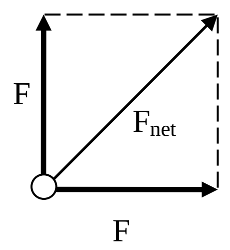
Newton's Laws: Forces and Motion

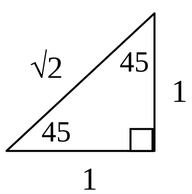
A force is a push or a pull. Force is a vector. It has a size and a direction. Forces add like vectors, not like scalars.

Example: Two forces, labeled F_1 and F_2, are both acting on the same object. The forces have the same magnitude $|\vec{F}_1| = |\vec{F}_2| = F$ and are 90^o apart in direction:



$$\vec{F}_{net} = \vec{F}_{total} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2$$

($\sum \vec{F}$ means "sum of all the forces on the object")



$$\Rightarrow F_{net} = \sqrt{2}\, F \qquad \text{(NOT 2F)}$$

Isaac Newton (British, 1642-1727) first figured out the precise relationship between forces and motion. " **...** for in those days I was in my prime of age for invention."

• Newton's First Law (NI): If the net force acting on an object is zero, then it has constant velocity. $\vec{F}_{net} = 0 \Leftrightarrow \vec{v} = \text{constant}$

• Newton's Second Law (NII):

$$\boxed{\vec{F}_{net} = m\,\vec{a}}$$

(Notice! F_{net}, not F, in this equation. There may be many forces acting on an object, but there is only one *net* force.)

The net force on an object causes the object to accelerate (change its velocity).

Units of force: $[F] = [m][a] = kg \cdot m/s^2 = 1 \text{ newton} = 1 \text{ N}$
A force of 1N is about 0.22 pounds. A small plum weighs about 1 newton.

Things to notice about NII:

- If $F_{net} = 0$, then a = 0 and velocity = constant. (1^{st} Law).
- The vector $\vec{a}$ has the same direction as $\vec{F}_{net}$.
- Magnitude of acceleration *a* is proportional to 1/m (at constant F_{net}).
- $\vec{F}_{net} = \sum \vec{F} = m\vec{a}$ is a **vector** equation
  $\Rightarrow \sum F_x = m a_x \;,\; \sum F_y = m a_y$

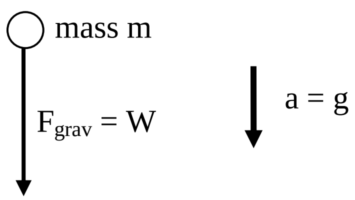
Before Newton, everyone thought (incorrectly!): "Force causes motion." WRONG!
(You can have motion without any force causing the motion. Example: glider on an air track.)

After Newton, "Force causes *changes* in motion." RIGHT!

DEFINITION: The WEIGHT of an object = the force of gravity on the object.

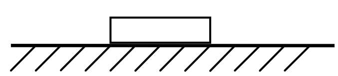
$W = mg$ Why?

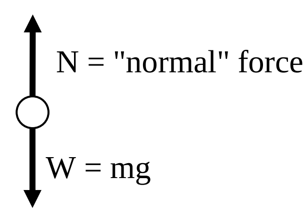
Recall this experimental fact: when object is in free-fall, meaning $F_{net} = F_{gravity}$, then a = g.
So in this situation (free-fall), $F_{net} = ma \Rightarrow F_{grav} = mg$



How big a force is 1 N? If m = 1 kg, $W = F_{grav} = mg = (1\text{ kg})(9.8\text{ m/s}^2) = 9.8$ N. A kilogram mass has a weight of about 10 N, which is about 2.2 lbs. The pound (lb) is the English unit of force: 1 lb = 4.44 N.

Example:
Glider on an air track with v = constant OR book at rest on a table. In both cases,
$a = 0 \Leftrightarrow F_{net} = 0$
N = force exerted on book by table
OR force on glider by air track





"normal" means "perpendicular"

Since $F_{net} = 0 \Rightarrow N = mg \Leftrightarrow$ upward force on book exactly cancels downward force on book. How does the table know to exert an upward force that is exactly the right size to cancel the weight of the book? Answer: the table is not perfectly rigid; it is flexible, it is slightly springy. The table is slightly compressed by the weight of the book, and it pushes back on the book, exactly like a compressed spring. The heavier the book, the more the table-spring compresses, and the more it pushes upward on the book.

Rules for drawing "Free-body diagram" or force diagram :
0) Draw a blob representing the object.
1) Draw only the forces acting **on** the object (not the forces which the object exerts on others).
2) Indicate strength and direction of forces on the object by drawing arrows coming out of the object.
3) Use symbols to represent the magnitudes of the forces (Don't worry about +/- signs. The forces arrows show the directions of the forces already.)

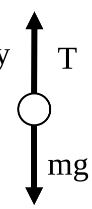
Example:
Object with mass m, hanging by a cord.
Magnitude of force exerted by the cord = tension T

Three different situations:

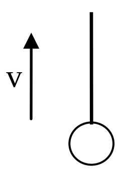
I. Hanging stationary: velocity v = 0 $\Rightarrow$

$a = 0 \Rightarrow F_{net} = 0 \Rightarrow$ $\mathbf{T = mg}$

free-body diagram:



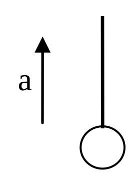
II. Pulled upward with constant v.



v = constant $\Rightarrow$ $a = 0 \Rightarrow F_{net} = 0 \Rightarrow$ $\mathbf{T = mg}$ **(again!)**

same free-body diagram as in case I

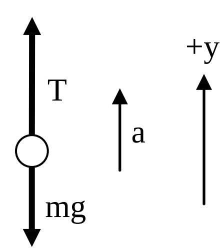
III. Object is **accelerated** upward (may be <u>moving</u> upward OR downward!)



Question: What is the magnitude of the tension T in the cord?

To analyze this situation:
Step 1: Draw free-body diagram showing forces (show direction of acceleration off to one side of diagram.)

Step 2: Choose a coordinate system with a (+)direction (almost always best to choose the direction of the acceleration as the +direction)

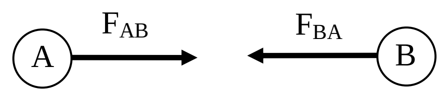


Step 3: Write down equations $\sum F_x = m a_x$, $\sum F_y = m a_y$

In this example, just the y-equation is needed:
$+T - mg = ma \quad \Rightarrow \quad T = mg + ma = m\,(a+g)$

• Newton's Third Law (NIII): If body A exerts a force on body B ($= \vec{F}_{BA} = \vec{F}_{\text{on B by A}}$), then B exerts an equal and opposite force on A ($= \vec{F}_{AB} = \vec{F}_{\text{on A by B}}$).

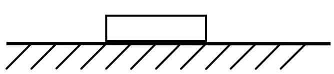
$$\underbrace{\boxed{\vec{F}_{BA} = -\vec{F}_{AB}}}_{\text{2 forces on 2 different objects}}$$

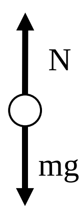


Forces always act between *pairs* of bodies. A force on one body is always being caused by a second body. NIII says that the force from one body on the other is always accompanied by a force from the other on the first, and the two forces of this "action-reaction" pair are always equal in magnitude and opposite in direction. Notice that the 2 forces of the action-reaction pair act on *different* bodies.

Example of equal and opposite forces that are NOT the action-reaction pair of NIII:

Book on a table:





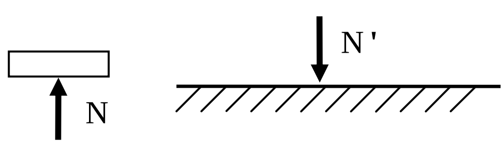
N = mg
(these are NOT the 2 forces of NIII, since both act on the same object.)

So which forces make up the action-reaction pairs in this situation ?

$\vec{N}$ = force on book due to table

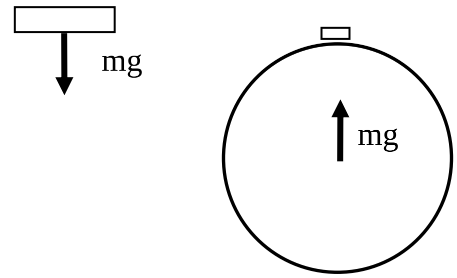
$\vec{N}'$ = force on table due to book

$\vec{N} = -\vec{N}'$, says NIII



$m\vec{g}$ = force on book due to planet earth (gravity)

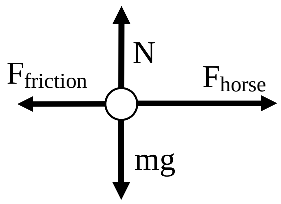
Book exerts same size force on whole earth upward.



The Story of the Clever Horse and the Cart: Horse says, "No matter how hard I pull on the cart, it will always pull back with an equal-sized force (according to NIII). Therefore, I cannot move the cart so I won't even try!"

Farmer's answer: "Your 1st sentence is correct, but irrelevant. Whether or not the cart moves depends only on the forces acting on the cart. The forces on you, the horse, are irrelevant."

Free-body diagram of forces **on** cart:



If $F_{horse} > F_{friction}$, then the cart will move. The forces on the horse are not relevant.

Where are we so far?

We have introduced Newton's 3 **Laws.** Laws are statements which are true always. There are no derivations of Newton's Laws; in particular there is no derivation of $\mathbf{F}_{net} = m\,\mathbf{a}$. These laws are taken as assumptions or axioms of the theory of Newtonian mechanics. We believe these laws are correct because all of the consequences of these laws are found to agree with experiment.

Remember, the philosophy of science is this: "The final test of the validity of any idea is experiment."

In Physics, the only statements that are true always are definitions (like $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$) and laws. You should memorize definitions and laws.

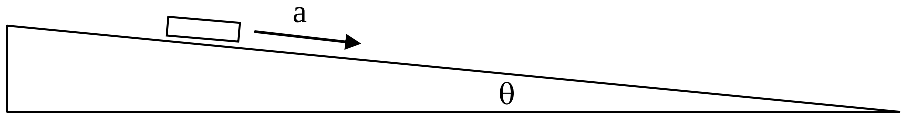
An equation like $x = x_o + v_o t + \frac{1}{2} a t^2$ is not a law, because it is not always true. (This particular equation is only true when acceleration a = constant.)

One loose end: We have stated that the acceleration due to gravity (near the surface of the earth) is always downwards with magnitude 9.8 m/s^2. So far, we have taken this as an experimental fact. But we have not yet explained *why* the acceleration of gravity is constant and has this value. That will come later.

Example: Drawing a free-body diagram and applying $\vec{F}_{net} = m\vec{a}$

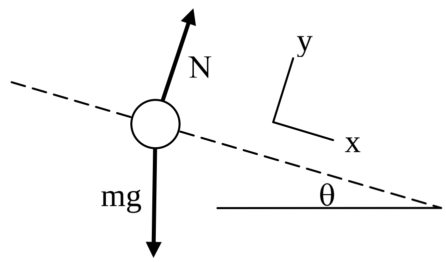
A glider on an air track tilted an angle θ to the horizontal. <u>No friction</u>.

- What is the magnitude of acceleration: a = ?
- What is the magnitude of the normal force: N = ?



Step 1: Draw free-body diagram.

(only 2 forces here: normal force N and weight W = mg)

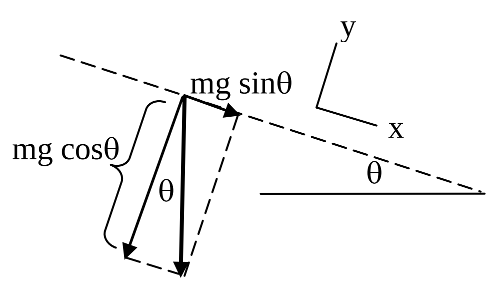


Step 2: Choose coordinate system. Here I chose tilted xy coordinates because I want the +x direction to be along the direction of the acceleration.

Step 3: Write down equations $\sum F_x = m\,a_x$, $\sum F_y = m\,a_y$

Notice that :
x-component of weight = +mg sin θ
y-component of weight = –mg cos θ (minus sign!)

$a_x = a$, $a_y = 0$ (acceleration is along +x-axis)



$\sum F_y = m\,a_y \Rightarrow \quad +N - mg\cos\theta = 0$

$\boxed{N = mg\cos\theta}$

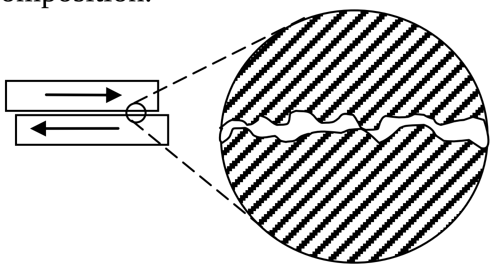
$\sum F_x = m\,a_x \Rightarrow \quad +mg\sin\theta = m\,a$, m's cancel so $\boxed{a = g\sin\theta}$

Notice that as θ → 0 (track becomes horizontal), cos θ → 1, sin θ → 0, N → mg, a → 0, as expected.

NOTICE that N is NOT equal to mg. The equation N = mg is only true in a very special situation: when the mass m is NOT accelerating and is sitting on a horizontal surface.

Dubson's Law: In general, N ≠ mg The statement N = mg is NOT a law. It is only true in special cases.

Friction ... is very useful! We need friction to walk. Friction is not well understood. The amount of friction between two surfaces depends on difficult-to-characterize details of the surfaces, including microscopic roughness, cleanliness, and chemical composition.

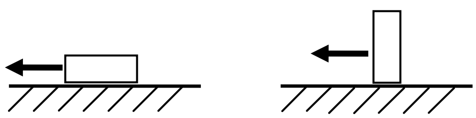


-- friction involves tearing, wear between microscopically rough surfaces.

If two metal surfaces are <u>atomically</u> smooth and clean (almost impossible to achieve), they will <u>bond</u> on contact = "cold weld".

Empirical observations about friction:

• the magnitude of the force of friction f between 2 surfaces is proportional to the normal force F_N, not the area of contact, ($f \propto F_N$).



Pull a block of mass m along a surface. Regardless of orientation, you get the same normal force (N = mg), and you get the same frictional force f.

Why not $f \propto$ area of contact? More area $\Rightarrow$ less weight per area.

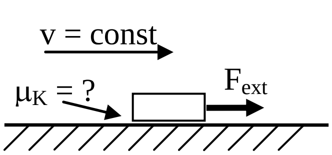
• static friction is different than sliding friction (also called kinetic friction). The maximum static friction usually larger than kinetic friction.

Kinetic Friction (also called sliding friction)

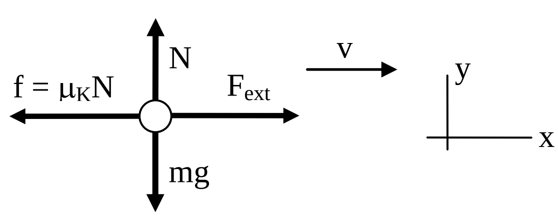
$$f = \mu_K N$$ (Not a law, just an empirical observation – usually, but not always, true)

μ_K = coefficient of kinetic friction = dimensionless number $0 < \mu_K < 1$ (usually)

Example: A block of mass m is being pushed along a rough horizontal table. One maintains a constant velocity v with a horizontal external force of magnitude F_{ext}. What is μ_K ?



Free-body diagram: direction of frictional force f is always <u>opposite</u> to the motion:



velocity = constant $\Rightarrow$ $a = 0$ $\Rightarrow$ $F_{net} = 0$ $\Rightarrow$

In y-direction: $N = mg$ In x-direction: $F_{ext} = \mu_K N$ So, $F_{ext} = \mu_K mg$, or...

$$\boxed{\mu_K = \frac{F_{ext}}{mg}}$$

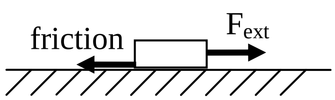
Static Friction

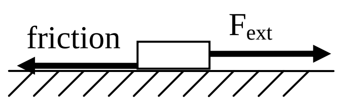
$f_{max} = \mu_S N$ (the maximum magnitude of the force of static friction is μ_S N)

μ_S = coefficient of static friction = dimensionless number $\mu_S > 0$

Usually, $\mu_S > \mu_K$ (maximum static friction is greater than kinetic friction)

Consider a book sitting on a table. You pull on a book with a small force to the right F_{ext}, but it doesn't move. There must be a frictional force to the left (otherwise the book would move).





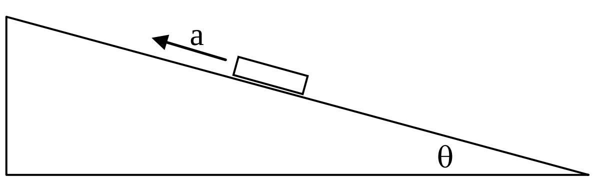
If you increase the external force and the book still does not move, the frictional force must be getter bigger to match.

If you make the external force big enough, the book will suddenly start to move. Just before the book moved, the static friction was at its maximum value. So the magnitude of the static friction force can be anything between zero and a maximum value, given by $f_{max} = \mu_S N$. The book will remain stationary until $F_{ext} > f_{max} = \mu_S N$. Then the book will start to slide.

Usually, $\mu_S > \mu_K \Rightarrow$ large force is needed to start an object sliding, but then a smaller force is needed to keep it sliding.

There is no good theory of friction $\Rightarrow$ μ's cannot be computed; instead, they are determined experimentally.

Example: Friction on an inclined plane

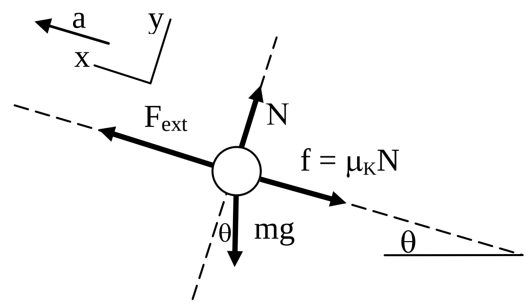


A mass m on an incline at angle θ, with sliding friction coefficient μ_K. What size external force F_{ext} is required to maintain an acceleration of magnitude *a* **up** the incline?

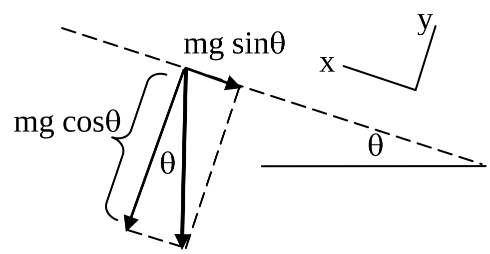
Step 1: Free-body diagram.

Step 2: Choose coordinate system. (Make direction of acceleration = + direction)

Step 3: $\sum F_x = m\,a_x$, $\sum F_y = m\,a_y$



Notice:
x-component of weight = –mg sinθ
y-component of weight = –mg cosθ



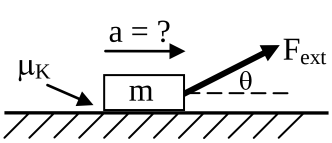
Y: $a_y = 0 \Rightarrow +N - mg\cos\theta = 0, \quad N = mg\cos\theta$

X: $a_x = a \;\Rightarrow\; +F_{ext} - \mu_K N - mg\sin\theta = m\,a$

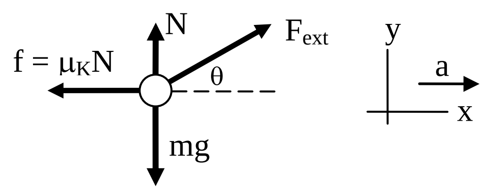
Combine X and Y equations:
$+F_{ext} - \mu_K\, mg\cos\theta - mg\sin\theta = m\,a \;\Rightarrow\; \boxed{F_{ext} = ma + \mu_K\, mg\cos\theta + mg\sin\theta}$

(Whew! a hard one.)

Example: A mass m on a flat table, with sliding friction coefficient μ_K, is pulled along the table by a force F_{ext} at angle θ. What is the (magnitude of the) acceleration *a*?



Steps 1,2:



Step 3:
Y-motion: $\sum F_y = m\,a_y,\; a_y = 0 \;\Rightarrow\; \sum F_y = 0 \Rightarrow\; \sum(\text{forces up}) = \sum(\text{forces down}) \;\Rightarrow$

$N + F_{ext}\sin\theta = mg$, $N = mg - F_{ext}\sin\theta$ (Do you understand the sinθ here?)

X-motion: $\sum F_x = m\,a_x,\; a_x = a \;\Rightarrow\; F_{ext}\cos\theta - \mu_K N = m\,a$

Now combine X and Y results:

$F_{ext}\cos\theta - \mu_K(mg - F_{ext}\sin\theta) = m\,a \;\Rightarrow\; \boxed{a = \underbrace{\frac{F_{ext}}{m}\left(\cos\theta + \mu_K\sin\theta\right) - \mu_K g}_{\text{notice that all terms have units of [a]}}}$

