

2020 Sp16. (SJP) Steve Pollock. 492-2495 Duane F1013 (10th floor).
Michael Calkins 735-4853, Duane F715 (7th floor)

Office hours: LOTS!! See web site.

Reading: Check out <http://www.colorado.edu/physics/phys2020>
Read Giancoli 16.1-4 for the 2nd class (we'll cover all of Ch. 16 in the next few lectures.)

1st class: Intro, introduction to "electrostatics"

2nd lecture: More on electrostatics, intro to Coulomb's Law

Welcome! 2020 is the 2nd semester of introductory algebra-based Physics. We'll be studying a lot of "modern physics" this semester, focussing especially on electricity, magnetism, and light.

Let's begin by reviewing some highlights from *last* semester. Physics 2010 can seem a little overwhelming, there were a lot of facts and ideas in that class! But they are all tied together with a few basic underlying principles, that reflect our universe and how it works: Phys 2010 was primarily about *mechanics*, how and why things move. The key elements of Phys 2010 were the following:

- 1) *kinematics*, the description of motion, velocity, and acceleration.
- 2) *dynamics*, the explanations of why things move as they do.

The essential ideas of dynamics are stated in Newton's laws:

Newton 1: Bodies will continue in a state of uniform (unchanging) velocity, unless acted on by a net outside force.

Newton 2: $F=ma$, forces cause acceleration, another word for change in velocity (because $a = \Delta v / \Delta t$)

This is a vector equation: forces (and acceleration) have direction and magnitude. To be a bit more accurate, Newton's second law really says

$$\sum \vec{F}_i = m\vec{a}, \text{ or if you like, } \mathbf{F}_{\text{net}} = m\mathbf{a}.$$

Newton 3: The force on A by B is equal and opposite to the (reaction) force on B by A, or: $\mathbf{F}_{AB} = -\mathbf{F}_{BA}$. (For every force applied on a body, there is an equal and opposite force applied to whatever caused that force)

Other important and useful concepts, extensions of Newton's laws:

1) *Total energy is conserved.*

Energy is a defined, mathematical quantity, which can take various different forms: kinetic energy, potential energy, thermal energy, chemical energy, electrical energy..

You really only focussed on the first two, last semester

Recall, kinetic energy is given by the formula $KE = 1/2 m v^2$
(m is mass, and v is velocity.) KE is a number, a scalar.

2) *Momentum is conserved* if there are no net external (outside) forces acting.

Momentum is also a defined quantity:

$m \cdot \mathbf{v}$ (mass*velocity) It's a vector, it has a *direction*.

Finally, we learned Newton's universal law of gravity: any two massive objects *attract* one another, with a force of magnitude

$$F = G M_1 M_2 / r^2.$$

(G is a constant of nature, r is the distance between the objects, M's are the masses of the objects)

Then you applied these ideas, and learned about e.g. gravity, friction, and contact (normal) forces; circular motion, static fluids, springs, simple harmonic motion (vibrations), waves, heat, and more.

Together, Newton's laws and the conservation laws constitute one of the most amazing and important intellectual breakthroughs in human history. Physics I was concerned with, and described (very successfully!) a lot of everyday stuff: baseballs, juggling balls, airplanes, air tracks, Olympic divers, Tai Chi masters, fireworks, the moon, helium balloons, beds of nails, barometers, fire extinguishers, pendula, water waves,... Most of this is "17th century" physics, understood by Isaac Newton, and *used* today by engineers, architects, physical therapists, etc.

Physics 2020 is about more modern physics, starting with (mostly) 19th century discoveries, especially about electricity, magnetism, and light, and moving into the 20th century (with quantum mechanics and relativity). Many of the *phenomena* described by this physics are *also* everyday, but perhaps a little higher tech than what we saw last semester. Now we want to describe and understand microwave ovens, light bulbs, stoves, computers, lasers, and so on.

Your intuitions may not serve you quite as well, much of what we'll be studying will feel a little more abstract, but it's all *very* real. By the end of the semester, you will *develop* some good intuitions about electricity, currents, lights, and atoms! We begin the semester in Ch. 16 of Giancoli.

Electric charge and Electric Fields: "Electrostatics"

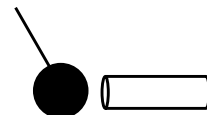
Last semester, our "favorite" force was gravity. But there are other types of forces that have been studied since Newton. We will spend a good portion of this semester studying *electric* forces (and then we'll move on to *magnetic* forces, different but related). These forces have been known about for 1000's of years, and most likely you have discovered for yourself that e.g. rubbing the carpet and touching someone can make annoying sparks, and rubbing a balloon on your shirt can make it stick to the wall (for a while). So, what's going on?

First, consider a pendulum, with a little "bob" hanging off it: It just hangs there, because of canceling forces of gravity (down) and tension (up). There is no net force on the bob: no acceleration.



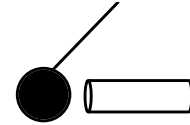
To make it move, we must apply a *force*. E.g., we could hit it with a bat, or tap it with our fingers. If you take a bat (or, say, a glass rod), and simply hold it still, near the bob, nothing at all happens (*Expt 1*). There is no force.

But what if you first RUB the glass rod with, say, a silk cloth? If you now bring the rod close, there IS a force, an attraction (*Expt 2*). (The bob moves, there MUST be a force!) But they're still not in contact, so what exactly is this force? If we carefully measure the motion (the angle the bob hangs, or its acceleration) then we can certainly *measure* the force, and learn about it...



What happens if we *touch* the ball with the rod (*Expt 3*)?

It depends on the room conditions, the type of ball, etc! But in some cases you will find that once they touch, the force changes; now the rod REPELS (rather than attracts) the pith ball. If we rub the rod with silk again, they will then repel each other still more strongly (*Expt 3b*), even without ever touching them again.



Suppose we remove the glass rod, and take a *plastic* rod, rub it with wool, and bring it near that same bob (*Expt 4*). The bob is once again attracted to the rod. And if we now let them touch (*Expt 5*), the direction of force should reverse, from attraction to repulsion.

These forces are very real, and quantitatively measurable. They have *nothing* to do with gravity or friction. They're not contact forces either. What is going on? Benjamin Franklin, and many other scientists around the mid 1700's, studied these kind of forces. They invented some terminology and ideas that were able to explain these observations (and many more), with a rather simple picture of what's going on.

Franklin called the forces "*electrostatic*", and postulated that rubbing materials can charge them up with electrostatic (or electric) charge. He argued that there are exactly two distinct kinds of charges in the world (two "flavors" of electric charge) and called them "positive" and "negative". (I suppose he could have called them "Fred" and "Ethel", but positive and negative turns out to be much more useful, as we'll see shortly) He called the charge carried by the rubbed glass rods "+" (positive), and the charge on the rubbed plastic rod "-" (negative) (back then it was an amber rod.)

He postulated the following simple rules:

- 1) Electric charges of the same sign repel each other
- 2) Electric charges of opposite sign attract each other.

There are more rules, but this is already the basis of electrostatics!

Where do these electric charges *come* from? Are they created when you rub things together? The modern view is, NO, they aren't created, they were already there. When you rub things you just manage to *separate* "+" and "-" charges. The important idea is that the world (and every material object in it) is made of *atoms*. Normal atoms are sort of like little planetary systems, with a heavy nucleus in the middle (kind of like the sun), but *positively* charged, and little light *negatively* charge electrons (sort of like planets) circling around. Opposite charges attract, remember, that's what holds the electron in orbit (Not gravity!)

Normal atoms have EQUAL amounts of "+" and "-" charge; they are electrically neutral. Neutral objects normally don't exhibit any electric forces. (remember **Expt 1.**)

Rubbing two different materials can pull some electrons from one to the other. Silk, e.g. has a stronger affinity for electrons than glass, so rubbing silk on glass tends to pull ("-") electrons OFF the glass, leaving the glass with an excess of "+". The rubbed glass rod is now positively charged. (It is really "-" electrons which LEFT, rather than "+" charges being added, but the end effect is essentially the same - do you see this?)

It is an important experimental observation (as well tested and confirmed as Newton's laws, or conservation of energy and momentum!) that *total electric charge in the universe is conserved*.

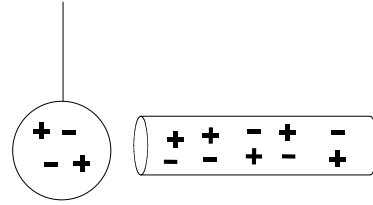
If you rub silk on glass, the amount of "+" charge on the glass (often called "Q", or just "electric charge") is *exactly* equal and opposite the amount of "-" charge on the silk. All you can ever do is separate + and - charges, but the total amount of charge (adding positive and negative, algebraically) is conserved.

That's why calling them "+" and "-" was such a good idea, because adding one "+" to one "-" gives you ZERO, a neutral system. (Similarly, adding 2 +'s to 2 -'s gives 0)

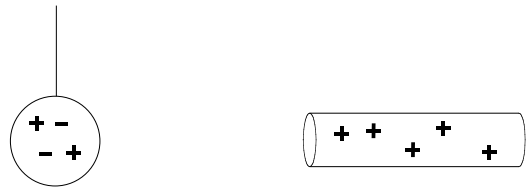
So let's try to understand those "rod and pithball" experiments now, with this microscopic picture.

Expt 1: Both ball and glass rod uncharged.

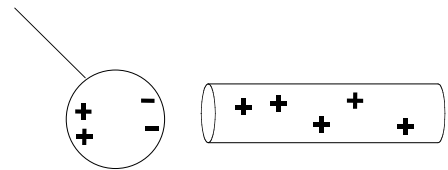
There are no net charges on the objects, so no noticeable forces. Microscopically, there ARE charges on the objects, but equal amounts of plus and minus (canceling, in each and every atom on the object), so they are neutral. There is zero net electric force between the objects.

**Expt 2: Rub glass rod, leaving it positively charged.**

We rubbed some electrons off (with the silk), so the rod is (net) positive. The silk is sitting somewhere out of the picture, with some extra "-"s on it: overall charge in the universe is conserved. (The pithball is still unaffected: the rod is far away)



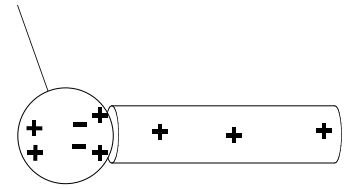
Now, what happens if we bring the rod CLOSER to the ball? Remember, like charges REPEL, opposite charges ATTRACT, so as the rod gets closer, the +'s on the pithball might get pushed just a little bit away, and the "-s on the pithball move closer. There is still no NET charge on the pithball (charge is conserved, they won't just go spontaneously flying off the pithball.) But as we'll see quantitatively very soon, electric forces get much stronger when the charges are closer (just like gravity). So, since the "-" charges on the pithball are closer to the rod, there is an overall (small) ATTRACTION between ball and rod.



(As we'll see later, charges don't really move around much on a pithball. But they can reorient just a little, that's all you need to get the effect described above)

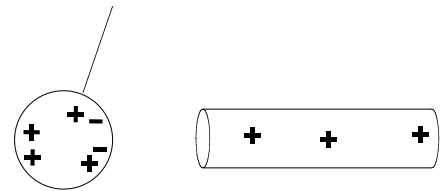
Expt 3: Touch the + charged rod to the pith ball.

Now the +'s on the rod, which repel each other, can spread out. That's what they want to do, they feel repulsive forces, they WANT to accelerate and move as far apart as possible. Before, they were stuck on the rod, but now they can spread out onto the ball. So they'll do that. Momentarily, we might imagine a picture like this for just an instant:

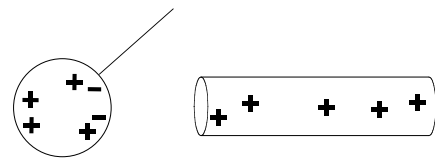


A few of the "+"s have physically moved over onto the ball. We call this "charging by contact" or "electrical conduction of charge", or just direct flow of electric charges.

But this picture won't last long, because notice that the ball is now net positively charged! There are more +'s than -'s, it has a net + charge, and so does the rod, still. And positively charged objects (like charged objects) repel! So, the pith ball will swing away, and we'll end up with REPULSION, something like this:

**Expt 3b: If you rub the rod some more:**

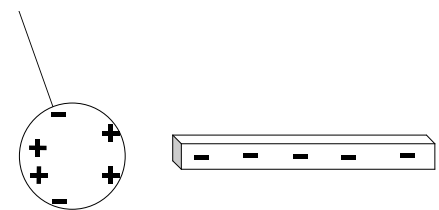
You will rub off even more electrons, leaving the rod even more positive, which will make the electrical repulsion even stronger still! (We will quantify this shortly, but it's a fact that the larger the charge, the more electrostatic force there is.



In this case, we are putting more charge on the rod, but NOT on the ball)

Expt 4: Take away the glass rod, rub a plastic stick with wool, and bring the plastic near the ball.

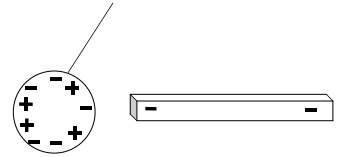
The pith ball hasn't been touched, it still has a net positive charge from before. But now we bring a NEGATIVELY charged object near and opposites attract: the (net) "+" ball moves towards the negative plastic stick.



(By the way, why does plastic get *negative* when you rub it? It's just because plastic has a larger electron affinity than wool, so plastic rubs electrons OFF the wool and keeps them. Electrons went from the wool to the plastic, leaving the plastic "-". (In principle you might have imagined that some "+" charges were rubbed off the plastic and put on the wool: from the pictures and experiments above you can't tell the difference, but that's just not what really happens.)

Expt 5: Finally, touch the plastic stick to the pith ball.

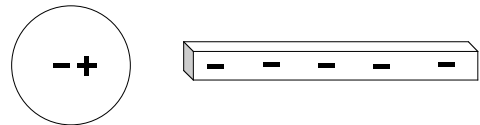
Once again, when they touch, you allow for charging by touch, or contact: a direct conduction of negative charges off the plastic stick, onto the ball. (The negatives want to get as far from each other as they can, like charges always repel!) So now the ball is net negatively charged (a little), and is repelled.



You can imagine countless experiments of this type, which you could perform to decide if this "picture" of what is happening is indeed a good one. (It has survived for over 200 years)

METALS are good conductors of electricity. Charges can easily flow through them. Metals have lots of highly mobile, *nearly* free electrons that can flow wherever they want (in or on the metal). Metals are shiny and malleable, e.g. gold, silver, copper, iron,... If you take even a thin wire of metal and connect two charged objects, charges flow easily through the metal. Like charges repel, opposite charges attract, so the net tendency is for things to try to "neutralize" as much as possible.

Most other materials are INSULATORS. (E.g. wood, rubber, concrete, ceramic,...) Electrons cannot flow through them, they're basically stuck to atoms. Rubbing these materials might drag a few electrons off, but basically they really don't like to move much. You can move charges a little inside insulators, and you can POLARIZE them. That means that they're still neutral, but the charges have separated a little, like this:

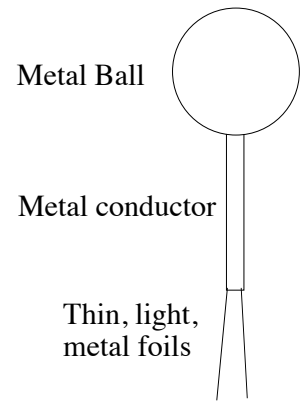


The plus and minus on the (neutral) pith ball can't really move much, but they can separate a little, forming a small DIPOLE with "-" on one side, and "+" on the other. That's because that rod over on the side is pushing the "-" away (likes repel) and pulling on the "+" (opposites attract) Because the "+" is a little closer, the PULL is a bit stronger than the push, and so the pith ball feels a small net attractive force. So a charged plastic rod can attract an insulator a little, by polarizing it, even without touching it (this is what was happening in **Exp't 2** above.)

Most materials are either metals, or insulators. A few special materials are somewhere in between, e.g. Silicon and Germanium, called "SEMI-CONDUCTORS". (More about them later, they're more complicated)

In the first lab, you will play with some experiments like these yourself. You'll use a simple little device for studying charges, called an **ELECTROSCOPE**. It looks like this:

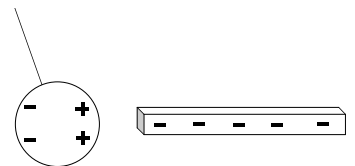
If it's uncharged, the metal foils just hang down, like pendulum bobs. But if you put some electric charges onto the metal ball, they repel one another and spread out all over the whole scope. Thus both foils get charged (with the same charge) and repel each other, and stick out a little. The more charge on the electroscope, the more the *foils* repel, and they stick out farther.



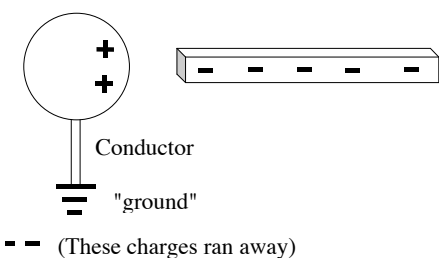
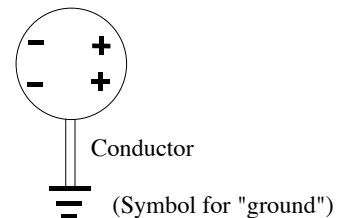
You can NOT tell what the sign of the net charge on the electroscope is simply by looking at it: If it's net +, the foils repel. If it's net -, the foils repel! So, all you can do is see **HOW** charged it is.

There are various ways to charge objects (like electroscopes, or pith balls). The simplest is to touch them with something else charged. Charges will repel each other, and spread out. This is charging by **CONDUCTION**. There's another way, which you can do without touching the objects together, called charging by **INDUCTION**:

Imagine a neutral pith ball we want to charge up. Suppose we have a "-" rod (e.g. plastic rubbed by wool) and we want the pith ball to become net "+". Touching (conduction) won't work, because that makes the ball "-". Instead, bring the rod close, but not touching. It polarizes the ball, like we've seen before. The pith ball is still neutral: if you pull the rod away, the ball is still uncharged.



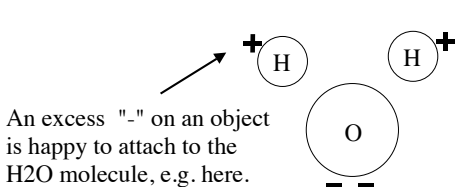
But what if you now connect the ball to the **GROUND** (or earth) with a good conductor. What happens? Remember, those "-"s on the ball want to get as far from the "-" rod as they can, because like charges repel.



Electrons are free to run along conductors, and they can get much farther away by going to the ground. So they do!
(Interestingly, it doesn't matter **WHERE** exactly you touch the conducting wire to the ball: if you give those electrons a chance to run away, they'll take it.)

At this point, the ball is "+." But if you remove the rod, LEAVING the ground wire connected, what would happen? Those electrons that ran away would be ATTRACTED back to the net "+" ball, and go right back, and the ball would be neutral again! So if you want to leave the ball charged "+" in the end, you must FIRST disconnect the ball from ground, and THEN you can pull away the rod. (Notice how we used a "-" rod to leave the ball "+" charged! Think through the process, picture how and why the charges flow at each step. Understand why the order of operations matters!)

A human body can be a decent conductor (I recall a Star Trek where aliens called us "ugly bags of mostly water". Salty water is a conductor!) In the lab, just touching the ball you can serve as the "conductor to ground" in the above pictures. (Metal wires are much BETTER conductors than you, but they don't contact the ball as efficiently, so it's a bit of a tossup.)



Air has water vapor in it, and water is a "polar molecule" - the H's tend to be a little "+", and the O tends to be a little "-" on the other side). Excess charges on objects can "leak" to the water vapor (and thus get to

ground) because the charges can attach to the appropriate side of H₂O molecules and drift away. This is one reason why you get shocked MORE in the winter - the air is dry, and so you stay charged up for longer, and you can also hold more charge before it leaks... (Electrostatics demos should work better winter semester!)

Air itself (without water vapor in it) is an insulator. If you build up enough charge, though, insulators can "break down", and allow the charge to flow. (E.g., you can tear apart N₂ or O₂ molecules in the air, leaving lots of charged "ions" that act sort of like the water vapor above, as a means to hook up and carry away charges to ground) Such a breakdown in air is pretty spectacular, that's what lightning is! Huge charges in the clouds break down a pathway through the air, and quickly dump their charge to the ground. (See the last page of my notes for this chapter if you'd like to learn a little more about lightning)

COULOMB'S LAW:

The explanation of charges above was *qualitative*, but not much later (late 1700's) Charles Coulomb made the story *quantitative*. What exactly is the force between electric charges? First, we must define a *unit* of electric charge. In SI (metric), we call one unit of charge a Coulomb, or C. One C of charge is a LOT. Our glass rods have charges of perhaps 10^{-6} C on them. A single electron has a charge of -1.6×10^{-19} C. We define a symbol "e", $e = +1.6 \times 10^{-19}$ C. ("e" is positive: an electron has charge -e, protons have charge +e. Neutrons have charge 0.) Since electron and proton charges are exactly equal and opposite, atoms (which have equal numbers of electrons and protons) are neutral.

Coulomb discovered (after many difficult experiments) a formula describing the force between any two (pointlike) charged objects a distance r apart:

$$\mathbf{F} = k \frac{Q_1 Q_2}{r^2}$$

This important law is called "Coulomb's law".

The constant "k" is 8.99×10^9 N m²/C². It's a constant of nature.

Example: if you have one C of charge, and another C of charge 1 m away, the force between the charges is $k \frac{Q_1 Q_2}{r^2} = (8.99 \times 10^9 \text{ N m}^2/\text{C}^2) * \frac{(1\text{C})(1\text{C})}{(1\text{m})^2} = 9$ billion Newtons, about a million pounds of force! (I told you a Coulomb was a LOT of charge!)

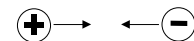
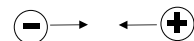
Coulomb's law tells you that if you DOUBLE one of the charges, the force doubles. If you double BOTH charges, the force quadruples!

If you move them twice as far apart, the force gets **weaker** by a factor of $2^2 = 4$. (These are precisely the kinds of experiments Coulomb did, to come up with that formula!)

Coulomb's law gives you the STRENGTH of the force. For directions you can use Ben Franklin's rules:

The force between charges is straight apart if the two charges have the same sign. (Like charges repel)

The force between charges is straight together if the two charges have opposite signs. (Opposite charges attract)



Notice that Coulomb's formula doesn't really care which you call "1" and which "2", the force is the SAME. That's just Newton's III law!

The force ON 1 BY 2 is equal (and opposite) to the force ON 2 by 1,

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

(If you plug the SIGNS of the charges into Coulomb's law, you get a "-" result for opposite charges, a "+" result for same sign charges. But I prefer to look at the pictures and draw the arrows, and use the formula JUST to get the magnitudes)

You should know that many books don't use the symbol "k", instead they replace k with $\frac{1}{4\pi\epsilon_0}$, i.e. k is $1/(4\pi\epsilon_0)$. So in those books,

Coulomb's law is written
$$F_{\text{elec}} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}.$$

We'll use ϵ_0 rarely, if at all, but you should know about it. (I'll give it to you if you ever need it on an exam)

Coulomb's law looks a little familiar, remember

$$F_{\text{grav}} = G M_1 M_2 / r^2, \quad \text{compared to} \quad F_{\text{elec}} = k Q_1 Q_2 / r^2.$$

The formula is indeed very similar.

One important difference: mass is always positive; F_{grav} is ALWAYS attractive. But there are 2 signs for charges: F_{elec} can attract or repel.

The *electric* force between two electrons is $k (-e) (-e)/r^2$.

Just for fun, let's compare this force with the *gravitational* force between them, $G (m_e) (m_e) / r^2$. Let's take the RATIO of these forces:

Dividing, we get $F_{\text{elec}}/F_{\text{grav}} = (ke^2/r^2) / (G m_e^2/r^2)$.

Notice the distance (r^2) CANCELS, and plugging in numbers

(m_e , the electron mass, is given in the front of Giancoli), I get $1.5E36$.

That means electric forces between electrons are a billion billion billion billion times bigger than the gravitational forces between them!!

Moral: electricity is VERY important for subatomic particles, but most physics in that realm can totally neglect gravity!

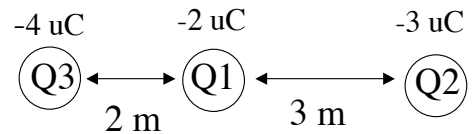
VERY IMPORTANT: Coulomb's law tells you the force between a pair of charges. If you have MORE than two charges, you must draw a picture, and add up all the Coulomb (electric) forces from all the other charges. *You CANNOT just add the magnitudes!* Forces add as vectors!

Example: You have 3 charges arranged as shown.

Let mass $M_1 = 1$ g, and charge $Q_1 = -2$ μC

(In these notes, I'll use the symbol "u" for the Greek letter "micro", or 10^{-6} , it should be a "mu", or μ).

Suppose $Q_2 = -3$ μC . $Q_3 = -4$ μC .



What is the force, and the resulting acceleration, of charge Q_1 ?

Answer: You must consider *both* forces on Q_1 separately, and then in the end add them as vectors. We want $\mathbf{F}_{\text{net}} = \mathbf{F}_{12} + \mathbf{F}_{13}$

(the force on 1 by 2 PLUS the force on 1 by 3)

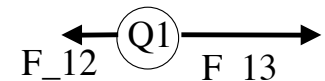
The *magnitudes* are straightforward, by Coulomb's law:

$$F_{12} = k Q_1 Q_2 / (r_{12})^2 = k Q_1 Q_2 / (3\text{m})^2 = 6.0\text{E-}3 \text{ N}$$

$$F_{13} = k Q_1 Q_3 / (r_{13})^2 = k Q_1 Q_3 / (2\text{m})^2 = 18.\text{E-}3 \text{ N}$$

What about the directions? Just draw the arrows!

(remember: like charges repel, opposites attract)



Look *carefully* at my labels and the directions, in this figure.

Do they make sense to you? (Do you see, e.g., that F_{12} is to the *left*?)

Set up a conventional coordinate system where right is "+x", left is "-x", then put the above together to get the total force in the x direction as

$$F_{\text{net}} = -6.0\text{E-}3 + 18\text{E-}3 = +12\text{E-}3.$$

(The answer is NOT the sum of 6 and 18, but the *difference*)

The resulting net force is positive, or in other words, to the right.

Acceleration = $F/m = +12\text{E-}3 \text{ N} / 1\text{E-}3 \text{ kg} = +12 \text{ m/s}^2$. (To the right)

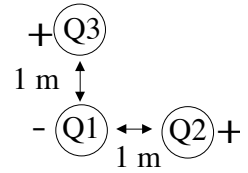
NOTE: All Coulomb forces always simply add up, as vectors. Charges NEVER block each other out. If I wanted to know the force on Q_2 , above, I'd find the forces from Q_1 and Q_3 and add them (as vectors) Q_1 doesn't, in ANY way, ever "block out" the force from Q_3 , even though it's in the middle!

This is called *superposition*. Electric forces "superpose" (add up) on top of each other. If you have several charges and want to know the forces somewhere, you merely ADD (or superpose) *all* the individual force vectors, one from each charge.

Another example:

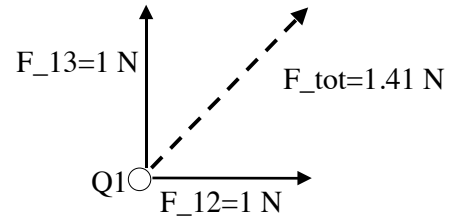
3 charges are arranged as shown. $Q1 = -1.1E-10$ C, $Q2 = Q3 = +1$ C. $Q1$ is 1 m from $Q3$, and 1 m from $Q2$.

What is the net force on $Q1$? (Assume $Q2$ and $Q3$ are pinned down.)

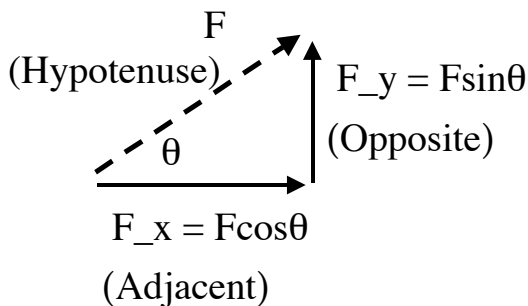


Answer: You must add F_{12} (the force on 1 by 2) and F_{13} (force on 1 by 3). First use Coulomb's law to get each of those separately (I picked the numbers so the forces come out to be 1 N each - check this!)

Then look carefully at the picture, think about the SIGNS, and decide the directions of those forces. I claim we have a force of 1N to the right added to a force of 1N up. The result is NOT 2N (!!) it's $\text{Sqrt}[1^2+1^2]=1.4$ N, in a diagonal direction!



(If the two forces in that example hadn't been perpendicular, I could NOT simply use Pythagorus to find F_{tot} , like above. I'd have to really add the vectors. So, here's a brief **REVIEW** of some of the important aspects of vectors: (But see Giancoli Ch 3.2-3.4 for more details if you need to.)



Any vector, \mathbf{F} , can be described by its x and y components, given by $F_x = F \cos(\theta)$, and $F_y = F \sin(\theta)$. This comes from "SOHCAHTOA": Sin is Opposite over Hypotenuse, Cos is Adjacent over Hypotenuse, Tangent is Opposite over Adjacent.

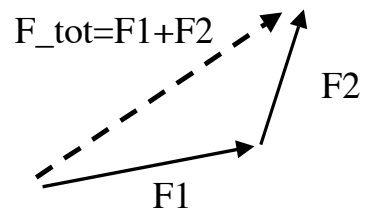
(My convention: a vector \mathbf{F} has magnitude F , written without bold font.)

You can also go the other way: given the x and y components of a vector, the hypotenuse (i.e. the magnitude) is given by Pythagorus:

$$F = \text{Sqrt}[F_x^2 + F_y^2]$$

And, given F_x and F_y , you can find $\theta = \arctan(F_y/F_x)$

When you add vectors, the graphical method is "tip to tail", as shown.



Mathematically, the way to add vectors is to remember that if

$$\mathbf{F} = \mathbf{F1} + \mathbf{F2}, \text{ then}$$

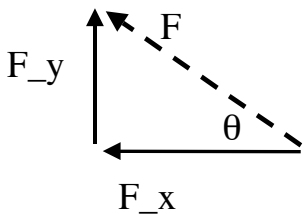
$$F_x = F1_x + F2_x$$

$$F_y = F1_y + F2_y$$

(i.e. the components just add like plain old numbers, but watch your *signs!*)

To add $\mathbf{F1}$ and $\mathbf{F2}$, first find the x and y components of $\mathbf{F1}$ and $\mathbf{F2}$ (with sin's and cos's) and then add these, to get F_x and F_y . Finally, use Pythagorus to get the magnitude of the total.

Often you do have to be a bit careful about minus signs.

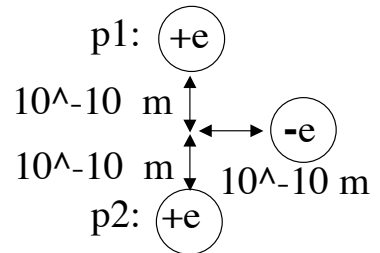


E.g. in this picture,
 $F_x = -F \cos(\text{theta})$
 $F_y = +F \sin(\text{theta})$

(Can you see the reason for that minus sign in the F_x equation? F_x is to the LEFT.)

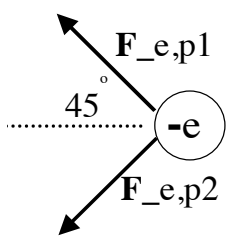
Another example of adding forces:

Two protons ($Q=+e$) and an electron ($Q=-e$) are aligned as shown. *What is the force on the electron?*



Answer: We need to figure out $\mathbf{F}_{tot} = \mathbf{F}_{e,p1} + \mathbf{F}_{e,p2}$

(the force on the electron from the first proton PLUS the force on the electron from the second proton.)



It's always helpful just to draw those forces, to help guide the eye. Since proton and electron have opposite charges, the forces are both attractive on the electron. You should convince yourself that the distance from the electron to *either* one of the protons is

$$r = \text{Sqrt}[10^{-10}\text{m}^2 + 10^{-10}\text{m}^2] = 1.4 \cdot 10^{-10}\text{m}$$

(Can you also see that the force vector is tipped at 45 degrees?)

Coulomb's law says $F_{e,p1} = k e e / (1.4E-10m)^2 = 1.2E-8 \text{ N}$

By symmetry $F_{e,p2}$ is the *same* (can you see this?)

But $\mathbf{F}_{tot} = \mathbf{F}_{e,p1} + \mathbf{F}_{e,p2}$ is NOT just $2*1.2E-8!$

You must add *vectors*, not magnitudes. To do this, use components:

$$\begin{aligned} (F_{tot})_x &= (F_{e,p1})_x + (F_{e,p2})_x \\ &= -(1.2E-8N)*\cos(45) + -(1.2E-8N)*\cos(45) = 1.7E-8 \text{ N} \end{aligned}$$

$$\begin{aligned} (F_{tot})_y &= (F_{e,p1})_y + (F_{e,p2})_y \\ &= +(1.2E-8N)*\sin(45) + -(1.2E-8N)*\sin(45) = 0. \end{aligned}$$

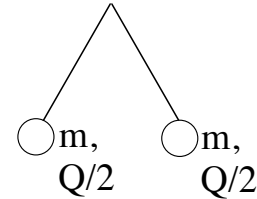
Think about why $(F_{tot})_y=0$. Think about signs, look at the picture!

The final result is thus $F_{tot} = 1.7E-8$, pointing directly leftwards.

Yet another example:

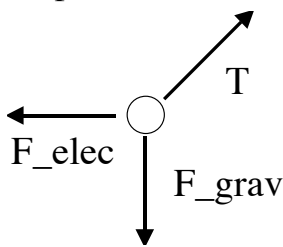
An electroscope is designed as two hanging pith balls, (each of mass m) as shown. A total charge $+Q$ is put on the scope, and has split itself evenly between the two balls. The balls are a distance " r " apart. (that's the distance between centers)

What is the tension in the wire holding one of the balls?



Answer: This problem is like those old Chapter 4 example of Newton's law (with electric forces too.)

The method to solve it is to *draw a force diagram* for eith one:say, the left-hand pith ball. There are THREE forces on it:



gravity (down, $|\mathbf{F}_g|=mg$),
electric (left, $|\mathbf{F}_{elec}| = k (Q/2)(Q/2)/r^2$) and
tension \mathbf{T} (points in the direction of the wire).

Draw the force diagram, like this, and use Newton II which says that in equilibrium (when nothing is accelerating, the balls just hang there) $\mathbf{F}_{tot}=0$, i.e.

$\mathbf{F}_g + \mathbf{F}_e + \mathbf{T} = 0$ or, solving for \mathbf{T} , which is what we're after:

$$\mathbf{T} = -(\mathbf{F}_g + \mathbf{F}_e)$$

(By symmetry, the magnitude of tension in the other wire will be the same, although the direction is different.)

In this problem, F_g and F_e are perpendicular, so adding them as vectors is relatively easy, the magnitude of the sum is, by Pythagorus,

$$T = \text{Sqrt}[F_g^2 + F_e^2] = \text{Sqrt}[\{mg\}^2 + \{k (Q/2)(Q/2)/r^2\}^2]$$

(Can you see this? If not, draw $F_{elec} + F_{grav}$ for yourself!)

ELECTRIC FIELDS:

Electrostatic forces are (like gravity) "Action at a Distance". It's a strange idea! One way people have developed to help get more comfortable with this idea is another concept: *force fields*. (Michael Faraday invented this in the 1800's.) It's strange too, but very useful.

A charge Q produces forces on ANY other charges, ◦ q
anywhere in the universe. Imagine putting a tiny charge q
somewhere, a distance r away.

Little q is pushed (by Coulomb's law), it feels a force ⊙ Q
 $F_{qQ} = kqQ/r^2$. (Read that as the Force on q by Q)

Faraday argued there's an "electric force field" surrounding Q .
It's like a "state of readiness" to push any other charge that happens to come along. The field isn't exactly physical; it's not something you can taste, or smell, or see. It just manifests itself if you put a charge (any charge) q somewhere (anywhere!) It's a curious but *very* useful concept.
(Thus, Faraday doesn't quite think about electric forces as "action at a distance": it's not so much that Q is pushing on q , far away, it's more like Q produces an electric force field everywhere, and it's that field *right wherever q is*, that finally pushes on q .)

Q_2
 Q_1
 Q_3

 Q_4

- Suppose you have a bunch of charges. If you bring in one more little charge, " q ", anyplace you like, say the little black spot, it will feel an electric force in some direction.
(You could figure it out by using Coulomb's law, adding the four separate forces, as vectors. You'll get **SOME** answer.)

But again, you can instead think of an "electric force field" at the point, which tells you exactly which way a little "test charge" q will be pushed IF you put it there. The E-field is present *whether or not* you bother putting q there. It is present at any (and every) point in space.

I'm going to first just define the E-field mathematically. (I'll justify and explain this definition on the next page) It should certainly be a vector (i.e. it has a size and a direction). It should tell you the force on ANY test charge q . We define $\mathbf{E} = \mathbf{F}/q$, or more carefully:

$$\boxed{\mathbf{E}(\text{due to } Q, \text{ at point } p) = \mathbf{F}(\text{on test charge } q, \text{ at point } p) / q .}$$

16-18 (SJP, Phys 2020)

From this def., the units of E will be Newtons/Coulomb, or N/C

I did NOT define $E = F$, instead I divided out the "test charge" q . Why? Because the E field is a property of space at the point p . It shouldn't matter how much charge I use to test for it. If I bring in " q ", I'll feel some force F . If I bring in " $2q$ ", Coulomb's law says I'll feel exactly twice the force, $2F$. But since $E=F/q$, in the second case (twice the force, twice the charge) the factors of two cancel, and E comes out the exact *same* no matter WHAT " q " is! That's what we want - E has some value at every point in space, whether or not there's any charge physically at that spot - and you can use it to *figure out* the force on *any* test charge of any size that you bring to that spot.

Analog/Interlude, to help motivate E fields:

Go to King Sooper's and buy some sugar. On day 1 you buy 2 pounds, and pay \$4. On day 2 you buy 3 pounds, and pay \$6. What you pay depends on how much you buy. So, it might seem complicated to try to predict how much you'll have to pay tomorrow, when you buy yet a different amount.

But then you notice that

$$\text{\$spent}/(\text{amount bought}) = \$4/(2 \text{ lbs}) = \$6/(3 \text{ lbs}) = \$2/\text{lb}.$$

There is a simple, underlying, universal, common UNIT PRICE.

So now you immediately know how much you'll pay, no matter *how* much you buy: Price = $(\$2/\text{lb}) * (\text{amount you buy})$

It's basically the same with E fields: E is like the "unit price per pound" (only here it's really "unit force per charge")

The force (price) on a test charge q seems complicated at first: different if you put in different q 's. But then you notice that

$$\text{Force} = (\text{unit price}) * (\text{amount}) = E * q .$$

Knowing E you can easily figure out the force on ANY q now!

That's one reason why E is useful - it's like knowing the "unit price" at the store.

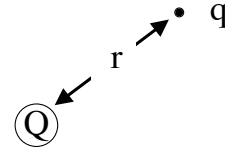
Bottom line: if you know what E is at any point in space, you can immediately figure out the force on ANY charge " q " placed at that point, because

$$\boxed{\mathbf{F} = q \mathbf{E}.}$$

(Just multiply both sides of the equation defining E by " q " to get this.)

Computing the \mathbf{E} field is pretty much the same as finding the force, just remember to divide out q . E.g., if you have a single charge Q somewhere, and want to know the \mathbf{E} field at some other point, just imagine putting a test charge " q " at that point, and use Coulomb's law:

F (on q , at point p) = $k Q q/r^2$, so $E = F/q$, or



$$\mathbf{E} = k Q/r^2.$$

(For a single charge Q , this tells $|\mathbf{E}|$, a distance r away)

The direction of \mathbf{E} is *radially outwards*, away from Q , if $Q > 0$.
 The direction of \mathbf{E} is *radially inwards*, towards Q , if $Q < 0$.

(That's just the statement that like charges repel, opposites attract)

Since $\mathbf{E} = \mathbf{F}/q$, the direction of \mathbf{E} is the same as the direction of the force on a *positive* test charge.

(To ponder: if \mathbf{E} is to the left at some point in space, what is the direction of the force on a + charge there? Now, how about on a - charge? Answers on next page)

\mathbf{E} is defined at all points in space, and depends **ONLY** on Q , not on the test " q ". Regions of space have a "possibility" of providing electric forces, even if no test " q " happens to be there at the moment. Similarly, gravitational fields exist at all points. There's a "g-field" in this room. It's downwards everywhere. If you **DO** release a pebble at some spot, the pebble immediately starts to accelerate down because of the "g-field". But the g-field is still there whether or not you put a "test pebble" at some spot to check.

By the way, you can't "see" E-fields, but you **CAN** see time-varying E-fields - that's precisely what light is! We'll talk about this more later .

If you have many charges, finding \mathbf{E} is little more than an exercise in adding vectors: you just imagine putting a test charge q down, and then, as before:

$$\mathbf{E} = \mathbf{F}_{\text{net}}(\text{on } q) / q.$$

Finding \mathbf{E} (or equivalently \mathbf{F}_{net}) can be a chore if there are lots of charges. But in the lab, finding \mathbf{E} is fairly easy -just put a test charge there, and measure the force on it! That's it.

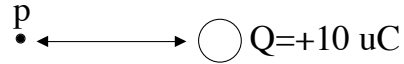
Directions: $\mathbf{F} = q\mathbf{E} \Rightarrow$ the direction of \mathbf{F} is the same as \mathbf{E} (if $q > 0$)

If e.g. \mathbf{E} points west (and if q is positive) \mathbf{F} is also west.

However, if q is negative, $q\mathbf{E}$ would be east (recall, multiplying a vector by a negative number flips the DIRECTION of the vector!)

Example: Put a +10 uC charge at the origin.

Look at a point p some distance to the LEFT of the origin.



What is the direction of \mathbf{E} there?

What is the direction of force on a + test charge there?

What is the direction of force on a - test charge there?

Answers: Remember, \mathbf{E} fields point away from + charges.

At point p , \mathbf{E} points left.

The direction of force on a "+" test charge is the same as \mathbf{E} , left.

(Like charges should repel. Q and " q " are both "+", they repel, it all makes sense)

The direction of \mathbf{F} on a "-" test charge opposes \mathbf{E} , i.e. to the right

(Makes sense. The q and Q are now opposite, so they attract)

If the charge " Q " had been -10 uC in the previous example, ALL of the answers would be reversed:

In that case the \mathbf{E} field at p would point right, TOWARDS the "-" charge.

The force on a "+" test charge would be the same direction as \mathbf{E}

(that's always true!), i.e. to the right. (Makes sense. The test charge is now different from Q , so the force is attractive.)

The force on a "-" test charge opposes \mathbf{E} (always true!) In this case, left.

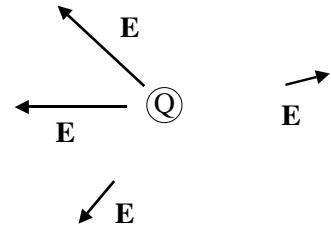
(Again, makes sense. The test charge is now the same as Q here, both are "-", so the force between them is repulsive. At point p , that means it points left, away from Q .)

This is all a bit confusing to write, but it's really not that bad if you just think about the above examples a little.

Example: Suppose we put a positive charge at the origin. Draw the E field at a few (randomly chosen) points.

The E field points radially outward (radially means directly away from the Q, like a "radius" of a circle.)

The farther away you get, the weaker it is (because force, and thus E, drops off like $1/r^2$)



This kind of drawing is a little tedious, and picking points at random doesn't seem like the best way of drawing an E field. But alas, E is defined *everywhere*, and it's a vector, so you really *can't* draw the field in any easy way!

There is a neat pictorial trick that people use to try to "visualize" E-fields. You draw "lines of force". This means, instead of drawing vectors at points, you draw lines, called field lines.

Field lines start and end at charges (always!)

The direction of the lines (really, the *tangent* to the lines) at any point tells you the direct of E. (The lines have arrows, to eliminate any ambiguity in direction)

The more lines you have (the *denser* they are), the *stronger* the E field. (If you double the charge, you double the "density of lines".)

Lines never cross (if they did, E wouldn't be defined at the crossing point. But there must always be a unique force on any test charge!)

Here are several examples.

Example 1: A single + charge.

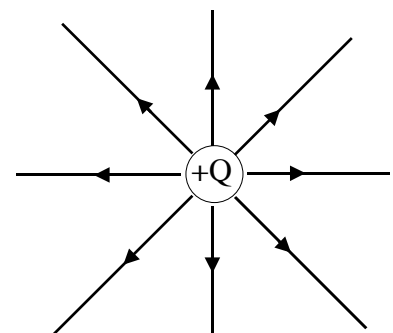
The location of the arrows is not significant.

The arrows all point away from the + charge (E fields go AWAY from positive charges)

The lines are all radially outward.

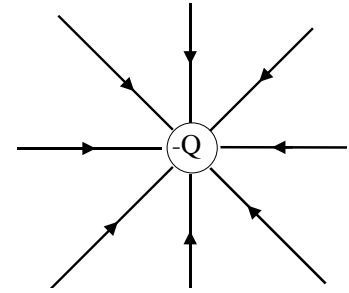
Notice that the lines are less dense further away from the charge, which tells you the E field is weaker out there.

(this is just Coulomb's law, E drops like $1/r^2$!)



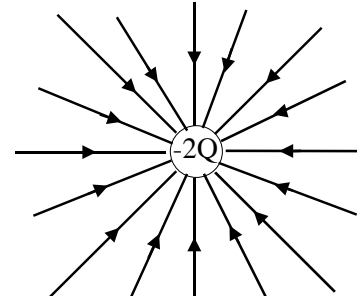
Example 2: A single - charge.

The arrows all point *towards* the - charge
 (E fields go towards negative charges)
 The lines are all radially inward.
 Notice that the lines are again less dense further away,
 (this is just Coulomb's law, E drops like $1/r^2$!)



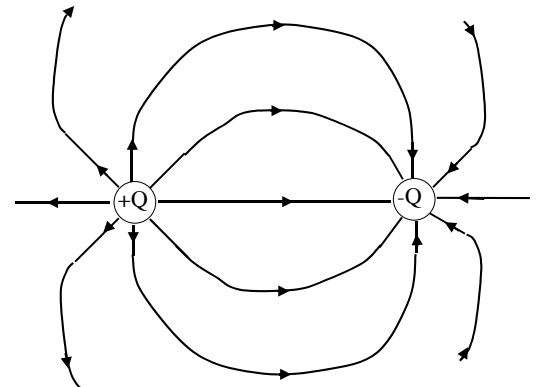
Example 3: A single charge of -2.

Just like the last one, but the density of lines is twice as much, because the charge is twice as big.



Example 4: A dipole
 (that means "two poles" : + on left, - on right)

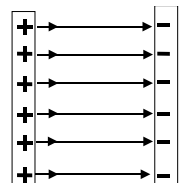
The lines are curved now. At any point in space, the E field is given by the *tangent* to the line. (Giancoli Fig 16-29a is better drawn version of this same fig!)
 As always, the arrows tell you the direction of E, which is the same as the direction that a *positive* test charge would move, if released at that point. (A negative test charge would go the opposite way!)



Look at the picture and convince yourself that the directions make sense: think about which way a "+" test charge would want to go given those two charges +Q and -Q.

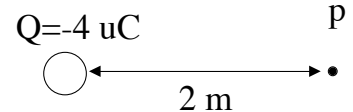
Example 5: Field lines between two infinite "lines" of uniform charge.

The lines are uniform density, the E field here is the same size and direction everywhere! Wherever a test charge may be, it is pushed to the right with the same force anywhere... (It may not be totally obvious to you that the E field is *uniform* here, but it is. This is a case where working it out from Coulomb's law is hard, because there are an infinite number of charges in the story! There are fancier tricks that can be used to deduce E fields which we won't cover in this course, that make this a surprisingly easy problem to solve quantitatively)



Example:

A -4 uC charge is at the origin, as shown. What is the E field at a point 2 m to the right of the origin?

Answer:

The force on a test charge q at point p would be $k Q q/r^2$
(that's just Coulomb's law)

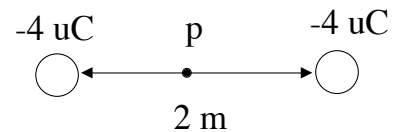
$E = F/q$, so

$$E = k Q/r^2 = (9 \times 10^9 \text{ N m}^2/\text{C}^2)(-4 \times 10^{-6} \text{ C})/(2 \text{ m})^2 = -9 \times 10^3 \text{ N/C}$$

The direction of the E field is towards the (" $-$ ") charge; to the left.

(A " $+$ " test charge at p would move to the left, opposites attract)

Example: Two -4 uC charges are set up as shown. What is the E field right in the middle, between them?



Answer: The force on a " $+$ " test charge at point p would be to the RIGHT from the right-hand charge, and to the LEFT from the left hand charge (opposites attract, the test charge I'm imagining is $+$.) Since I picked p in the middle, the forces exactly *cancel*, $F_{\text{net}}=0$, so $E=0$ at p . If I shift a little away from the center, however, the forces won't exactly cancel, and there will be a nonzero E field. To find it at any point "off-axis" (anywhere besides on the line shown between the charges) I'd have to add the two forces as *vectors*, a bit of a pain.

In the end, the field lines look exactly like the ones shown in Giancoli 16-29b (except my 2 charges are " $-$ ", so the arrows would all be reversed.)

Notice in that figure there is zero density of lines right in the middle, which corresponds to the answer $E=0$ we just got.

Think about what you get a little away from the center, and see if the field lines in Giancoli's picture make physical sense to you.

Field lines are very useful - they're a pictorial way to "visualize" the force that any test charge would feel anywhere in space. But, if you want to be quantitative, you usually need to compute (or be told) the value of E fields, it's hard to get numerical values from field line graphs.

E fields and metals

I said \mathbf{E} is defined everywhere in space. What is the electric field \mathbf{E} *inside* a chunk of metal? I claim, in steady state, the answer must be zero! Why? Because metals are filled with free electrons that can move around (they conduct) - so if \mathbf{E} was NOT zero at some point inside, then the force on an electron there would be nonzero - the electron would start to move. They would continue to move, building up a "counter" \mathbf{E} field, a canceling field, until finally everything settles down, nothing moves, $\mathbf{E}=0$ throughout the metal (so $\mathbf{F}=0$) No more motion! (Steady state MEANS everything has settled down, no charges are moving, so they feel no net force on them anymore)

The same argument tells you that, in steady state, if there's a nonzero \mathbf{E} field outside of a chunk of metal, it will always be perpendicular to the surface right at the edge. If it wasn't perpendicular, there would be a component of \mathbf{E} parallel to the surface, and electrons right at the surface would then flow (surface currents), until that piece of the \mathbf{E} field got cancelled out. (Look back at the figure I drew in Example 5. If those charges resided on metal plates, the \mathbf{E} field lines would have to be perpendicular to the plates - just as I drew it)

Some end of chapter comments:

There are tons of practical consequences of the above simple statements. Just one example: if you make a metal box and put it in a region of large \mathbf{E} field, the electrons in the metal quickly (essentially instantly) rearrange on the surface to make $\mathbf{E}=0$ everywhere INSIDE the box. If you then hollow out the box, it makes no difference, $\mathbf{E}=0$ inside, still. If there's a lightning storm (large, potentially fatal \mathbf{E} fields all around you), a relatively safe place to be is inside a metal car, because the \mathbf{E} field INSIDE the car (box) is zero. (It would be better if the car was *entirely* metal) If you're in a fancy fiberglass-body car, too bad - fiberglass doesn't conduct, it's not a metal, so the above arguments fail to hold. At least you'll stay dry...

Giancoli ends this chapter with a cool section on electric forces in DNA: biophysics is an incredibly interesting and rapidly growing field of study. The interplay of basic laws of physics in biological systems allows us to understand, and control, living systems in ways that were unimaginable just a few years ago. Electric fields make forces on charges, shaping and controlling \mathbf{E} field allows you to manipulate charged objects (which ultimately means just about everything!)

Aside: a little lightning physics: (<http://thunder.msfc.nasa.gov/primer>)

The story of lightning is pretty complicated, but here's a summary of some of the basic ideas of "ordinary" lightning: In a thundercloud, "+" charges tend to get swept up to towards the top, "-" are down low (the *mechanism* for this charge separation is not yet fully understood - nice to know that the physics of something as elemental as lightning still has some scientific mystery associated with it!) and the large electric fields near those charges begin to break down the air molecules, forming paths of "ionized" (broken down) air, which is a much better conductor of electricity than normal air. This process is called forming a "stepped leader", because it tends to occur in steps or jumps of 50-100 m. The path is pretty random, although it tends to work its way down towards the ground (which is inductively charged the opposite way). Meanwhile "positive streamers" (which are like the stepped leaders only they're working their way UP from the ground, usually from tall or sharp objects - we'll see why later) are forming near the ground. When a "-" stepped leader path meets a "+" streamer path, there's now a *continuous* path of good conductivity between ground and cloud, and BOOM there's a lightning bolt.

The "flash" arises from the high temperatures associated with the energy being dumped into the air from all those charges passing by, and also from electrons getting re-attached to molecules after they're stripped off. The flash usually works its way UP from the point where the leaders joined the streamers, up towards the cloud (so in this sense, lightning goes UP from the ground to the cloud!) There can be one flash, or a couple of quick strokes through one path (typically 4, each lasting about 30 microseconds). The "boom" of thunder is the shock wave of the air as this superheated column (20,000 C, hotter than the surface of the sun!) expands and then recontracts. Since sound travels so much slower than light, you hear the boom later than you see the flash. If you count, you can estimate how far away the bolt was - sound goes about 1 mile in 5 secs...

