Electromagnetic Waves

Last semester, we studied Classical Mechanics. The fundamental laws (axioms) of Classical Mechanics are called Newton's Laws, and we were able to write them all down and understand them in their full, complete form.

This semester, we are studying a subject called Classical Electromagnetism. There are four fundamental laws of electromagnetism, called Maxwell's Equations (after the Scottish physicist James Clerk Maxwell). In this course, Faraday's Law is the only one of Maxwell's Equations which we shall actually write down in complete form. The other 3 laws are a bit too mathematically complex to write down in full detail, but we have seen simplified versions of these laws. In words, Maxwell's 4 equations are:

(1) Electric fields are created by charges. (The full form of this equation is called Gauss's Law. We have seen this equation in a simplified form: $|\vec{E}|_{\text{due to Q}} = k\dfrac{|Q|}{r^2}$.)

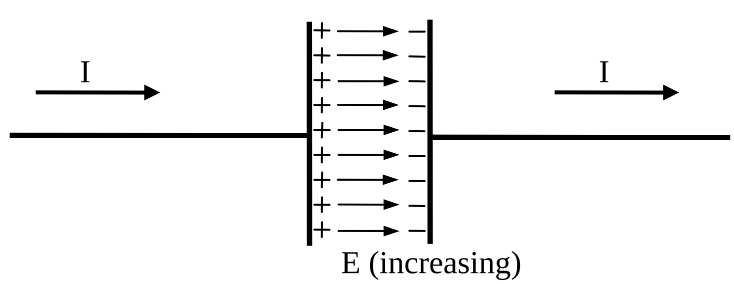
(2) Magnetic fields are created by currents. (This equation is called Ampere's Law, and we have seen this equation is the simplified form $|\vec{B}|_{\text{due to straight wire}} = \dfrac{\mu_0 I}{2\pi r}$.)

(3) Magnetic field lines always form closed loops. (This equation has no standard name.)

(4) Electric fields are created by *changing* magnetic fields. (Faraday's Law).

Actually, all four of these laws had been discovered experimentally before Maxwell started his research in the 1850's. So why do we call them Maxwell's Equations?

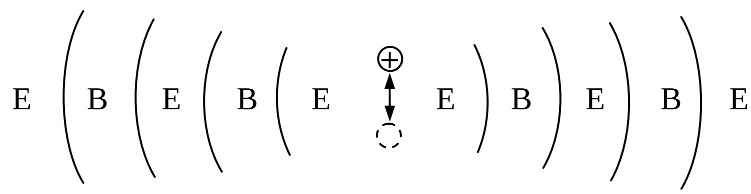
Maxwell made a change to Ampere's Law, equation (2). Maxwell argued, on theoretical grounds, that Ampere's Law must be incomplete; it needs a modification. Maxwell's noticed that there are situations in which a electric current inevitably involves a *changing* electric field. For instance, if a capacitor is being charged up by a steady current, then there must be an increasing electric field between the plates, due to the increasing charge brought to the plates by the steady current. Maxwell's showed that, in order to properly describe such situations, Ampere's Law must be modified so



that it reads "Magnetic fields are created by currents *and* by changing electric fields." This new form of Ampere's Law (now called the Ampere-Maxwell Law) appealed to Maxwell's sense of aesthetics. There was now a pleasing symmetry in the equations:

- changing B-fields create E-fields (Faraday's Law)
- changing E-fields create B-fields (Ampere-Maxwell Law)

Maxwell realized that because of this symmetry, the equations predicted a peculiar kind of self-sustaining interaction between E and B fields. Maxwell thought: Suppose you have a charge q and you *shake* it, back and forth. The q creates an E-field, but when you shake the charge, you are changing the E-field in the space around it. This changing E-field creates a B-field. But now you just created a B-field where there was none before, so you have a changing B-field (it changed from zero to non-zero). This changing B-field will create an E-field, and that newly created E-field will create a B-field, which will create an E, which will create a B, which will … (the process will go on, forever). Maxwell showed that the equations predicted the existence of an *electromagnetic wave* which travels outward from the shaking charge:



Maxwell computed the speed of this strange, new electromagnetic wave and found that the speed was given by a simple formula: $\text{speed } v = c = \dfrac{1}{\sqrt{\varepsilon_o \mu_o}} = 3.0\times10^8 \text{ m/s}$.

This number is the same as the speed of light! Maxwell had shown that light was an electromagnetic wave! Before Maxwell, scientists had no clear idea what light is. This was a great synthesis, a bringing together of previously separate fields of physics: electricity, magnetism, and optics. Before Maxwell, no one knew what light was. It was known that light was some kind of wave (we will see the evidence for this later), but no one knew what kind of wave it was. Maxwell figured it out.

Light is an electromagnetic wave which is created by accelerating electric charge.

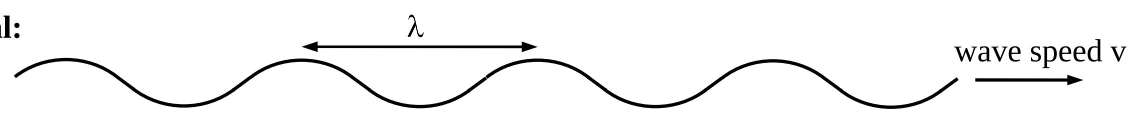
Review of Waves:

Wave vs. particle: When a particle moves from location A to B, mass is transported. When a wave travels from A to B, no mass is transported. A wave carries energy, momentum, and information, but not matter.

Before electromagnetic waves were discovered, all the kinds of waves that were known required a *medium* to carry the wave. Water waves, sound waves, and waves on a string are disturbances in a medium (water, air, string). But EM waves can travel in vacuum — no medium necessary. This was extremely surprising to 19th century physicists. So surprising, in fact, that they could not believe it. Rather than believe the evidence of experiment, scientists clung to their notions about how all waves should behave. They invented (made up) a medium to carry the EM wave, and gave it the (appropriately fairy-tale-like) name of the *luminiferous aether*. But there is no aether. The EM wave is unlike any other kind of wave. The EM wave is its own medium. It rolls out it own red carpet as it goes.

Traveling Waves can be categorized as:

sinusoidal:



or

impulse:

Traveling waves can also be categorized as:

transverse: displacement of the medium is perpendicular (transverse) to the direction of the wave velocity (like a wave on water or a string or drumhead).

or

longitudinal: displacement of the medium is parallel to the direction of the wave velocity (like sound wave or a slinky that has been push-pulled).

Longitudinal Wave:

Sinusoidal waves have a wavelength (λ) and a frequency (f).

period T = time for 1 wavelength to pass by.

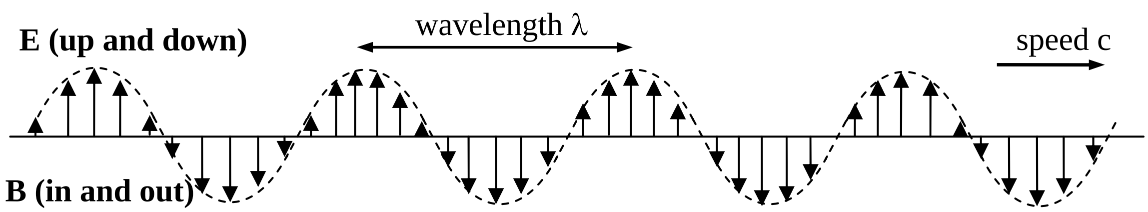
frequency f = 1 / T = rate at which wavelength's pass by.

Example: T = 0.1 s $\Rightarrow$ f = 1 / 0.1 s = 10 s^{-1} = 10 Hz (meaning 10 wavelengths go by each sec)

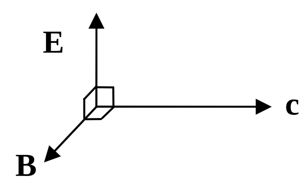
Wave speed is $v = \dfrac{\text{distance}}{\text{time}} = \dfrac{1 \text{ wavelength}}{\text{time for } 1\ \lambda \text{ to go by}} = \dfrac{\lambda}{T}$ $\qquad v = \dfrac{\lambda}{T} = \lambda f$

For light waves, speed v = c, this is written $\boxed{c = \lambda f}$

EM waves are transverse waves: the E- and B-field vectors are both perpendicular to the direction of the wave. Drawing an EM wave in space is quite difficult; the E and B-fields are everywhere and intimately mixed. The figure here shows the E-field along a particular line, at a moment in time.



The E and B-field are perpendicular to each other like so:



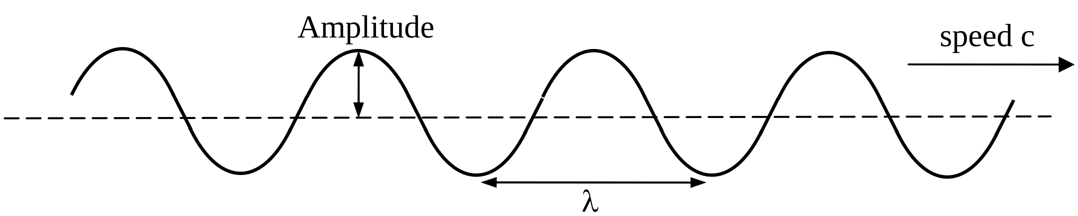
Electromagnetic radiation (light) can have *any* wavelength. But our eyes are sensitive only to a narrow range of wavelengths between 400 nm and 700 nm. Different wavelengths in this range of *visible light* correspond to different colors. Wavelength = 700 nm light appears red to us, 400 nm light appears violet, and the wavelengths in between correspond to all the colors of the rainbow (ROYGBIV). All wavelengths outside this narrow band are invisible to human eyes.

All EM radiation is caused by shaking (accelerating) electric charge. The more rapidly the charge is shaken (the higher the frequency of the shake), the shorter the wavelength of the light, since $\lambda = \dfrac{c}{f}$. Now we can understand why all things glow (give off light) when they get hot. When something is very hot, its atoms are jiggling furiously. Atoms are made of charges (electrons and protons), and the jiggling charges emit EM radiation.

Different wavelength ranges are given names:

Wavelength λ	**Name**	**Use/occurrence**
$< \approx 0.01$ nm	Gamma-rays	Radioactivity
≈ 0.01 nm $\rightarrow \approx$ nm	X-ray	medical
$\approx$ nm $\rightarrow$ 400 nm	Ultraviolet(UV)	Sunburns, "black" lights
400nm $\rightarrow$ 700 nm	Visible	Human seeing
700nm $\rightarrow \approx$ 1mm	Infrared (IR)	"Heat rays"
$\approx$ cm	microwave	Communications, microwave ovens
$\approx$ m $\rightarrow$ km $\rightarrow \infty$	radio	Radio, TV

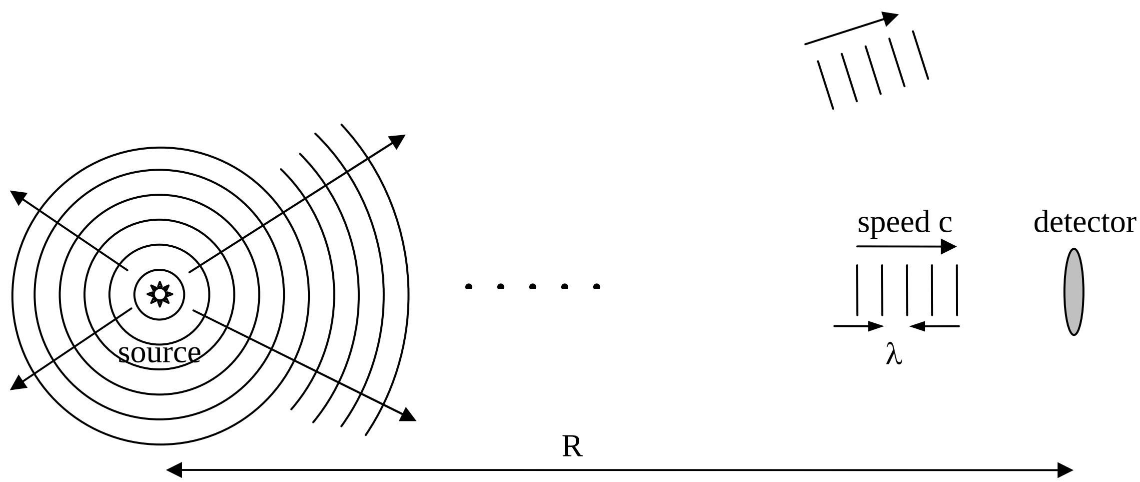
The *amplitude* of the wave, E_{max}, is a measure of the brightness or *intensity* I of the wave.



An electromagnetic wave carries energy. You can feel the energy hitting your face when you face the sun. The intensity I is defined as the energy per time *per area* impinging on a surface:

$$\text{intensity} = \frac{\text{power}}{\text{area}}, \qquad \boxed{I = \frac{P}{A}} \qquad \text{units}[I] = W/m^2$$

Consider a small source of monochromatic (single-wavelength) light, emitting energy at a rate P (in J/s). If the source sends light out in all directions uniformly, then the *wavefronts* are expanding spherical shells. A small detector (like an eyeball), very far from the source, will sense a small portion of the wavefronts. Over the small region that is detected, the wavefronts are nearly flat planes. Such a beam of light is called a *plane wave*.



Example of intensity calculation: Suppose a small source of EM radiation emits power P_o isotropically (uniformly in all directions). What is the intensity of the light at a distance R from the source? Answer: Consider the energy U_o emitted from the source during a very brief time interval t_o. When that energy is carried by the EM wave a distance R from the source, the energy is spread out uniformly over a sphere of area $A = 4\pi R^2$, and the energy per area is $U_o / (4\pi R^2)$. The energy per time per area, the intensity, is then $I = \dfrac{U_o / t_o}{A} = \dfrac{P_o}{4\pi R^2}$. Notice that the intensity falls as 1 / (distance)^2. The more distant a light source, the dimmer it appears.

Example of power detected from a distant source: What is the power entering an observer's eye from a 100-W tungsten-filament lightbulb a distance R = 20 m away? Only about 3% of the power from an incandescent light bulb comes out as visible light (the rest is heat). The diameter of a human eye pupil is about 2 mm.

Answer: The visible light power from the bulb is P_o = 3 W. At R = 20 m, the intensity is

$I = \dfrac{P_o}{4\pi R^2} = \dfrac{3\,\mathrm{W}}{4\pi (20\,\mathrm{m})^2} = 6.0\times10^{-4}\ \mathrm{W/m^2}$. The power entering a detector of area A is

$$(\text{power detected}) = \frac{\text{power}}{\text{area}}\cdot(\text{detector area}) = \text{intensity}\cdot(\text{detector area}) = \frac{P_o}{4\pi R^2}\cdot A\,.$$

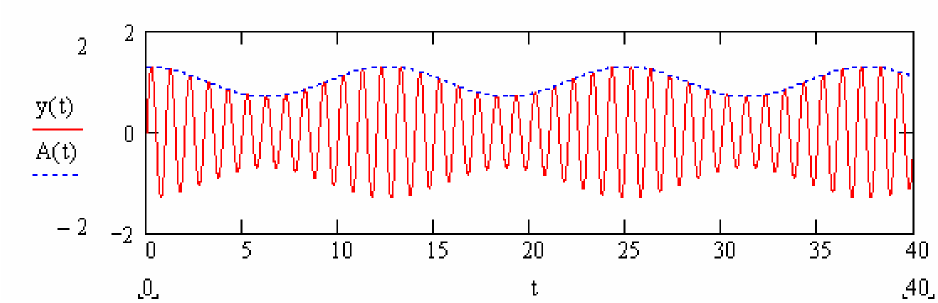
In this case, A = area of human iris = $\pi r^2 \approx 3\cdot(1\mathrm{mm})^2 = 3\times10^{-6}\ \mathrm{m}^2$. So the power detected is $(6\times10^{-4}\ \mathrm{W/m^2})(3\times10^{-6}\ \mathrm{m^2}) = 1.8\times10^{-9}$ W (not much power – the eye is extremely sensitive!)

TV transmissions are in the "radio" range of wavelengths: λ_{TV} = 1 to 5 m, frequency f ≈ 10^8 Hz = 100 MHz. On TV, different channels corresponds to different frequencies. For instance, channel 6 is allotted the frequency range f = 82 – 86 MHz (wavelength range λ = 3.4 – 3.7 m)

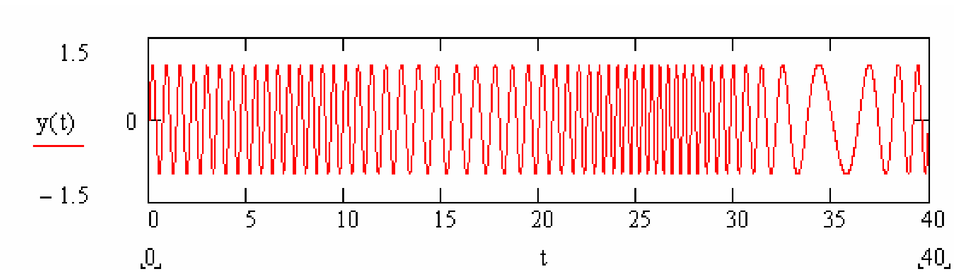
The signal (audio and video information) is carried by a range of frequencies (Δf = "bandwidth") centered on a "carrier frequency" f_o.

In audio-only radio transmissions, the signal's information is encoded either as

Amplitude Modulation (AM)



or Frequency Modulation (FM)



Analog television signals are always sent as FM.

Digital TV signals are sent in an entirely different format: the picture is encoded as a series of numbers (0's and 1's).