Special Relativity

After Newton and his mechanics, after Maxwell and his E&M, came three great revolutionary new theories of the 20^{th} century. Albert Einstein was responsible for 2 and one-quarter of these theories.

1) Special Relativity, a theory of space and time (Einstein 1905)
2) General Relativity, a theory of gravity (Einstein, 1916)
3) Quantum Mechanics, a theory of the behavior of atoms (Planck, Einstein, Bohr, Heisenberg, Schrodinger, Born, Dirac, Pauli, …, 1900-1928)

Comment about the word "theory": In science, the word "theory" means "a self-consistent model which is consistent with all known experimental facts and which makes specific predictions which can be tested by further experiment." This is a very different meaning than the common use of the word : In street talk, the word "theory" seems to mean "conjecture" or "some random notion", as in "it's just a theory". This is exactly the opposite of the meaning of the word in science: In science, a "theory" is the most complete, reliable form of knowledge (about the physical universe) that we possess.

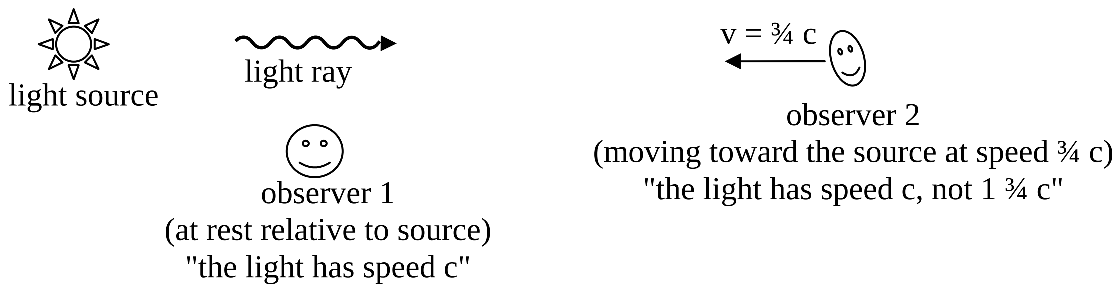
Special Relativity is based on 2 postulates (axioms)

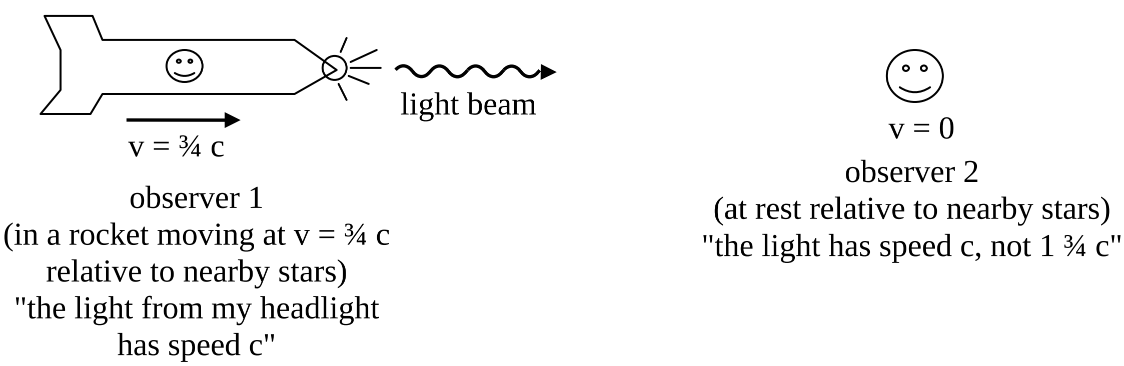
I. All the laws of physics are in the same in every inertial frame of reference. Inertial frame = one moving at constant velocity = one which is not accelerating, not rotating.

II. (the weird one). The speed of light is the same for all observers regardless of the motion of the observer or the motion of the source of the light.

Postulate I says that there is no way to determine the velocity of your inertial frame, except by comparing your motion to the motion of other bodies. Relative velocity is meaningful; absolute velocity has no meaning. Example: two space ships drift by each other in intergalactic space. Who's to say which one is moving and which one is at rest? The only meaningful statement that can be made is that they are moving relative to each other.

Postulate II just seems crazy. It says that $c = 3\times10^8$ m/s for everyone, no matter what.





How can all these different observers, moving relative to each other, all measure the same speed for light? Something has to give. What gives is our "common-sense" notions of time and space. Special relativity says that the different observers do not agree on how fast time is passing and they also do not agree on how far apart things are. Since speed = distance/time, measurement of speed always involves measurements of distance and time.

The laws of electricity & magnetism (Maxwell's equations) led Einstein to these postulates.

Maxwell's equations predict that the speed of light is $c = \dfrac{1}{\sqrt{\mu_o \varepsilon_o}} = 3\times10^8\ \mathrm{m/s}$, always!

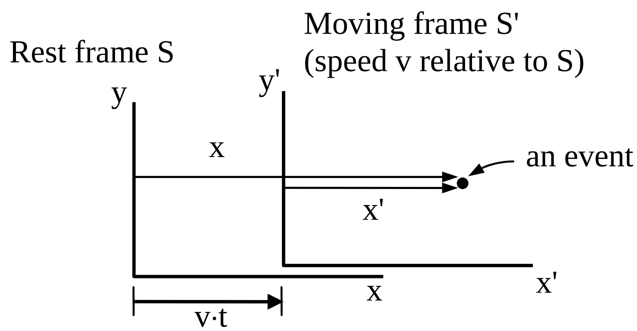
Maxwell's equations give no mention of the motion of the source or observer. Pre-relativity physicists interpreted this situation (wrongly!) like this: Suppose I send out a light ray, at speed c, and then I <u>chase</u> the ray at nearly the speed c. "Obviously", in my moving reference frame, I will then observe "slow light". In this moving frame, in which there is slow light ($v < c$), Maxwell's Equations can't be valid, since they predict that light has speed $v = c$. There must be a special reference frame, a "still space" frame, in which Maxwell's equations are valid and light has speed c. (Remember: this is all <u>wrong</u>. I'm just telling you how pre-relativity physicists

thought.) Light in space must be like sound in air. The speed of sound is 345 m/s *relative to the still air*; likewise it must be that the speed of light is c, *relative to still space*.
In the 19th century, physicists imagined that space was filled with a mysterious substance, the "aether" (or "ether"), through which light traveled like sound through air.

There were two serious problems with this old view:
1) It contradicted experiment. No slow light was ever observed. Two scientists named Michelson and Morley built a device designed to detect changes in the speed of light due to the motion of the earth around the sun, through the aether. The Michelson-Morley experiment gave a null result; they found no changes in the speed of light. The MM expt has been called "the most important null result in the history of science".
2) There was no consistent theory of slow light. Theorists were unable to modify Maxwell's equations so that they would produce sensible results in frames moving relative to the aether.

The solution to this conundrum was provided by Albert Einstein, age 26, in 1905. Einstein said: There is nothing wrong with Maxwell's equations; they work in <u>all</u> inertial reference frames; so the prediction that the speed of light is always *c* is correct. The problem is with our usual concepts of space and time.

Definition: **event** = location + time = (x, y, z, t)
The same event can be observed from different reference frames:(x, y, z, t) vs. (x', y', z', t')



(origins coincide at t = 0)

"Classical" or Galilean transformation relates (x, y, t) to (x', y', t') :

$$\boxed{\begin{aligned} x' &= x - v\cdot t \\ y' &= y \\ t' &= t \end{aligned}}$$

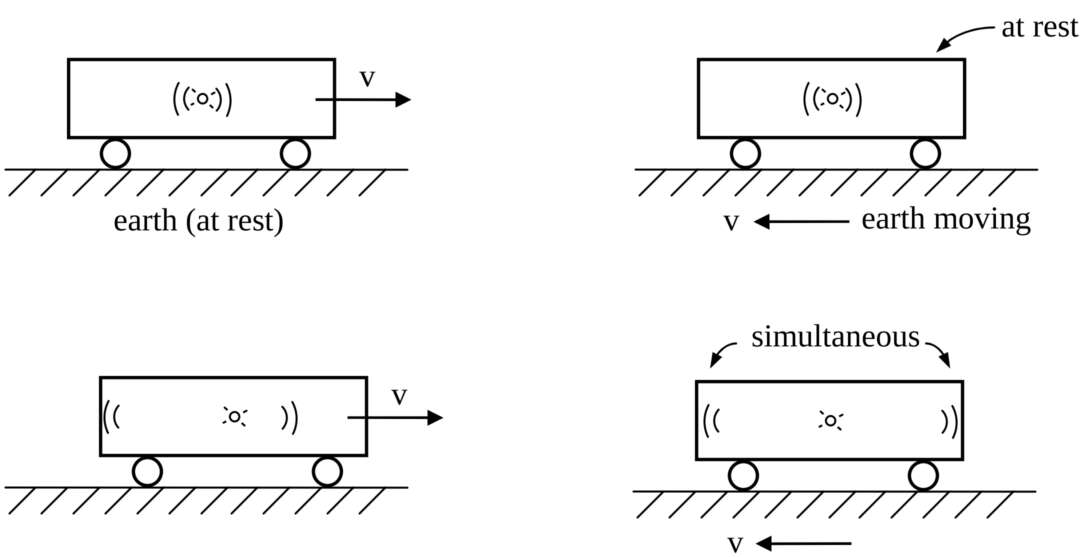
The Galilean transformation is correct in the limit v << c, but is wrong for large v ($t \neq t'$)

In the classical (non-relativistic) view, everyone experiences the same time ($t = t'$) regardless of their speed. But Postulate II requires that time is different in different frames of reference ($t \neq t'$). Einstein showed that the notion of simultaneity is not absolute. Events that are simultaneous in one reference frame are not simultaneous in other reference frames.

Definition: Two events are **simultaneous** if they occur at the same time but in different locations. Events simultaneous in S are not simultaneous in S' as the following thought experiment shows.

Suppose a flash bulb goes off in the center of a moving train car. In the frame of the train car (the "train frame"), light reaches the front and back of the car simultaneously. But in the frame of the earth, light reaches the back of the car first, because the train is moving forward to meet the light.

Earth Frame **Train Frame**



The two events (light hits back of car, light hits front of car) are simultaneous in the train frame, but are not simultaneous in the earth frame. The notion of absolute simultaneity is incompatible with the constancy of the speed of light (in both frames light moves forward and backward at speed c) . If different observers disagree about whether two events happen at the same time, it must be that their clocks are don't agree.

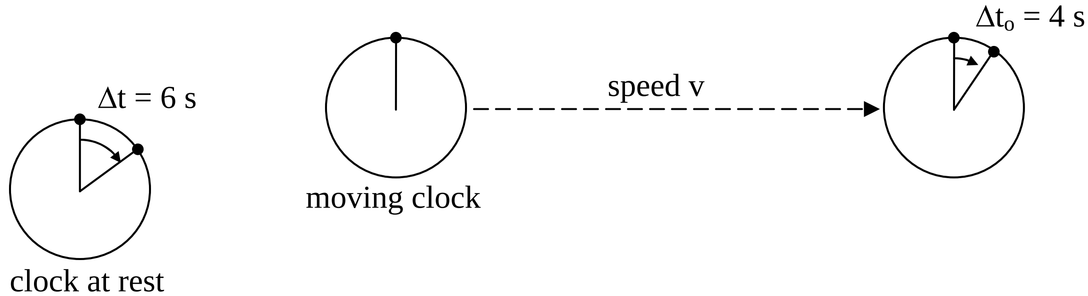
The postulates of Special Relativity (all laws the same in inertial reference frames and constancy of c) leads to several amazing predictions:

- Weird prediction 1: Moving clocks run slower
- Weird prediction 2: Moving meter sticks are shorter
- Weird prediction 3: Nothing can go faster than the speed of light. Nothing!
- Weird prediction 4: Mass m can be converted into energy E, and energy E can be converted into mass m according to $E = mc^2$.

All of these predictions have been exceedingly well-verified by experiment. Special Relativity is correct. (Sorry, Star Trek fans: warp drive is fiction.)

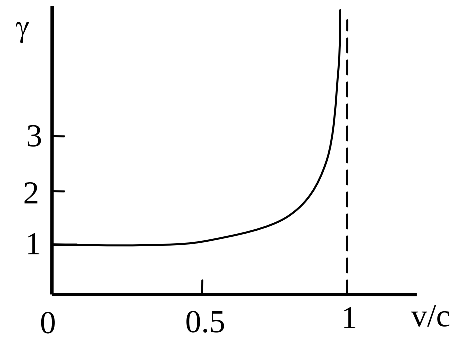
Time Dilation: "Moving clocks run slower". If a clock is moving at speed v, relative to a "rest frame", and the hands on the clock record that some time interval Δt_o , say 4 seconds, has passed, then the amount of time Δt that has elapsed in the rest frame will be more than 4 seconds, say 6 seconds. The moving clock will be running behind, that is, slow (since it only reads 4 s, when 6 s have passed for observers on the ground.) The ratio of the elapsed times is given by the "gamma factor" (greek letter gamma, γ)

$$\Delta t = \gamma\, \Delta t_0 , \text{ where } \gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} ,$$



(The proof of this is given in the appendix). Notice that if v is small compared to the speed of light, then γ is very close to 1. So, unless the moving clock is moving really fast (v close to c), then $\gamma \cong 1$ and $\Delta t \cong \Delta t_o$, meaning the clocks agree nearly exactly. Also notice that gamma is always greater than 1. The moving clock always runs slower, not faster.

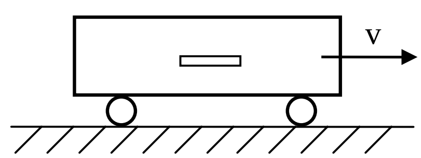
v	v/c	γ
30 km/s	0.0001	1.000000005
3000 km/s	0.01	1.0005
30,000 km/s	0.1	1.005
1.5×10^8 m/s	0.5	1.16
2.69×10^8 m/s	0.9	2.29
0.99c	0.99	7.09
0.99999c	0.99999	223



Even if the moving clock has a speed of 30 km/s (which is faster than the space shuttle), the moving clock is running slow by only a few parts in a billion. If the moving clock is moving at 90% of the speed of light, then it has slowed by more than a factor of 2. But if you are going at 99.999% of the speed of light, then your time has slowed by more than a factor of 200.

Lorentz Contraction: "Moving sticks are shorter". If a meter is stick is moving past you at speed v, how do you determine its length? Answer: You measure the location of the front and back ends <u>at the same time</u>, that is, simultaneously. But simultaneity is not absolute. Events that are simultaneous in one frame are not simultaneous in another. In this case, the two events are
Event 1: the location of the stick's front end at some particular instant of time, and
Event 2: the location of the stick's back end at some instant of time.

Suppose we have a meter stick on a moving train. In the Train frame, the stick is at rest and has length L_o = 1 m. L_o is called the "proper length"; it is the length measured in the frame in which the stick is at rest. In the Earth frame, the stick is moving with speed v and has a measured length $L \neq L_o$. In the Earth frame, in which the stick is moving, the length $L < L_o$.

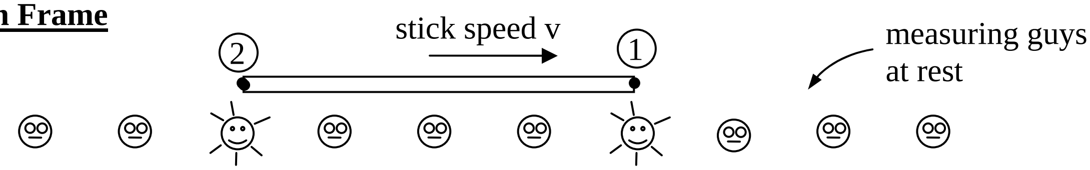


$$\boxed{L = \frac{L_0}{\gamma} = L_0\sqrt{1-(v/c)^2}}$$

What is going on here? This just seems nuts. If two guys, each holding a meter stick, run past each other, then each guy reports that his own stick is 1 meter long, but the other guy's stick is shorter. Each guy says the other guy's stick is shorter. How can they both be right??

To understand this seeming paradox, we must think carefully about how we can make measurements of moving objects. You cannot measure the positions of moving things with just one observer, because when the observer looks (with eyes) at far-away things, he sees them not as they are "now" but as they were when light left them. To measure the length of a moving stick, you need an array of observers, each with a known location, a clock, and a notepad to record events. Each observer only records events that happens right next to him, so as to avoid any light-propagation-delay effects.

Earth Frame

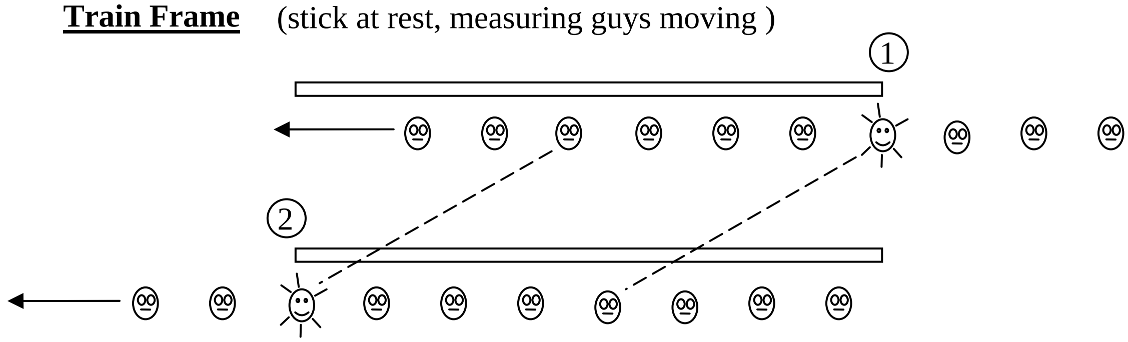


Observers all agree to record whether a stick end is at their position at a pre-arranged time t_o.

Event 1: position of front of stick at time t_o.
Event 2: position of back of stick at time t_o.

Now, the same two events as observed from the Train frame:

Train Frame (stick at rest, measuring guys moving)

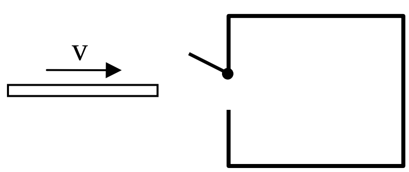


In the earth frame, events 1 and 2 are simultaneous. In the stick/train frame, events 1 and 2 are not simultaneous: 1 occurred first, then 2. Observers on the train say: "Of course you earth observers measured the stick to be short. You did it incorrectly. You measured the position of one end, then you waited a while, and then measured the other end.

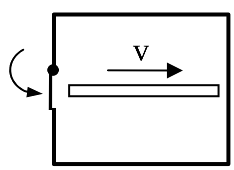
So is the moving stick "really" shorter? The *measured* length is really shorter. Whether you say the measured length is the same as the actual length is a matter of semantics.

The Barn and Ladder Paradox. A farmer wishes to get a long ladder into his barn, but the ladder is longer than the barn. It won't fit in. So the farmer shoots the ladder into the barn. If the ladder goes really fast, it will be Lorentz-contracted and will fit in the barn. Once in, the farmer can close the barn door, trapping the ladder. But wait! In the frame of the moving ladder, it is the barn which is Lorentz-contracted, and the ladder cannot possibly fit in. So does it fit in, or not?

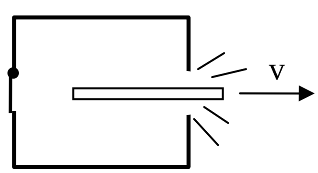
Barn Frame



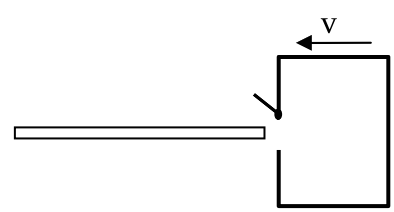
1. Barn door closes.
Ladder is trapped!



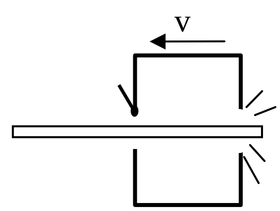
2. Ladder crashes thru back.



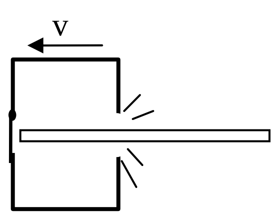
Ladder Frame



1. Ladder crashes thru back.



2. Barn door closes.



There is no paradox, because the same events occur in both frames, but the sequence of events is different.

Example: Interstellar travel. A rocket ship journeys to a star 50 light-years away, traveling at speed v = 0.999c. How long does the journey take according to Earth frame observers, and according to rocket frame observers?

1 light-year = distance light travels in one year.

$v = d / t$, $d = v \cdot t$, $\Rightarrow$ 1 light-yr = d = $c \cdot t$ = $(3\times10^8 \text{ m/s})(3.15\times10^7 \text{ m/s})$

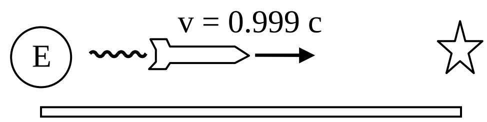
1 lt-yr = 9.5×10^{15} m $\leftarrow$ Never need this!!

Instead, write $\boxed{\text{1 lt-yr} = c \cdot \text{1yr}}$ = a distance

Imagine a long ruler held in space between the earth and the star:

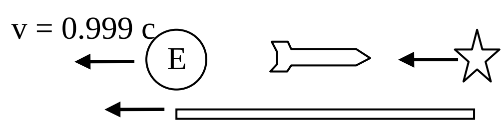
Earth frame: travel time =

$$t = \frac{d}{v} = \frac{50\, c\cdot \text{yr}}{0.999\, c} \underset{\text{(c's cancel)}}{=} \frac{50 \text{ yr}}{0.999} = 50.05 \text{ yr}$$



distance = L_o = 50 lt·yr

Rocket frame: Method I: In the rocket frame, the long ruler between the earth and the star is moving at speed v = 0.999 c , so it is Lorentz-contracted to length $L = L_o / \gamma$.



$L = L_o / \gamma$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.999)^2}} = 22.37$$

$L = L_o / \gamma$ = 50 lt ·yr / 22.37 = 2.24 lt·yr

Time for ruler to go by = travel time = $t = \frac{d}{v} = \frac{2.24\, c\cdot \text{yr}}{0.999\, c} \underset{\text{(c's cancel)}}{=} \frac{2.24 \text{ yr}}{0.999} \cong 2.24 \text{ yr}$

Method II. The clock on the ship (at rest in the Ship's frame) is a moving clock in the Earth's frame. $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{50.05 \text{ yr}}{22.37} = 2.24 \text{ yr}$

The Ship and the Earth both agree that their relative speed is v = 0.999 c. But according to the Earth frame, the journey takes more than 50 years, while the traveling astronauts record that the journey took only 2.24 years.

There are two practical problems involved in this kind of fast, interstellar travel: (1) To get a ship up to the speed of 0.999 c, it has to be accelerated from rest and humans cannot withstand large accelerations. (2) It takes a **gigantic** amount of energy to get a ship going that fast.

Energy is mass. Another consequence of the postulates of Special Relativity (here presented without proof) is the equivalence of mass and energy.

Definition: rest mass m_o = mass of particle when it is at rest.

S.R. predicts that the energy locked up in the rest mass m_o = "rest mass energy" =

$$\boxed{E_o = m_o c^2} \quad \text{[notice units are like KE = (1/2)mv}^2\text{]}$$

Physicists usually use units of eV (rather than joules) in relativity calculations.

Rest energy of a proton = $E_o = m_o c^2 = (1.67\times10^{-27}\text{ kg})(3\times10^8\text{ m/s})^2$ =

$1.50\times10^{-10}\text{ J}\times(1\text{ eV} / 1.6\times10^{-19}\text{ J}) = 9.39\times10^8\text{ eV} = 939\text{ MeV}$ $(1\text{MeV} = 10^6\text{ eV})$

Proton rest energy is m_oc^2 = 939 MeV. Proton rest mass is m_o = 939 MeV/c^2

(Physicist often get sloppy and say "proton mass is 939 MeV")

Electron rest energy = 0.511 MeV (about 2000 times less that that of proton)

For comparison, single chemical bond energy is about 0.1 – 2 eV.

Relativistic Kinetic Energy When a particle with rest mass m_o is moving with speed v, the total energy of the particle is

$$E_{tot} = \gamma m_o c^2 = \frac{m_o c^2}{\sqrt{1-(v/c)^2}} = \text{(rest mass energy) + KE} \qquad \text{(Proof not shown.)}$$

In relativity, KE $\neq (1/2)\,mv^2$ <u>except</u> at low speeds ($v << c$).

Relativistic KE is defined as KE $= E_{tot} - m_oc^2 = \gamma m_o c^2 - m_o c^2 = (\gamma - 1)\, m_o c^2$

$$\Rightarrow \quad KE_{rel} \equiv (\gamma - 1)\, m_o c^2$$

[One can show that in the limit $v << c$, $(\gamma - 1)\, m_oc^2 \rightarrow (1/2)\, mv^2$.]

Notice that in the limit $v \rightarrow c$, $\gamma \rightarrow \infty$, KE $\rightarrow \infty$. So S.R. predicts that it takes an infinite amount of energy to accelerate any mass up to the speed c!

$$\boxed{\text{c is a universal speed limit.}}$$

Example: Energy for interstellar travel. Have a spaceship with mass $m_{ship} = 10^5$ kg (1 human $= 10^2$ kg + food/ship/fuel = 1000 × m_{human}) How much energy is required to accelerate the ship up to 0.999c ($\gamma = 22.4$)?

$KE_{ship} = (\gamma - 1)\, m_o c^2 = (21.4)(10^5\ \text{kg})(3\times10^8\ \text{m/s})^2 = 1.9\times10^{23}$ J

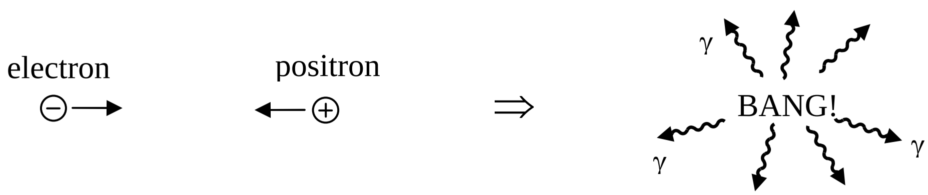
How much energy is that? US power consumption = $P \approx 3$ TW $= 3\times10^{12}$ W (10^{12} W = terrawatt)

Energy consumption in 1 year $= E = P \cdot t = (3\times10^{12}\ \text{W})(3.2\times10^7\ \text{s}) \approx 10^{20}$ J

KE_{ship} is the total US energy consumption for 2000 years!

The only way to store such a large amount of energy and then release it on demand is matter-antimatter annihilation. Antimatter is matter made of anti-particles. For every kind of particle (electron, proton, etc.), there is an anti-particle that is identical in every way, except that it has the <u>opposite</u> charge. anti-electron = positron = just like the electron, except q = +e.

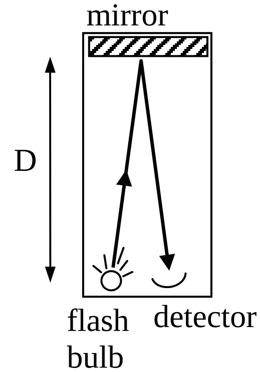
When an electron and a positron collide, they combine and spontaneously *annihilate* each other, converting 100% of their rest energy ($2mc^2$) into gamma-rays (short-wavelength EM radiation).



If we have a mass m of matter (protons and electrons) and another mass m of antimatter(anti-protons and positrons), then we have a total stored energy of $2mc^2$. How much anti-matter is needed to store $KE_{ship} = 1.9\times10^{23}$ J ?

$2mc^2 = KE_{ship} \Rightarrow \quad m = KE / (2c^2) = (1.9\times10^{23}\ \text{J}) / [(2)\,(3\times10^8\ \text{m/s})^2] \approx 1\times10^6$ kg

$m = 1\times10^6$ kg . This is 10 times as much as the mass of the ship itself. So you actually need much more fuel, because you are using fuel to accelerate fuel as well as the payload.



Appendix: Derivation of the time dilation formula $\Delta t = \gamma\, \Delta t_o$. A "light clock" is a device that can, in principle, keep perfect time. It consists of a box of height D with a flash bulb at the bottom and a mirror at the top. The clock's "tick" is a flash from the bulb, which travels upward, bounces off the mirror and returns to a detector next to the flash. When the detector receives the return flash, bulb flashes again, and the cycle repeats: tick, tick, tick...

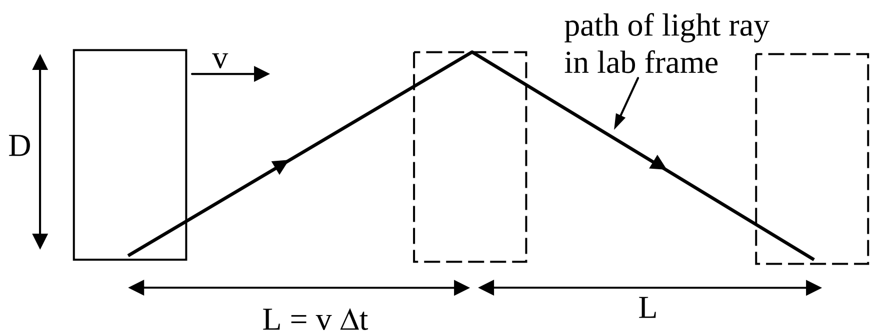
Consider the time interval between these two events:

Event 1: light emitted by bulb, and Event 2: light hits mirror

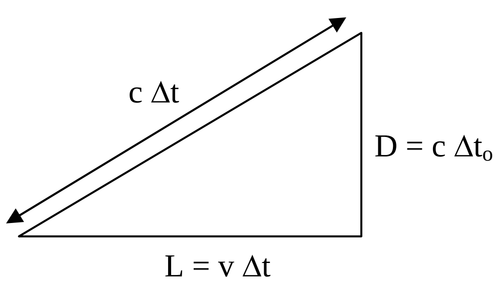
$$\text{speed} = \frac{\text{distance}}{\text{time}} \;\Rightarrow\; c = \frac{D}{\Delta t_0}\;, \quad \Delta t_0 = \frac{D}{c}$$

Δt_o = time interval in the frame in which the clock is at rest; the time internal according to the clock at rest is called the "proper time".

In the "Lab frame", this light clock is moving through the lab at speed v.



Δt = time interval in the lab frame. In the lab frame, the light went further, at the <u>same</u> <u>speed</u> c, so it must have taken longer! $\Delta t > \Delta t_o$.



$$(c\,\Delta t)^2 = (v\,\Delta t)^2 + (c\,\Delta t_0)^2$$

$$(\Delta t)^2\,(c^2 - v^2) = c^2\,(\Delta t_0)^2$$

$$\Delta t = \Delta t_0 \sqrt{\frac{c^2}{(c^2 - v^2)}} = \Delta t_0 \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$