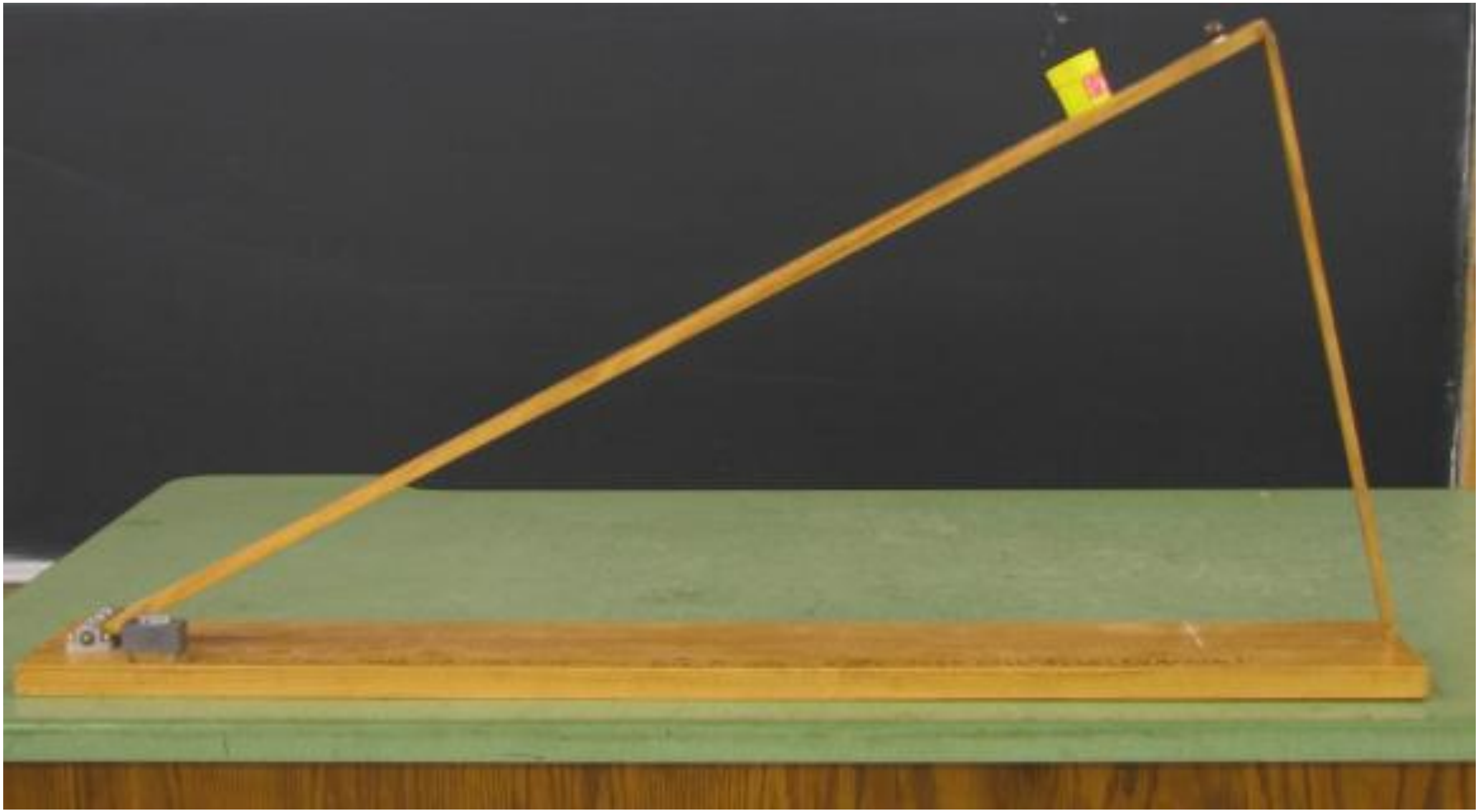
- **Exam seating:**
  - if your TA is Rosemary Wulf or Andrew Hess, your exam is here, G1B30.
  - if your TA is Jake Fish or Clarissa Briner, your exam is next door, G1B20.
- More details about the exam are on the course website:

http://www.colorado.edu/physics/phys2010/phys2010_sp14/exams.html

Rotational Dynamics

Translation	↔	Rotation	
x	↔	θ	angle of rotation (rads)
$v = \frac{\Delta x}{\Delta t}$	↔	$\omega = \frac{\Delta \theta}{\Delta t}$	angular velocity (rad/s)
$a = \frac{\Delta v}{\Delta t}$	↔	$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{a_{tan}}{r}$	angular acceleration (rad/s^2)
F	↔	$\tau = r F_\perp$	torque (N m)
M	↔	$I = \sum m r^2$	moment of inertia (kg m^2)
$F_{net} = M a$	↔	$\tau_{net} = I \alpha$	Newton's 2nd Law

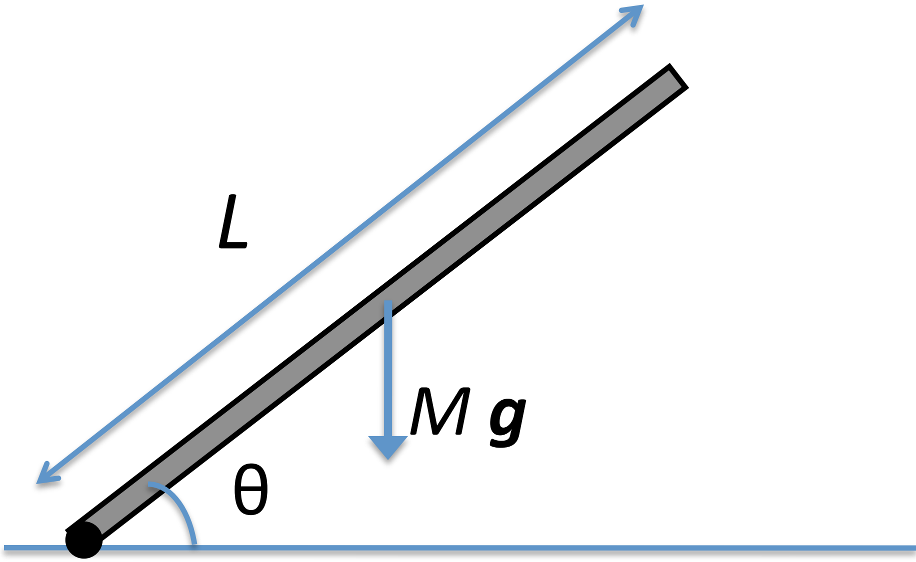
Falling Chimney Demonstration



Clicker Question

Room Frequency BA

A rod of length *L* and mass *M* makes an angle θ with a horizontal table.



1) What is the magnitude of the torque **τ** exerted on the rod by gravity?

A) ML

B) Mg sin θ

C) MgL

D) 0.5 MgL cos θ

Hint: for a uniform extended rod, the torque due to the force of gravity is applied at the center of mass, i.e., the middle of the rod.

$$\tau = F_{g,\ perp} (L/2) = Mg \cos\theta \ (L/2)$$

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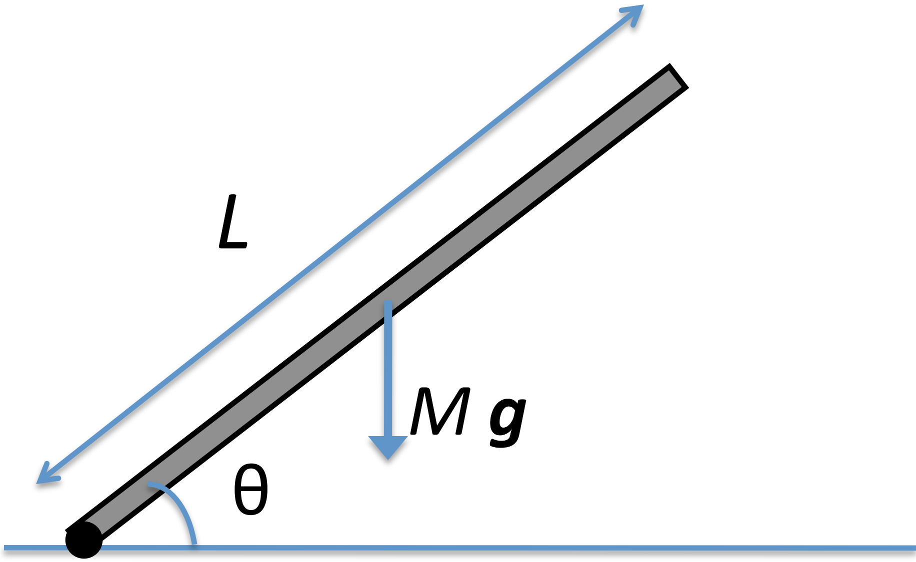
A rod of length *L* and mass *M* makes an angle θ with a horizontal table.



1) What is the magnitude of the torque **τ** exerted on the rod by gravity? 0.5 Mg L cos θ

2) What is the angular acceleration **α** of the rod when it is released? (Note: the moment of inertia is ***I* = *M L*²/ 3**)

A) L(cosθ)/3g

B) (3g/2L) cos θ

C) 3g/(Lcos θ)

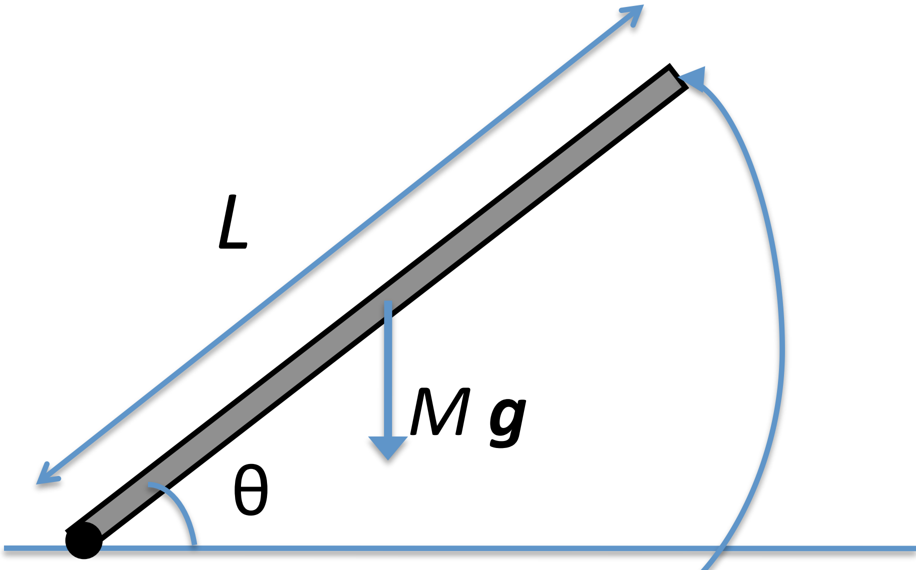
D) gL/3

$$\alpha = \tau/I = (3/2)\, MgL \cos\theta / ML^2 = (3g/2L) \cos\theta$$

Clicker Question

Room Frequency BA

A rod of length L and mass M makes an angle θ with a horizontal table.



1) What is the magnitude of the torque **τ** exerted on the rod by gravity? 0.5 Mg L cos θ

2) What is the angular acceleration **α** of the rod when it is released? (Note: the moment of inertia is $I = M L^2 / 3$) $(3g/2L) \cos \theta$

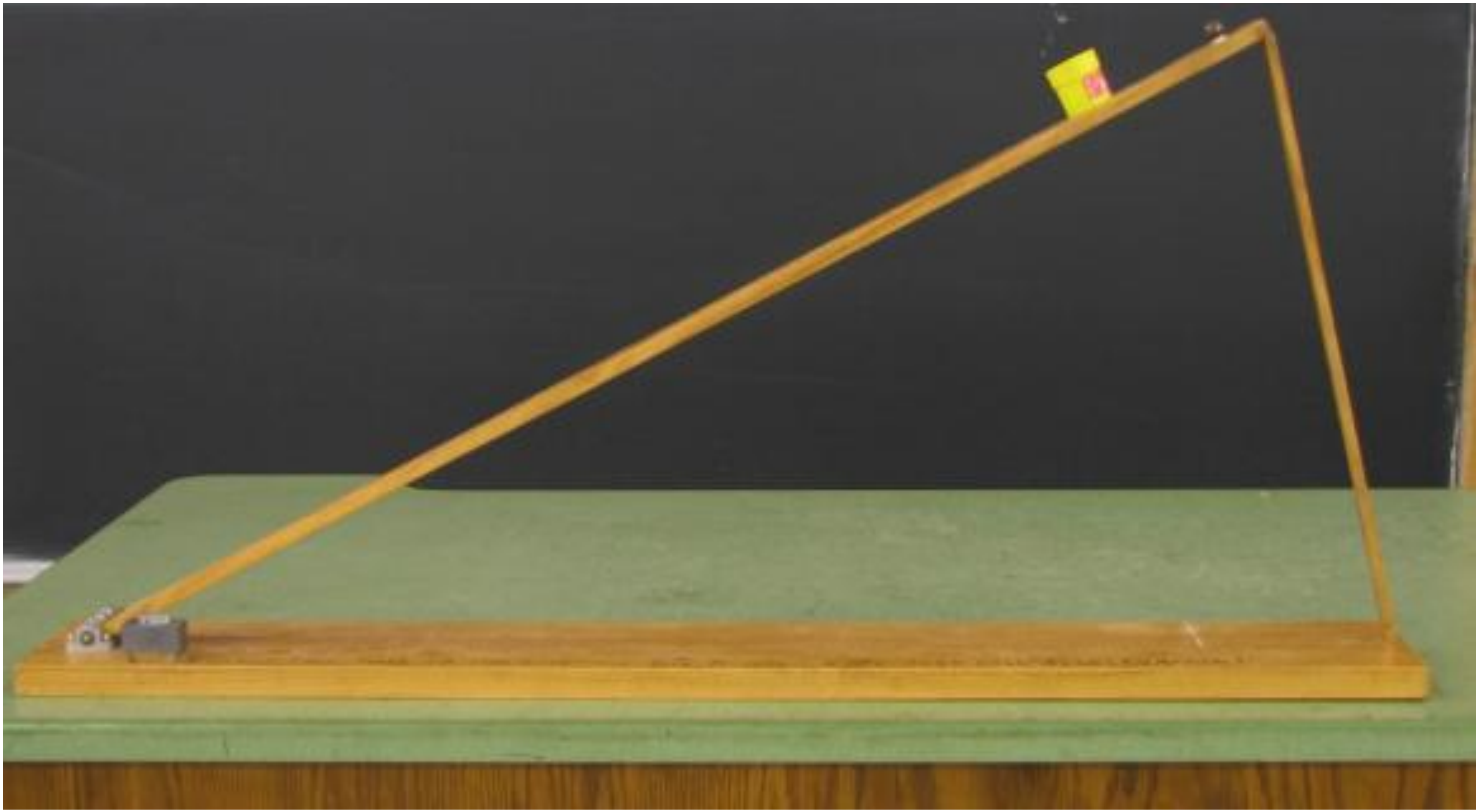
3) What is the tangential acceleration a of the far end of the rod?

A) $(3/2)\, g/L^2 \cos \theta$ B) $2g/3 \sin \theta$ C) $1.5\, g \cos \theta$ D) $gL/3$

$a = L\,\alpha = (3g/2) \cos \theta$

Note: $a_y = a \cos \theta = 1.5\, g \cos^2\theta$ **> g** if $\cos^2 \theta > 2/3$!

Falling Chimney Demonstration

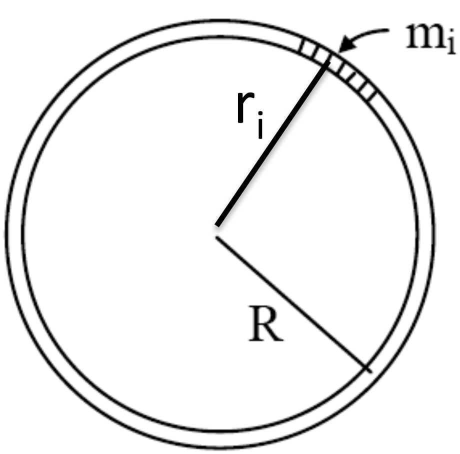


Torque and Moment of Inertia

$$\tau \equiv rF_{\perp} = I\alpha$$

$$I_{\text{point}} = mr^2$$

Consider the moment of inertia of a **hoop** of total mass M and radius R:



$$I = \sum_i m_i r_i^2 = \left(\sum_i m_i\right) R^2 = M R^2$$

(since $r_i = R$ for all i)

$I_{point} = mr^2$

$I_{hoop} = MR^2$

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Torque and Moment of Inertia

$$\tau \equiv rF_{\perp} = I\alpha$$

$$I = \sum_i m_i r_i^2$$

$$I_{\text{hoop}} = MR^2$$

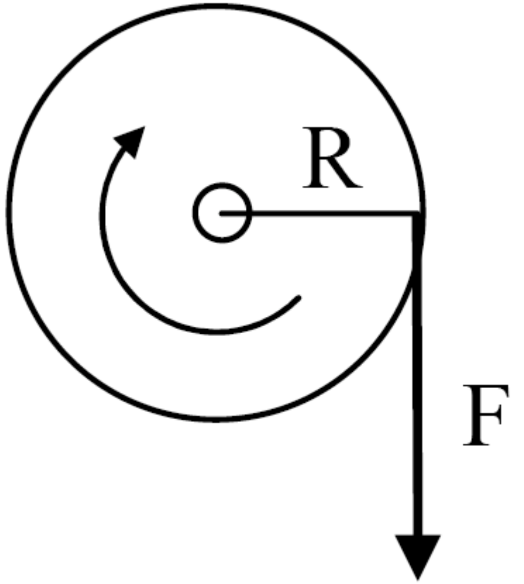
A force *F* is applied to a **hoop** of mass *M* and radius *R*. What's the resulting **magnitude** of the angular acceleration?

A) RF/M

B) F/MR^2

C) MR^2

D) F/MR



$$\tau = I_{\text{hoop}}\alpha = RF$$

$$\alpha = \frac{RF}{I_{\text{hoop}}} = \frac{RF}{MR^2} = \frac{F}{MR}$$

Darth Maul’s Double-Bladed Light Saber

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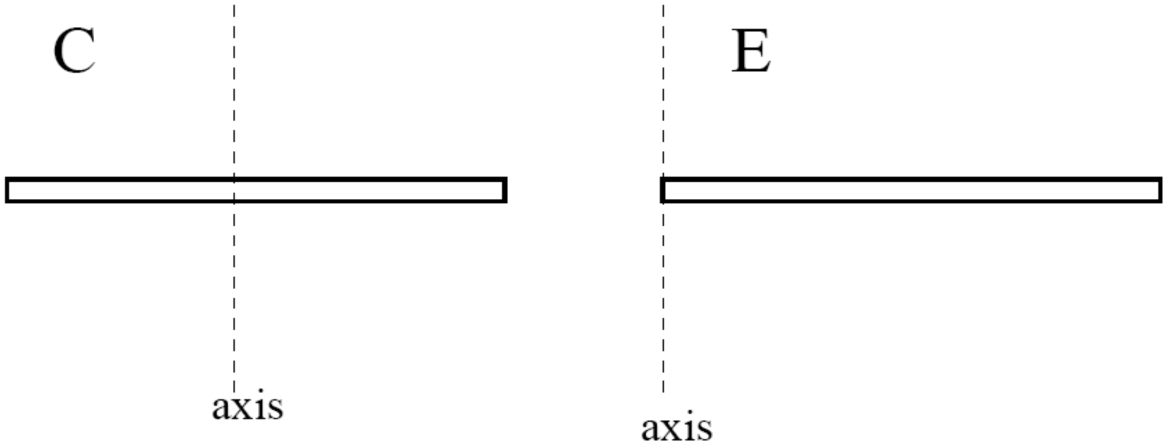
Torque and Moment of Inertia

$$I = \sum_i m_i r_i^2$$



Consider a uniform rod with an axis of rotation through is center and an identical rod with an axis of rotation through on end. Which has a larger moment of inertia?

A) $I_C > I_E$ B) $I_C < I_E$ C) $I_C = I_E$

If more mass is further from the axis, the moment of inertia increases.

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Torque and Moment of Inertia

$$\tau \equiv rF_{\perp} = I\alpha$$

$$I = \sum_i m_i r_i^2$$

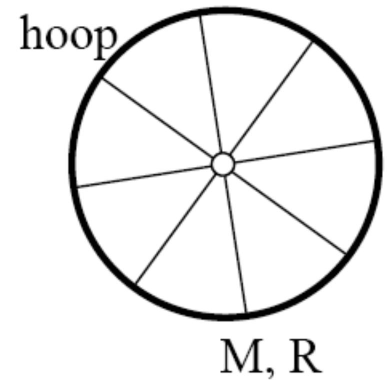
$$I_{\text{hoop}} = MR^2$$

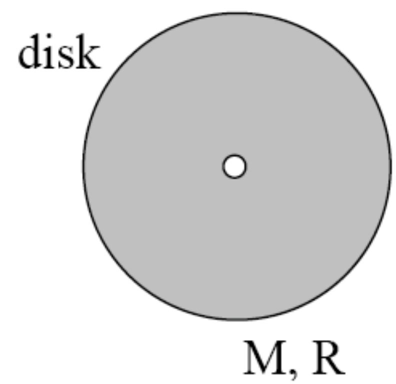
Two wheels have the same radius *R* and total mass *M*. They are rotating about their fixed axes. Which has the larger moment of inertia?

A) Hoop

B) Disk

C) Same

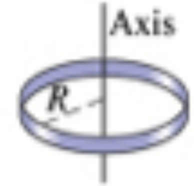
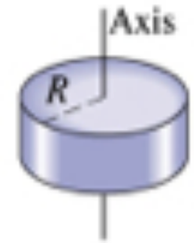
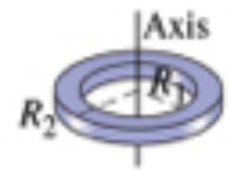




The hoop's mass is concentrated at its rim, while the disk's is distributed from its center to its rim. So, the hoop will have the larger moment of inertia.

Moment of Inertia

$$I = \sum_i m_i r_i^2$$

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R	Through center		MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}MW$
(c)	**Solid cylinder,** radius R (disk)	Through center		$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center		$\frac{2}{5}MR^2$

Rotational Dynamics

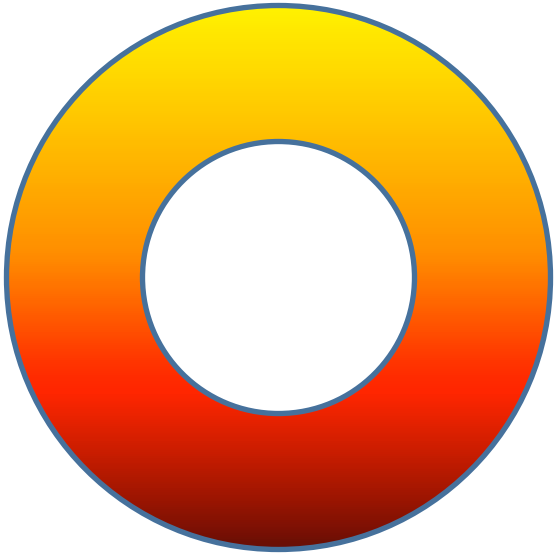
Translation	↔	**Rotation**	
x	↔	θ	angle of rotation (rads)
$v = \dfrac{\Delta x}{\Delta t}$	↔	$\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{v}{r} = \dfrac{2\pi}{\tau} = 2\pi f$	angular velocity (rad/s)
$a = \dfrac{\Delta v}{\Delta t}$	↔	$\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{a_{tan}}{r}$	angular acceleration (rad/s^2)
F	↔	$\tau = r\,F_{\perp}$	torque (N m)
M	↔	$I = \sum m\,r^2$	moment of inertia (kg m^2)
$F_{net} = M\,a$	↔	$\tau_{net} = I\,\alpha$	Newton's 2nd Law
$KE_{trans} = (1/2)M\,v^2$	↔	$KE_{rot} = (1/2)\,I\,\omega^2$	Kinetic Energy (joules J)

$$\Delta KE + \Delta PE = \text{constant} \quad \leftrightarrow \quad \Delta KE_{trans} + \Delta KE_{rot} + \Delta PE = \text{constant}$$

(Conservation of Mechanical Energy)

$$p_{tot} = \sum_i m_i v_i = \text{constant} \leftrightarrow \quad L_{tot} = \sum_i I_i \omega_i = \text{constant}$$

Rotational Kinetic Energy



Does this object have translational kinetic energy?

No, zero net translational velocity of the object.

However, there is motion of each piece of the object and thus there must be kinetic energy.

Each piece of the donut has a velocity $v = \omega\, r$.

$$KE = ½\, mv^2 = ½\, m\, (\omega\, r)^2$$

$$KE = ½\, I\, \omega^2$$ Rotational KE