Giancoli CH 17: Electric Potentials and Capacitors:

We've been talking about electric forces, and the related quantity $\mathbf{E} = \mathbf{F}/q$, the E field, or "force per unit charge". In 2010, after talking about forces, we moved on to work and energy (Ch. 6)

Quick Review of work and energy:

The work done by a force, F, moving something through a displacement "d", is Work = F*d, or more carefully, $W = F_{\parallel}d = Fd\cos(\theta)$.



E.g. if you (an "external force") lift a book (at constant speed) up a distance d,



Newton II says $\mathbf{F}_{net} = m\mathbf{a}$, i.e. $\mathbf{F}_{ext} - \mathbf{F}_{g} = 0$ (because, remember, if speed is constant => \mathbf{a} =0) or $\mathbf{F}_{ext} = mg$.

You do work $W_ext = F_ext^*d = +mgd$

(The + sign is because θ is 0 degrees, your force is UP, and so is the displacement vector) The gravity field does W_field = -F_g*d = -mgd

(The minus sign is because θ is 180 degrees, the force of gravity points DOWN while the displacement vector is UP)

The NET work (done by all forces) is $W_{ext+W_{field}} = 0$, that's just the *work-energy principle*, which says $W_{net} = \Delta KE$ (=0, here)

You did work. Where did it go? NOT into KE: it got "stored up", it turned into *potential energy (PE)*. In other words, F_ext did work, which went into *increased* gravitational potential energy.

For gravity, we defined this potential energy to be PE = mgy, so $\Delta PE = mg(y_{final} - y_{initial}) = + mgd (=W_{ext})$

(The change in PE is all we ever cared about in real problems)

Now, let's *drop* the book, and see what happens. There is no more "external force" touching the book (like "me" in the previous example), only gravity acts. (Neglect friction)

Energy conservation says

$$PE_i + KE_i = PE_f + KE_f$$
, i.e. $mgd + 0 = 0 + \frac{1}{2}mv_f^2$

This formula gives a quick and easy way to find v_f . The concept of energy, and energy conservation, is very useful! Another way of rewriting that equation is

$$(PE_f - PE_i) + (KE_f - KE_i) = 0,$$

i.e. $\Delta PE + \Delta KE = 0,$ or $\Delta E_{tot} = 0$
(End of quick review of work and energy, see Ch. 6 for more details)

There is an electric "analogue" of the above examples:



Consider 2 charged parallel metal plates (called a "capacitor"), a fixed distance d apart.

Between the plates, **E** is uniform (constant),

and points *from* the "+" *towards* the "-" plate.

Imagine a charge +q, initially located near the bottom plate.

The force on that charge is $\mathbf{F}_{E}=+q\mathbf{E}$ (down, do you see why?). (Let's totally neglect gravity here.) Now LIFT "q" from the bottom to the top, at constant speed:

Now En 1 q nom the bottom to the top, at constant speed

You do work $W_ext = F_ext^*d = +qEd$ The *Electric* field does $W_field = -F_E^*d = -qEd$. (Do you understand those signs? Think about them!)

Just like the previous case: you did work, but where did it go? As before, it didn't turn into KE, it turned into potential energy. We say the charge's electrical potential energy has increased: $\Delta PE = qE(y_{final} - y_{initial}) = + qE d (=W_{ext})$ (where y is the distance *above* the negative plate)

We lifted the charge from a region of LOW PE (near the "-" plate) to a region of HIGH PE (near the "+" plate). (Note: "up" and "down" are *irrelevant* here, you could turn the picture on its side or even upside down. It's not gravity in this story, it's 100% electrical energy.)



Just like we defined $\mathbf{E}=\mathbf{F}/q$ (dividing out q gives force/unit charge) let's now define something we call "electrical potential" or just "potential" = $\overline{\mathbf{V} = \mathbf{PE}/q}$.

• Calling this quantity "potential" is really a pretty bad name, because this "potential" is quite DIFFERENT from "potential energy".

• Potential has units of [energy/charge] = [Joules/Coulomb] = J/C. We call 1 J/C = 1 Volt = 1 V

(People use the symbol "V" for the *unit* volt, as well as for the *quantity* itself. Another bad choice, but we have to live with it)

A change in potential is called a "potential difference",

 $\Delta V = V_f - V_i = \Delta P E / q$, and from this we see $\Delta P E = q \Delta V$.

Example: A car battery maintains 12 V between the terminals. If the headlights contain a 36 W bulb, how much charge is the battery moving through the bulb each second? (And, how many electrons is that?)

<u>Answer</u>: 36 W = 36 Watt = 36 J/s. Each second 36 Joules of energy are dissipated in a bulb. This energy all comes from the loss of potential energy as charges

+ -

flow from one terminal, through the bulb, to the other terminal. If a charge "q" drops 12V, the energy lost is $\Delta PE = q\Delta V$, or q*12V. Each second, 36 J are lost, i.e.

 $36 \text{ J} = q^{*}12 \text{ V}$, or q = (36 J)/(12 V) = (36 J)/(12 J/C) = 3 C.

That's a lot of electric charge being moved by a car battery! The number of electrons going through the bulb each second is 3C/(1.6E-19 C/electron) = 2E19 electrons. (A heck of a lot)

I was a little sneaky about signs (the charge of an electron is negative): just think about it.

Here's a related question for you: given that it's (negative) electrons that flow out of a battery, which way do they go? from the "+" terminal through the bulb to the "-", or the other way?

(The answer is from - to +. Electrons are repelled from "-", and attracted towards "+".)

For a parallel plate capacitor, we just found (two pages ago) $\Delta PE = + qE d$, so $\Delta V = \Delta PE / q = (qEd) / q = E d$.

Here's another sketch of a capacitor:



With gravity, you can choose to call "zero" potential energy wherever you want. You might choose sea level, or the tabletop, or the ground. It's the same story with electricity: you can pick any spot you want and call the electrical potential energy 0 there. We usually call this point "the ground"! Let's call point "a" in the diagram above "the ground" or "0 potential".

Now put a charge "+q" at the point "b" in that figure. It will have a potential given by $V(at point b) = E^*d$.

It has a potential *energy* at point b of PE = +q*V(at b) = +qEd.

It has "+" potential energy there, which makes sense. It's like a pebble up in the air, it can do work, just let it go! (The upper plate repels a "+" charge, the lower plate attracts it: if you let it go it will run "downhill" in energy, from high potential to low...)

Notes:

• E points from high V to low V (always!)

"+" charges want to head towards low V, if you'll let them

"-" charges want to head towards HIGH V, if you'll let them (!)

• We can talk about the "potential at a point", or the "potential energy at a point", but the numerical value depends on where we chose to call 0.

We can talk about "potential differences" between points, and then it does NOT matter where we chose to call 0!

What if you're near a point charge, Q, rather than a capacitor?

 $\stackrel{+Q}{\frown}$ $\stackrel{r}{\longrightarrow}$ $\stackrel{"a"}{\bullet}$ $\stackrel{What's the potential at the point "a"?}{\bullet}$

1st, where do we want to call "0 potential". It's not so obvious here. A standard choice is "far away", off at infinity. Out there, PE=0, V=0, seems reasonable!

Now we need to think about moving a test charge "q" from far away (where PE=0, V=0), to the point "a". Because the work you do bringing it from PE=0 to the point *is* precisely its potential energy! (It's like how much work you do lifting a book from the ground, i.e. PE = 0, up to a height d: it's mgd, the final potential energy)

Now, Work = F^* distance, and $F=kQq/r^2$. Unfortunately, this force *changes* as you move in from far away (r is changing). So, you really need calculus to figure out the work. The answer, though is very simple (and maybe you can even guess it, just multiply $F^*r...$)

 $W_ext = k Q q/r$ (Notice, that's an r downstairs, not an r^2!)

So the PE at point "a" is exactly that, PE(at "a") = k Q q/r, or

V(at "a") = PE(at "a")/q = k Q/r.

(Note that we *chose* V=0 to be off at infinity, to get that formula.)

If Q and q are both "+", then PE=kQq/r >0. (This makes sense: two positive charges want to "fly apart", they'll DO work if you'll let them. The system has *positive* potential energy. Like a rock up in the air...)

Also, just like with capacitors, the potential V is big ("+") when you're near a positive charge Q. (the closer, the bigger.)

If Q and q are *opposite* signs, then PE<0. This is also correct: you would have to do work ON opposite charges to "pry them apart", the system has a negative potential energy! We might say the system is "bound".

<u>What if there are a bunch of point charges, what's the potential?</u> The voltage at any point is just the sum of the voltages arising from each of the individual particles (this is "superposition" again) It's really quite *easy* to find the voltage at a point because of this!



finding **E** at the origin would be a *lot* more trouble.

Example: A test charge "+q" is moved from point "a" to "b" in the figure. (There are two other charges present, -Q and +Q, fixed in position at the corners of a square, as shown.)

How much work does this take?

external work at all. (Depending on how you move, you might do some +work part of the way, and -work part of the way, but in the end, you do zero total work going from this particular "a" to "b".)

Trying to figure out the work by thinking of "force*distance" along the path would be HARD, because force changes all the time. Using voltages makes this much easier to figure out.

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It's often useful to find *all* the points in a diagram that have the same voltage. E.g., consider a capacitor again.



"equipotential" (equal potential) line.

Everywhere along the top surface, the potential is the same (V=Ed).

Everywhere along the bottom, the potential is the same (V=0).

And *every* point on that dashed line has the same potential. (something between 0 and Ed)

We call such a line an

If you move a test charge along an equipotential line (e.g. the horizontal dashed line in the figure) the potential is the *same* everywhere, so *no* work is required. It's like walking along a flat surface where there's no change in the gravitational potential. Or, e.g. *traversing* sideways on a ski slope.

There can't ever be a component of **E** parallel to an equipotential line. (if there *was* a nonzero **E**_parallel, you'd do work moving along it, since $W = \mathbf{F}_{parallel*d} = q \mathbf{E}_{parallel*d}$) This means that in drawings, **E** field lines are always perpendicular to equipotential lines.



The equipotential lines are shown as "dashed" in this figure.

They're like contour lines on a topo map, which show "gravitational equipotential" lines. (Constant height on a topo means constant PE_grav).

Anywhere along a dashed line, the potential is constant.

• *Inside* any (static) conductor, we know E=0. That means no work is required to move charges around anywhere inside, or along the surface. So metals are equipotentials throughout their volume!

• Real life is 3-D, those "lines" are really "surfaces"...

More examples of equipotential lines:



Remember, voltage near a charge +Q is given by V = kQ/r.

The farther away you get, the lower the voltage.

If "r" is fixed (i.e. a circle), V is constant. (Again, in 3-D, these would be equipotential surfaces, rather than lines.

In this case, they'd be spheres surrounding the charge Q)

Another example: here's a dipole, with a couple of equipotential lines shown. (Look back in Ch. 16, we sketched the electric field lines before. Now we're adding in the "equipotential surfaces)



We usually choose to define that center equipotential line as V=0. Remember, it's up to you to pick where V=0 is, and that line extends out to infinity. That's a pretty common choice. (Far away from everything, the potential is considered zero.)

To find the *numerical* value of the potential on one of those dashed surfaces, you'd do a quick calculation just like 2 pages ago - find the distance r1 to +Q, and r2 to -Q, and then V = kQ/r1 + k(-Q)/r2.

Anywhere on the center line, you're equidistant from both charges, so r1=r2, and the two terms cancel, V = 0. It's all consistent...

ENERGY, and units:

Example: Release a proton (charge +e), from point a in the figure. Suppose the voltage difference between the plates is 5000 V (a typical voltage in e.g. a normal TV set.) *How fast is the proton going at point b?*

Answer: I'd use conservation of energy here:

 $PE_i + KE_i = PE_f + KE_f$, i.e. $+ eV_a + 0 = + eV_b + \frac{1}{2}mv_b^2$ Solving for v_b we get

Solving for v_b we get $v_b = \sqrt{2e(V_a - V_b)/m} = \sqrt{2(1.6 \cdot 10^{-19} \text{ C})(5000 \text{ J}/\text{ C})/(1.67 \cdot 10^{-27} \text{ kg})} = 10^6 \text{ m/sCan}$ you *check* that the units worked out o.k.?)

Note that $V_a - V_b = +5000 \text{ J/C}$, i.e. V_a is a HIGHER potential. However, the *change* in potential of the proton as it moves is $\Delta V = V_b - V_a = -5000 \text{ J/C}$.

Think about the signs: objects spontaneously move to LOWER potential energy if they can. That means "+" objects like to go to lower *potential (i.e. lower voltage)*.

What was the final KE of the proton in this example? It's easy enough to find, using conservation of E:

$$KE_{f} = KE_{i} + PE_{i} - PE_{f} = 0 + eV_{a} - eV_{b} = e \cdot (5000V)$$
$$= 1.6 \cdot 10^{-19} \,\mathrm{C} \cdot (5000 \,\mathrm{J} \,/ \,\mathrm{C}) = 8 \cdot 10^{-16} \,\mathrm{J}$$

Many people prefer to change units here, like converting 2.54E-2 m to 1 inch (metric to non-metric) We can define a new *unit* of energy:

1 eV = 1.6E-19 J

The "eV" is also called an "electron Volt", but it is NOT a volt (which is a unit of potential, or J/C, recall) It's just a name, "ee-vee"!

The eV is defined so as to be the energy loss of a particle of charge "e" (like a proton) dropping across 1 Volt of potential.

Since energy change is q*Delta V, this is an energy change of (+e)*(1 V) = (1.6E-19 C)*(1 J/C) = 1.6E-19 J, just like we said.

In the little problem at the top of the page we could've done it without a calculator if we'd used eV's instead of J. Namely,

 $KE_f = KE_i + PE_i - PE_f = 0 + eV_a - eV_b = e \cdot (5000V) = 5000 eV$ You can check that this agrees by doing a simple unit conversion: $5000eV = 5000eV * \frac{1.6 \cdot 10^{-19} \text{ J}}{1eV} = 8 \cdot 10^{-16} \text{ J}$, which is what we got above.



CAPACITORS:

Any time you have two metal surfaces near each other, separated by air (or an insulator), you have a capacitor. My favorite example is the "parallel plate capacitor". We've already looked at it quite often.

A capacitor can be charged or not. When we say a capacitor has "charge Q", we mean +Q on one plate, and -Q on the other, like in the picture here. (Of course, the *net* charge is really zero, but we still say it's "charged"!)



If we say "a capacitor has voltage V", we really mean it has a difference in voltage between the plates $\Delta V = Ed$.

We drop the " Δ " for convenience, i.e. V=Ed. (It's a bad notation, but standard)

The more charge Q you put on the plates, the stronger the E field between the plates, hence the bigger the voltage drop. In fact, if you double Q, you discover that E doubles too (and so must V.)

That means $Q \propto V$, or Q = CV.

C is a constant, and we call it the "*capacitance*". (Of course, you can also turn it around and say C = Q/V.)

A capacitor can store charge (and energy) for you - that's why they're useful. Think of a closet with lots of bowling balls up on the shelf at high gravitational potential. You can let them fall down whenever you like, to do lots of work (or damage) later. Your closet shelf has only a limited capacity to hold bowling balls. Similarly, metal plates of a given voltage difference have a certain limited capacity to hold charge, and that's what "C" tells you.

A big capacitance (large C) means you hold lots of charge with a little voltage. A small C means you can only hold a little bit of charge, for a given voltage (a bit like a small shelf in the closet). Of course, given C, you can make Q as big as you want by making the potential bigger. (So our "closet" metaphor breaks down...)

The units of capacitance are [Coulombs/Volt] = C/V. We call 1 C/V = 1 Farad = 1 F. (The units are getting confusing now, because 1 V = 1 J/C, so 1 F=1 C^2/J too.)

• 1 F is a really big capacitor! It would hold 1 C (a lot of charge!) with only one volt difference between the plates. Most normal capacitors are more like micro or nano Farads.

For large parallel plates of area A, the E field between them is given by a simple formula (which I won't derive)

$$E = 4\pi k \frac{Q}{A}$$

The formula says the more Q you have, the bigger E is. (That at least makes some sense, doesn't it?) Also, the bigger the area of the plates, the more the charge is spread out, and that should weaken the E field, so the A dependence in that formula also seems reasonable. The formula doesn't depend on "d" (how far apart the plates are.) This is perhaps a bit surprising, but it's correct: E is UNIFORM, it doesn't change with distance from either plate.

Earlier, we defined
$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$$
, so you can also say $E = \frac{1}{\varepsilon_0} \frac{Q}{A}$.

We know V=Ed for a parallel plate capacitor. Combining this with the formula for E (just above) gives $V = 4\pi k \frac{Qd}{A} = \frac{1}{\varepsilon_0} \frac{Qd}{A}$.

Finally, C = Q/V (that defines C.) So that last equation for V yields (can you check the algebra yourself?)

$$C = \frac{1}{4\pi k} \frac{A}{d} = \varepsilon_0 \frac{A}{d}$$

(This formula is *only* for parallel plate capacitors)

Since V=Ed, and E is uniform, the *closer* the plates, the smaller the voltage drop. Smaller V (for a fixed charge) means C=Q/V is BIGGER. If you can gets the plates closer together, you get a better (bigger C) capacitor! So that "1/d" dependence in the formula makes some physical sense.

Similarly, if the area of the plates is bigger, the formula says you have more "capacity" to hold charge. That makes good intuitive sense to me; you have more area to spread those charges out onto, so you can easily hold more charge.



If you connect a wire up to a capacitor (like on the left) the charges on the capacitor plates are free to travel. The + and - attract one another, the capacitor will discharge quickly, possibly with some nice sparks. There's energy in there!

It takes work to charge up a

capacitor!

Consider e.g. a battery charging up a capacitor. A battery "wants" the plates to be at a certain voltage difference V. (E.g., a 12 V car battery always wants things connected to it to be 12 V apart.) To do this, it forces charges to go from one plate onto the other plate, fighting against an E field all the while.

How much work does the battery do, charging the capacitor up from 0 to V?

The first charge moves easily, it takes almost no work, because at first the plates have no charge. (There's no E field, and charges are pretty much free to move around wherever they want.)

On the other hand, if the capacitor is fully

charged (with a voltage V across it) you'd expect that moving Q charges across that voltage V would cost energy = Q*V. The real answer is the average of 0 and QV (on average, while charging , the voltage is V/2), so

Energy to charge up = Energy stored in capacitor = U = QV/2.

The symbol "U" is, for some reason, often used for energy. In this case, it's electrical potential energy. (Don't confuse "potential energy" with "potential"!)

Since Q=CV, we can rewrite this as $U = CV^2/2$, or $U=Q^2/(2C)$. Depending on what's held constant, these forms are sometimes useful.



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Example: A 12 V car battery is hooked up to a capacitor with plates of area 0.3 m², a distance 1 mm apart. *How much charge builds up on the plates?*

<u>Answer</u>: The battery will charge the capacitor up until the voltage is 12 V. Since Q=CV, we only need to find C. But we know that $C = \varepsilon_0 A/d$. So we're all set:

$$Q = CV = \varepsilon_0 \frac{A}{d} V = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \left(\frac{0.3 m^2}{10^{-3} m}\right) 12V$$
$$= 32 \cdot 10^{-9} \frac{C^2}{Nm} V = 32 nC$$

(I used 1 V=1 J/C: check for yourself that that nasty combination of units simplifies like I claimed, to Coulombs)

How much energy is stored in the capacitor now? U=Q*V/2 = 32E-9 C * 12V / 2 = 0.2 micro Joules. Not so much.

<u>Aside:</u> Where exactly is the energy stored, in a capacitor? The answer is that it's stored in the E field! Wherever you have electric fields, there is stored energy. The energy is stored in the "space" between the plates, in the form of electric field energy.



DIELECTRICS:

Any insulating material (paper, plastic, etc) can be called a dielectric. Most real capacitors have dielectric materials between the plates.

This helps to keep the plates apart! After all, the plates are oppositely charged, and so attract each other strongly. If they ever touched, the capacitor would discharge, or "short out", and be useless.

So, with a dielectric in there, you can make "d" quite small, and remember that helps make the capacitance bigger. But the main reason for putting a dielectric in there is

something different: it actually decreases the E field in the capacitor! Why?

Real materials (like dielectrics) are "polarized" by a strong E field. That means the E field in the capacitor effectively pulls some "-" charge

towards the top of the dielectric (nearer the "+" plate), and some "+" towards the bottom (nearer the "-" plate) In the region throughout the middle, it looks like some of the "Q" around you has been weakened, or shielded, or canceled out.

The net effect is that E is reduced throughout the dielectric. And since V=Ed, the voltage between the plates is reduced.

Since C=Q/V, if Q is fixed and V is reduced, C gets bigger. In this way, *dielectrics make the capacitance bigger*.

(Your capacitor can hold MORE charge for a given voltage, with a dielectric in there, because the surface of the dielectric effectively shields out some of the E field from the middle region)

Recall the formula $C = \frac{1}{4\pi k} \frac{A}{d} = \varepsilon_0 \frac{A}{d}$.

That's if there's no dielectric. If there IS a dielectric, we just argued C is bigger. It turns out for most dielectric materials, C is bigger by some constant factor which depends only on the material, i.e.

$$C_{\text{with dielectric}} = K \frac{1}{4\pi k} \frac{A}{d} = K \varepsilon_0 \frac{A}{d}$$

K will be some constant for any given dielectric material (Giancoli has a table) Bigger K means you get a bigger C. (Paper, e.g. has K=3, roughly.)





Capacitors are everywhere: in circuits, radios, computers, TV's... It's handy to have a "charge storage" device! The final (brief) topic of this chapter is a real world application of capacitors: *CRT*'s, or "cathode ray tubes".



A "cathode ray" is an old-fashioned name for electrons. The "cathode" is a heated piece of metal, set at a very low voltage. The "anode" is set at a high potential (so the "cathode" and "anode" basically form a capacitor) Electrons boil off the hot cathode, and then they are accelerated towards the high voltage ("+" charged) anode. The anode is a grid with lots of holes, so many electrons can fly right on by and cruise towards the screen.

They pass through a pair of capacitors (vertically and horizontally oriented) which have a voltage that "sweeps". As electrons pass through these capacitors, they feel the force from the E field, which bends the path of the electron. Since the voltage is swept, the electrons are also swept. They fly on by, and hit the screen, which glows where the electrons hit. So you see them sweeping by, and this makes the whole screen glow. By turning the anode on and off, you can make the electrons go through or not, thus making bright or dark spots, which allows you to make an image.

This device is used in old computer monitors and TV, oscilloscopes, EKG traces, you still see them all over. We'll play with an oscilloscope in lab, where you can control the voltages by hand and mess around with manipulating electron beams...