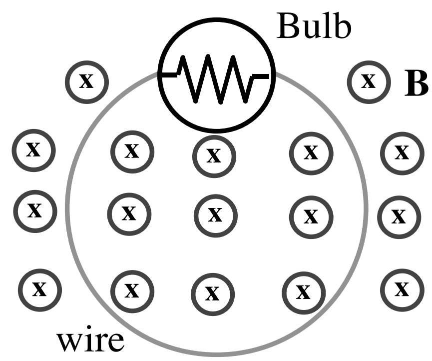
<u>Faraday's Law:</u>

A steady **E** field pushes charges around, makes *currents* flow. We've used the word "EMF" for this occasionally, an EMF is any voltage difference capable of generating electric currents.
Think of EMF = ΔV (=E Δx, remember that relation between V and E?)
(Note: batteries have an EMF, but resistors do NOT. Even though an R can have a voltage difference across it, it is not *generating* it! Resistors don't make currents spontaneously flow, batteries can.)

Michael Faraday, a British physicist (at the same time as Joseph Henry, an American, but Faraday published first) about 180 years ago discovered a remarkable new property of nature:

<u>Changing magnetic fields (not steady ones) can make EMF's.</u>
In other words, a time-varying **B** field can make currents flow.

Imagine a wire loop sitting in a **B** field, like this:



If the **B** field is steady then there is NO CURRENT, the bulb is dark.

But, if the **B** field *changes* with time, the bulb lights up, a current flows through that wire (!)

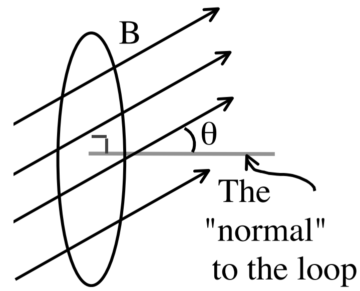
You might do this by e.g. just moving a big magnet closer, or farther away (yes, weakening the **B** field is still a *change*)... or move the coil itself closer (or farther) from the magnet face.

There's no battery here, no external voltage source, but the bulb still glows! This effect is surprising, it's something new...

Faraday spent only 10 days of (intensive) work on these experiments, but they changed the world radically.
This is how most of modern society's electricity is now generated!

Faraday worked out an equation (Faraday's Law) which quantifies the effect (how *much* current do you get?)
But before we can write it down, we need to first define one relevant quantity we haven't seen yet.

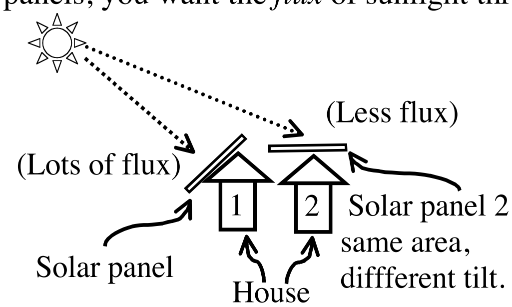
Imagine a **B** field whose field lines "cut through" or "pierce" a loop.
Define θ as the angle between **B** and the "normal" or "perpendicular" direction to the loop.
We will now define a new quantity, the **magnetic flux through the loop**, as



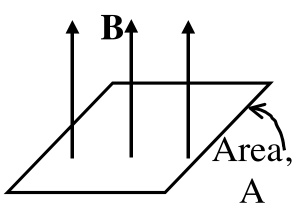
Magnetic Flux, or $\Phi = B_\perp A = B\,A\cos\theta$

$B_\perp$ is the component of B perpendicular to the loop:
$B_\perp = B\cos\theta$.
The UNIT of magnetic flux = $[\Phi]$ = T m^2 = Weber = Wb.

Flux is a useful concept, used for *other* quantities besides B, too. E.g. if you have solar panels, you want the *flux* of sunlight through the panel to be large. House #2 has poorly designed panels. Although the AREA of the panels is the *exact same*, and the sunshine brightness is the *exact same*, panel 2 is less useful: fewer light rays "pierce" the panel, there is less FLUX through that panel.

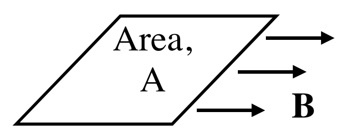


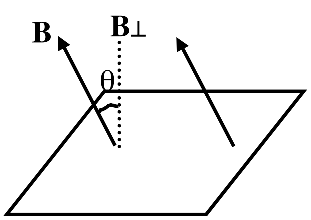
Examples of calculating magnetic flux:



Here (picture to the left) $\Phi = B\,A$, because B is perpendicular to the area. ($\theta=0$)

Here (picture to the right), $\Phi = 0$, because B is parallel to the area. ($\theta=90$.)
No flux: the B field lines don't "pierce" this loop at ALL, they "skim" past it... (That's zero flux!)





Here, (picture to the left), $\Phi = B\,A\cos\theta$. The flux is reduced a bit because it's not perfectly perpendicular.

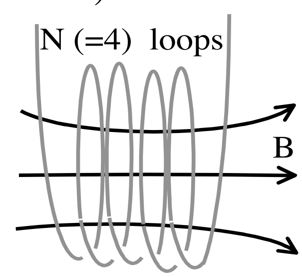
Faraday's Law: The induced EMF in any loop is

$\boxed{EMF = -\Delta\Phi/\Delta t}$. (Φ is magnetic flux, t is time.)

• If you put a loop into a B field, and then *change the flux* through that loop over time, there will be an EMF (basically, a voltage difference) induced. Current flows, if you have a *conducting* loop.

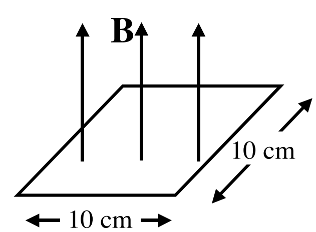
• The formula says it is only the *change* in flux through the loop that matters. A huge B field (lots of flux) does NOT make the EMF, it's the *change* in B with time that does the trick.
• This equation has not been derived - it's just an experimental fact!

• Units are $\frac{\mathrm{Wb}}{\mathrm{sec}} = \frac{\mathrm{T\,m^2}}{\mathrm{sec}} = \left(\frac{\mathrm{N}}{\mathrm{A\,m}}\right)\frac{\mathrm{m^2}}{\mathrm{sec}} = \frac{\mathrm{N\,m}}{\mathrm{A\,sec}} = \frac{\mathrm{J}}{\mathrm{C}} = \mathrm{Volt}$ (yikes!
It's a mess, but it works out. The formula gives the correct units.)



• If you were to "pile up" N loops on top of each other, the effective flux will be increased by a factor of N, the formula becomes <u>$EMF=-N\Delta\Phi/\Delta t$</u>. (Do you see why?)

• Since $\Phi = B\ A \cos\theta$, *you can change the flux in many ways: you could change B,* or *area,* or *the angle between **B** and the loop*.



<u>Example:</u> **B** is perp. to this loop, θ=0, as shown.
(Remember, θ is the angle from the *normal*)
The area is A= (0.1m)^2 = .01 m^2
Suppose B is 1 Tesla, as shown, and then you turn it off, taking a time of 2 seconds to do so...

Faraday's law says there will be an "induced EMF", or voltage, around the loop,
$|EMF|=|\Delta\Phi/\Delta t|$ = [(1 T * 0.01 m^2) cos(0) - 0 / (2 sec) = .005 V

If you had N=1000 coils (loops) of wire, all stacked (coiled) up around that same perimeter, you'd get |EMF|=5 V, enough to light up a small bulb. But remember, you'd only have this voltage for those 2 seconds while B was changing! Once B reaches 0 (and presumably stays there), there is no more *change*, and so |EMF| goes back to 0.

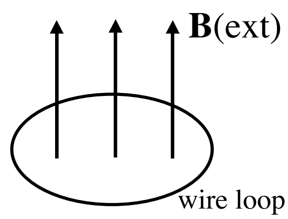
What's that *minus sign* about in Faraday's law?
Don't plug it in blindly - it's only there as a *reminder*, you must *figure out* the direction of the induced current flow, or voltage difference (the direction of the EMF) by **Lenz's Law:**

• Induced EMF tries to cause current to flow. If current flows, it will create a new (usually small) **B** field of its own, which we will call **B**(induced). (You'll need to remember RHR #1b: how currents in a loop produce **B** fields)

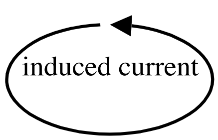
• I will call the original or "outside" field: **B**(external)
The direction of **B**(induced) *opposes the change in the original* **B**.
√*Note*: **B**(induced) *does NOT* necessarily oppose **B**(external)(!!)
It is opposite the CHANGE of **B**(external) (or more accurately, the change of flux). **B** is a vector, you really have to *think* about the direction of the *change* of that vector....

Lenz's law is a mouthful! It tells you the direction that the induced current will flow. Nature creates a **B**(induced) to *fight the change*.

Example: Consider a **B**(ext) that is up, and pierces a wire loop, as shown. It might be caused by a big old magnet or something. If **B**(ext) stays constant, there is no change, *no current* spontaneously flows around the loop.

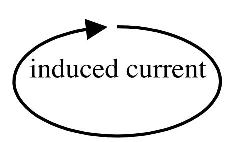


If **B**(ext) starts to decrease, nature will try to *fight that change*.
(Remember, if an "up vector" is decreasing, the change is DOWN)
Lenz's laws says a current will flow (or try to flow) to induce an upwards B field, to try to keep things as they were.
B(induced) may be small: it probably won't succeed, but it *tries*.
The direction of induced current is shown to the left.



B(induced) points up, opposite the *change* in **B**(ext). (Here, this just happens to be the same direction as **B**(ext) was originally, but that's *irrelevant*.)

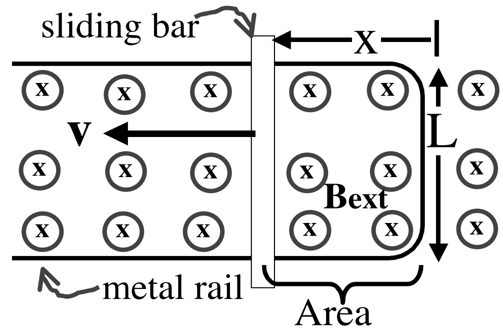
If **B**(ext) instead starts to *in*crease with time, then to fight that change you will induce a *downward* **B**, as shown.



You should look at Giancoli Fig 21.7 and work out *all* those examples for yourself. (DO NOT read the answers first!! Think about them - it takes practice to get Lenz's law.)

Example: A metal bar slides along conducting metal tracks in a uniform **B** field pointing *into* the page.

Push the bar to the left (as shown), and consider the conducting loop consisting of rail + slider.



The area inside that loop is increasing, and so flux through the loop (B*A) is also increasing. (A = L*x, and x is increasing with time)

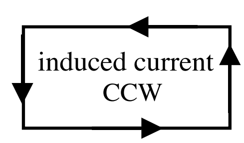
$|EMF| = |\Delta\Phi/\Delta t| = |B\,\Delta A / \Delta t| = B\,L\,\Delta x / \Delta t = B\,Lv$

(Because $v = \Delta x / \Delta t$ is the speed of the sliding bar)
That means current flows around the loop, by Faraday's law.
If you put a light bulb somewhere in that circuit, it'd glow.
The bigger B is (or the faster you slide the rod), the more current.

Now, what *direction* will the current flow, CW or CCW?
This requires Lenz's law!

The external flux is *into* the page and *increasing* with time. So the change in flux is *into* the page. (Do you see that?)
Lenz's law says current will start to flow to *fight the change*.

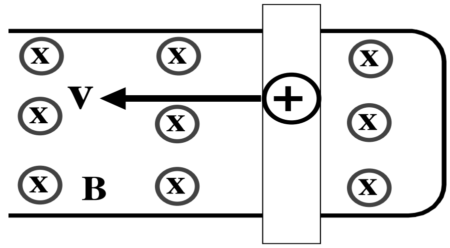
That current will induce a new **B** that points *out* of the page. By RHR #1b, that means CCW.



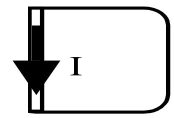
Note: It's not that **B**(induced) points out of the page because **B**(ext) is into the page. That's a coincidence. It's opposite the CHANGE in flux, not oppose the direction of flux.
E.g., If instead you push the slider to the right, **B**(ext) is of course the same, but now the flux is *decreasing* with time, that's opposite: the **B**(induced) will also be opposite, i.e. the current flows CW!!

Giancoli Fig 21-9 is similar. Try to figure out the direction of the induced current there, for yourself, and only *then* check the text to see if you got it right.

There is a totally different way, kind of "Chapter 20 style", to reach the same conclusion about the direction of induced current in the previous example.



Consider a small + test charge sitting somewhere in the slider. It (and of course every other atom, electron, etc. too) moves along with the slider to the left, with velocity **v**.
It sits in a uniform **B** field. That means it feels a force (F = q v B), and the direction is given by RHR #2, try it yourself, convince yourself it is DOWN.



But it's a test charge in a conductor - it's free to move.
What that means is the **B** field thus forces test charges down the slider, which means a current I down - exactly the direction we got before (from Lenz's law) Cool - a rather different way of looking at it, but the same result.

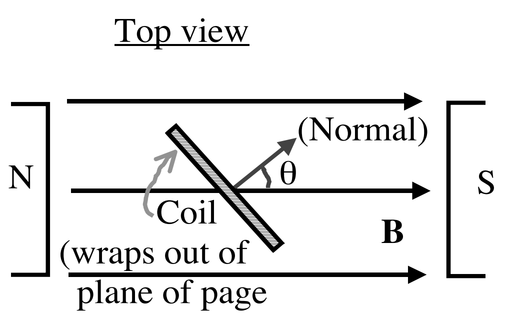
<u>Final comment:</u> We just saw there is an (induced) current flowing in the slider, and this current sits in a **B** field. Ch. 20 says any current in a **B** field will feel a force F = ILB.
Work out the direction for yourself (!), using RHR #2.
I claim the force in this example is to the *right*.

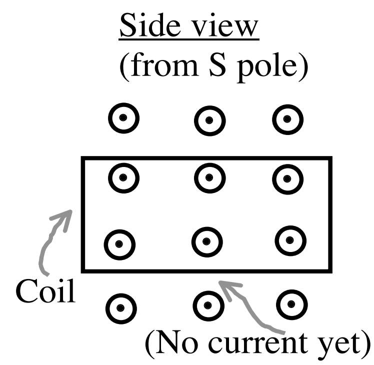
That means the **B** field tries to slow down the slider. It's kind of like magnetic friction. If you did not continue to push that slider to the left, the induced current feels this force that would slow the slider down to a halt.

We call any induced current like this, caused by conductors moving in magnetic fields, ***eddy currents*** (maybe because they look a little like water eddy's in a river?)

Eddy currents always cause slowing or "braking" forces. They behave in some ways like magnetic friction. This effect is *used* to slow down some kinds of trains - it's a "retarding" force proportional to velocity. By changing the resistance in the rest of the track you can change the magnitude of the current, hence the force - and it's easy to control electronically. Eddy currents have many other industrial applications, including in detecting coins in coin vending machines!

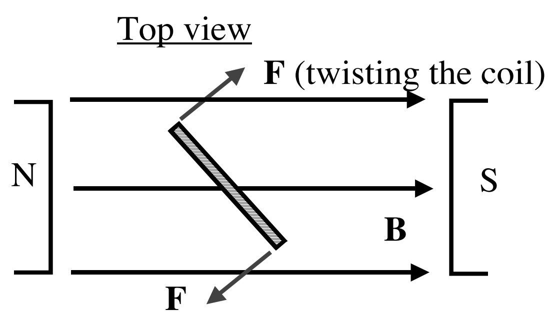
Different example: Put a *coil* of metal into a fixed **B**(external)





We've seen this setup before, in Ch. 20. (Remember, if you run a current I through that coil of wire, RHR #2 says there are forces that twist the loop. That's a MOTOR, putting current into it causes mechanical motion.)

But you can also do the opposite: suppose you (or a waterfall, or a steam engine...) mechanically force the loop to start to rotate.
What happens then?



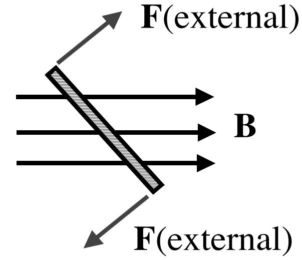
The flux through the coil, $\Phi = B\ A\ \cos\theta$, is *changing*, because θ is now steadily changing.

Faraday's law says
$|EMF| = |\ \Delta\Phi/\ \Delta t\ | = B\ A\ \Delta\cos\theta/\ \Delta t.$ (B and A are both constant!)

There is an induced EMF, a current spontaneously starts to flow in the loop. If you have wires leading out from the loop (like in the picture of the motor in Ch. 20) current flows to the outside world.

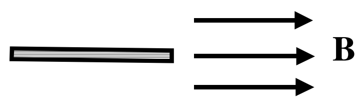
This is an ***electrical generator.*** It's just like a motor, only opposite: mechanical motion causes current. (You can even use the *same apparatus* either way, as a motor or as a generator.)

What is the *direction* of the current in the last example?
At the moment shown, the flux through the coil is to the right, and it is *increasing*.



Think about this (it's 3-D spatial imagery)
The B field isn't changing, but it's "piercing" the loop more and more efficiently, around the moment shown.
Let me exaggerate to convince you:

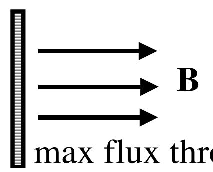
just before :



No flux through loop

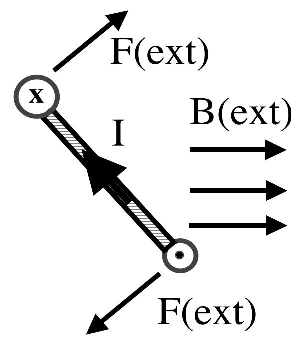
The flux is *increasing to the right*.

just after :

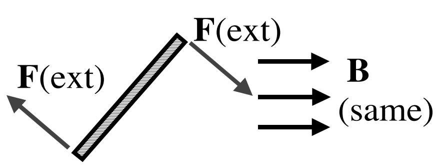


max flux through loop

Lenz' law says the induced current will fight this change, i.e. you will create a **B**(induced) that must point to the *left*. By RHR #1b,
that means the current at the moment in question flows around the loop as shown here.



But now let's look again just a little later...

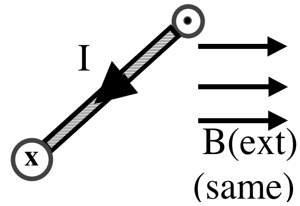


The flux is still, at this moment, to the right. But, now it is *decreasing* with time. (In a moment, there won't be any flux, when the loop is again parallel to the B field)

If flux is "right but decreasing", that means the *change* is leftwards.

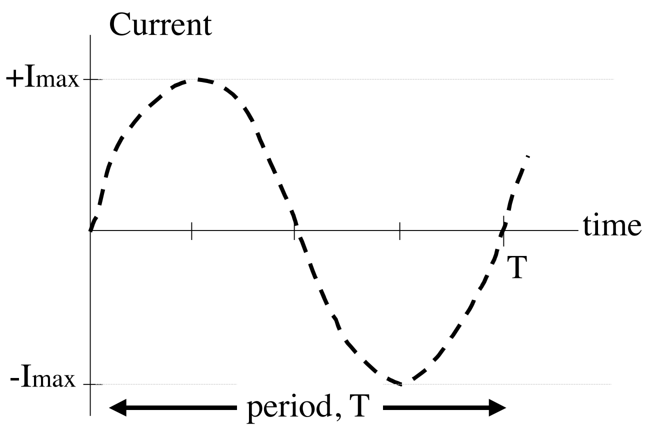
To fight the change, you need to make a current which will make a "*rightwards*" B field, like this:

Notice how the current has flipped its direction.



Bottom line: You do work on the coil to rotate it. In return, you get an electrical current out. This is precisely how our big power plants work - large coils being rotated in a big fixed B field (or sometimes, the magnet gets rotated around a fixed coil.)

The current is flipping direction each partial rotation of the coil (see the previous page for this story). If you graph current coming out of the coil as a function of time, it looks like this:



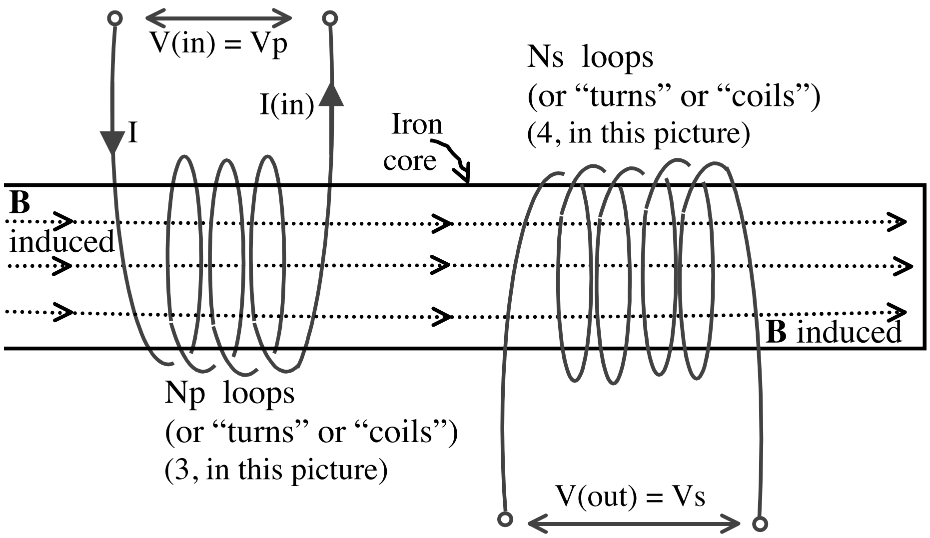
The frequency is exactly the same as the mechanical frequency of the rotating loop. In the US, that means you must turn that loop 60 times/sec, or 60 Hz.

Suppose you *stopped pushing*. You might imagine that if there was no friction, the loop would keep turning, giving you "free electricity". No such luck. There is still a current flowing, and this current is in an external **B** field, so it feels a force. RHR #2 tells the direction (work it out, looking at the pictures on the previous pages.)
At all times, the resulting forces make a torque that acts to *slow down* the loop. This is the "eddy currents" story again. Induced currents are caused by conductors moving in a **B** field. They will always act to slow things down.

This is a law of nature. If the forces ever acted in the *other* direction, i.e. to speed things up, you'd be getting something for nothing, violating conservation of energy. (Lenz's law, that minus sign in Faraday's law, is basically the statement of conservation of energy)

Transformers:

These are clever (and simple) devices to transform AC voltages.
You need two coils:
the "**input wire**" or "*primary winding*",
and the "**output wire**" or *"secondary winding"*.
The wires are usually wrapped around iron. There are no moving mechanical parts needed.



If V(in) is steady (I(in) is steady) then **B**(induced) through the coils is steady. The flux Φ through the secondary coil is therefore steady, and Faraday's law says V(out)=0: *no output current, or voltage.*

Moral: transformers do *nothing* if the input is steady, or DC.

But, if V(in) is AC, then the induced **B** keeps flipping direction, which means Φ is changing, and Faraday says we *will* induce an EMF in the secondary coil. That means there is a V(out).

Faraday says, specifically,
V(in) = Np $\Delta\Phi/\Delta t$,
V(out) = Ns $\Delta\Phi/\Delta t$.
The flux Φ is the *same* through both coils, because an iron core will guide all the **B** field lines through the secondary. (B and Area are thus the same for both coils...)

Dividing those last two equations gives

V(in) / V(out) = Np / Ns . (But, only if Vin is AC!)

or, of you prefer, V(out) = V(in) * (Ns/Np) .

A **"step up" transformer** has more secondary windings, Ns > Np, which means V(out) > V(in).
You get more voltage out than you had to start with.

A **"step down" transformer** has Ns < Np, so V(out) < V(in)

A step up transformer *almost* looks like something for nothing: V(in) could be small, and you could get a huge Voltage out (!?) Well....yes, V(out) can indeed be greater than V(in), but

Power(out) = Power(in).

or if you prefer

V(out)*I(out) = V(in)*I(in)
This is conservation of energy. A good transformer is very efficient, but energy is still conserved!

I(out) = [V(in)/ V(out)] * I(in) = (Np / Ns) * I(in) .

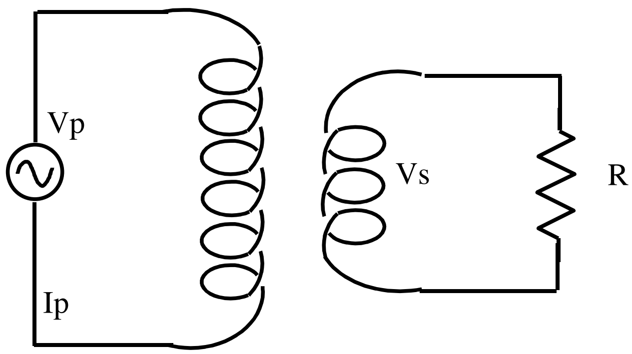
Step up transformers do increase the voltage, but at the cost of decreasing the current you get out. In reality, you always lose a little power to heating, eddy currents in the iron, etc.
A good transformer might give you P(out) = 99% P(in)...

Many household devices use step *down* transformers, e.g. chargers, which don't need much voltage (certainly not 120V for a little 1.5 V battery), but they want lots more current than the 15-30 A your wall sockets are limited to.

Other household devices use step *up* transformers, e.g. your TV set needs much higher voltages than 120V, but not much current is needed.

Transformers are a wonderful invention, and allow for much of our electrical technology and distribution system.

Example: A 100:1 step down transformer:



This is the generic symbol used for a transformer.

"100:1 step down" means Np/Ns = 100:1

Suppose in this circuit that Vp=120 V (it's plugged into a regular wall socket), and Ip=15 A (a typical value you might get out of normal wall sockets before the circuit breaker blows.)
Then, Vs = (1/100) Vp = 1.2 V.
Is = (100)* Ip = 1,500 A.

You could never get 1500 A directly out of the wall!
This transformer puts lots of currents out, just at a lower voltage.
Notice that Power(in) = 120V * 15 A = 1800 W
Power(out) = 1.2 V * (1500 A) = 1800 W. (Energy is conserved)

Example: If you wanted to melt a 12 cm long nail by running current through it, could you do it by just wiring it straight into your wall plug?
Using resistivity for iron, I'd estimate the nail's resistance to be
R = ρ * L / A = (10^-7 Ω m) * (0.12 m / (10^-5 m^2) = 1.2E-3 Ω.
(I got the area by π r^2, with r about 2 mm, kind of a fat nail?)
If you plugged this into the wall (like I did the pickle), you'd expect
I = V/R = (120 V) / (1.2 E-3 Ω) = 100,000 Amps. Yikes! You'd blow the fuse, nothing would happen. But if this was the "R" in the figure above, then I = (1.2 V / 1.2E-3 Ω) = 1000 A, no problem.
(You'd be drawing 10 amps from the wall, do you see why?)
But it's still putting out 1000 A*1.2 V = 1200 W of power,
which almost surely would melt a nail!

The power company (Excel) sends currents over long distances. They do lose power in the lines due to resistance, P(loss) = I^2*R.
They'd prefer to transmit a small current, to avoid this power loss (because the resistance of the lines is something constant) But you and I demand a certain average power = V*I. They want to supply this to us. They want to give us all the V*I we ask for, but send less current through the cables, so they use VERY HIGH voltages! (Large V*small I gives the power we ask for, while small I through the lines means less loss.)

They have step up transformers at the power plants, which can take the voltage up to a million volts on some long distance power lines. At the edge of cities, they have step down transformers (at transformer stations, there's one not far from my house) which convert it down to lower voltage, perhaps 2000 V. Then, right at your house, there's one last step-down transformer to take it down to 120 V (or actually 220V).

Example: Say my house uses 10^4 W of power. (A little high, but not an unusual number for a big household, during the daytime) Suppose the transmission lines have a total resistance, all the way from power station to my house, of 1 Ω. (That's pretty small)
If they just transmitted at 120 V all the way, I'd use
I = P/V = (1E4 W/120 V) = 83 A.
This current has to go through their lines, losing power of
I^2*R = (83A)^2*(1 Ω) = 7,000 W.
Yikes! 70% of the power I need is wasted in heating their lines!

But if they used 120,000 V instead (using transformers), then
I = P/V = (1E4 W/ 120,000 V) = 0.083 A.
This current goes through their lines, losing power of
I^2*R = (.083A)^2 * (1Ω) = .007 W. Nothin'!