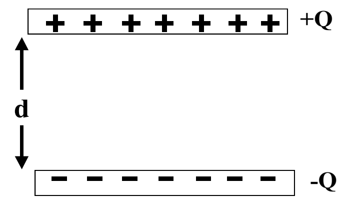
CAPACITORS:

Any time you have two metal surfaces near each other, separated by air (or an insulator), you have a capacitor. My favorite example is the "parallel plate capacitor". We've already looked at it quite often.

A capacitor can be charged or not. When we say a capacitor has "charge Q", we mean +Q on one plate, and -Q on the other, like in the picture here. (Of course, the *net* charge is really zero, but we still say it's "charged"!)



If we say "a capacitor has voltage V", we really mean it has a difference in voltage between the plates $\Delta V = Ed$. We drop the "Δ" for convenience, i.e. V=Ed. (It's a bad notation, but standard)

The more charge Q you put on the plates, the stronger the E field between the plates, hence the bigger the voltage drop. In fact, if you double Q, you discover that E doubles too (and so must V.)
That means $Q \propto V$, or $\boxed{Q=CV.}$
C is a constant, and we call it the "*capacitance*".
(Of course, you can also turn it around and say C= Q/V.)

A capacitor can store charge (and energy) for you - that's why they're useful. Think of a closet with lots of bowling balls up on the shelf at high gravitational potential. You can let them fall down whenever you like, to do lots of work (or damage) later. Your closet shelf has only a limited capacity to hold bowling balls. Similarly, metal plates of a given voltage difference have a certain limited capacity to hold charge, and that's what "C" tells you.

A big capacitance (large C) means you hold lots of charge with a little voltage. A small C means you can only hold a little bit of charge, for a given voltage (a bit like a small shelf in the closet). Of course, given C, you can make Q as big as you want by making the potential bigger. (So our "closet" metaphor breaks down...)

The units of capacitance are [Coulombs/Volt] = C/V .
We call 1 C/V = 1 Farad = 1 F.
(The units are getting confusing now, because 1 V = 1 J/C, so
1 F=1 C^2/J too.)

• 1 F is a really big capacitor! It would hold 1 C (a lot of charge!) with only one volt difference between the plates. Most normal capacitors are more like micro or nano Farads.

For large parallel plates of area A, the E field between them is given by a simple formula (which I won't derive)

$$\boxed{E = 4\pi k \frac{Q}{A}}$$

The formula says the more Q you have, the bigger E is. (That at least makes some sense, doesn't it?) Also, the bigger the area of the plates, the more the charge is spread out, and that should weaken the E field, so the A dependence in that formula also seems reasonable. The formula doesn't depend on "d" (how far apart the plates are.) This is perhaps a bit surprising, but it's correct: E is UNIFORM, it doesn't change with distance from either plate.

Earlier, we defined $\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \cdot 10^{-12} \frac{\mathrm{C}^2}{\mathrm{Nm}^2}$, so you can also say $E = \frac{1}{\varepsilon_0} \frac{Q}{A}$.

We know V=Ed for a parallel plate capacitor. Combining this with the formula for E (just above) gives $V = 4\pi k \ \frac{Qd}{A} = \frac{1}{\varepsilon_0} \frac{Qd}{A}$.

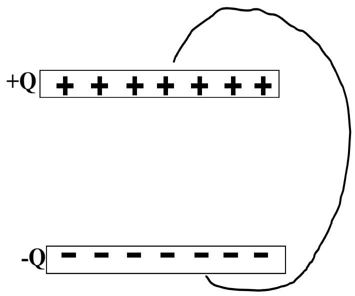
Finally, C = Q/V (that defines C.) So that last equation for V yields (can you check the algebra yourself?)

$$\boxed{C = \frac{1}{4\pi k} \frac{A}{d} = \varepsilon_0 \frac{A}{d}}$$

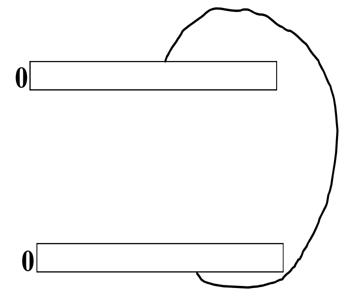
(This formula is *only* for parallel plate capacitors)

Since V=Ed, and E is uniform, the *closer* the plates, the smaller the voltage drop. Smaller V (for a fixed charge) means C=Q/V is BIGGER. If you can gets the plates closer together, you get a better (bigger C) capacitor! So that "1/d" dependence in the formula makes some physical sense.

Similarly, if the area of the plates is bigger, the formula says you have more "capacity" to hold charge. That makes good intuitive sense to me; you have more area to spread those charges out onto, so you can easily hold more charge.

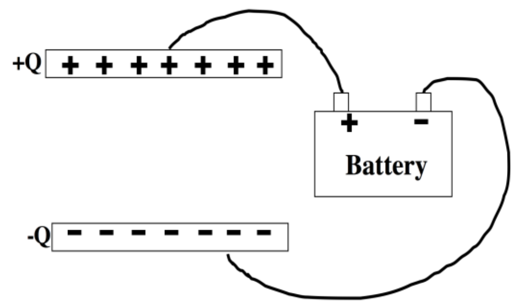


If you connect a wire up to a capacitor (like on the left) the charges on the capacitor plates are free to travel. The + and - attract one another, the capacitor will discharge quickly, possibly with some nice sparks. There's energy in there!



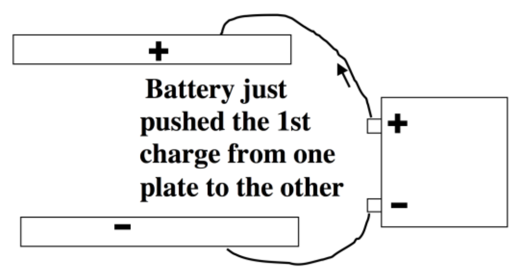
<u>It takes *work* to charge up a capacitor!</u>

Consider e.g. a battery charging up a capacitor. A battery "wants" the plates to be at a certain voltage difference V. (E.g., a 12 V car battery always wants things connected to it to be 12 V apart.) To do this, it forces charges to go from one plate onto the other plate, fighting against an E field all the while.



How much work does the battery do, charging the capacitor up from 0 to V?

The first charge moves easily, it takes almost no work, because at first the plates have no charge. (There's no E field, and charges are pretty much free to move around wherever they want.)



On the other hand, if the capacitor is fully charged (with a voltage V across it) you'd expect that moving Q charges across that voltage V would cost energy = Q*V. The real answer is the average of 0 and QV (on average, while charging , the voltage is V/2), so

$$\boxed{\text{Energy to charge up} = \text{Energy stored in capacitor} = U = QV/2\ .}$$

The symbol "U" is, for some reason, often used for energy.
In this case, it's electrical potential energy.
(Don't confuse "potential energy" with "potential"!)

Since Q=CV, we can rewrite this as U = CV^2/2, or U=Q^2/(2C).
Depending on what's held constant, these forms are sometimes useful.

Example: A 12 V car battery is hooked up to a capacitor with plates of area 0.3 m^2, a distance 1 mm apart.
How much charge builds up on the plates?

Answer: The battery will charge the capacitor up until the voltage is 12 V. Since Q=CV, we only need to find C. But we know that
$C = \varepsilon_0 A/d$. So we're all set:

$$Q = CV = \varepsilon_0 \frac{A}{d} V = 8.85 \cdot 10^{-12} \frac{\mathrm{C}^2}{\mathrm{Nm}^2} \left(\frac{0.3\ \mathrm{m}^2}{10^{-3}\ \mathrm{m}} \right) 12\mathrm{V}$$

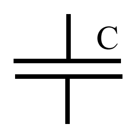
$$= 32 \cdot 10^{-9} \frac{\mathrm{C}^2}{\mathrm{Nm}} \mathrm{V} = 32\ \mathrm{nC}$$

(I used 1 V=1 J/C: check for yourself that that nasty combination of units simplifies like I claimed, to Coulombs)

How much energy is stored in the capacitor now?
U=Q*V/2 = 32E-9 C * 12V / 2 = 0.2 micro Joules. Not so much.

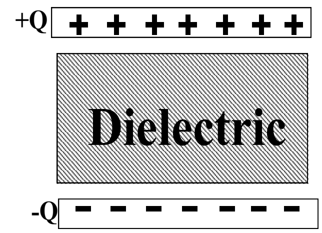
Aside: *Where* exactly is the energy stored, in a capacitor?
The answer is that it's stored in the E field! Wherever you have electric fields, there is stored energy. The energy is stored in the "space" between the plates, in the form of electric field energy.

In diagrams, we will represent capacitors with a simple symbol like this, even if the capacitor in reality isn't physically "parallel plate". (Sometimes the "C" is left off too)



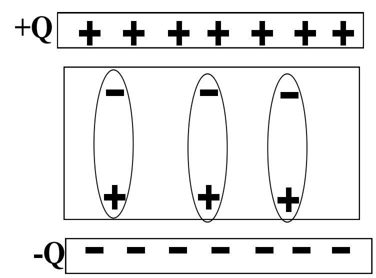
DIELECTRICS:

Any insulating material (paper, plastic, etc) can be called a dielectric. Most real capacitors have dielectric materials between the plates. This helps to keep the plates apart! After all, the plates are oppositely charged, and so attract each other strongly. If they ever touched, the capacitor would discharge, or "short out", and be useless.



So, with a dielectric in there, you can make "d" quite small, and remember that helps make the capacitance bigger.
But the main reason for putting a dielectric in there is something different: it actually *decreases* the E field in the capacitor! *Why?*

Real materials (like dielectrics) are "polarized" by a strong E field. That means the E field in the capacitor effectively pulls some "-" charge towards the top of the dielectric (nearer the "+" plate), and some "+" towards the bottom (nearer the "-" plate) In the region throughout the middle, it looks like some of the "Q" around you has been weakened, or shielded, or canceled out.



The net effect is that E is reduced throughout the dielectric. And since V=Ed, the voltage between the plates is reduced.

Since C=Q/V, if Q is fixed and V is reduced, C gets bigger.
In this way, *dielectrics make the capacitance bigger.*

(Your capacitor can hold MORE charge for a given voltage, with a dielectric in there, because the surface of the dielectric effectively shields out some of the E field from the middle region)

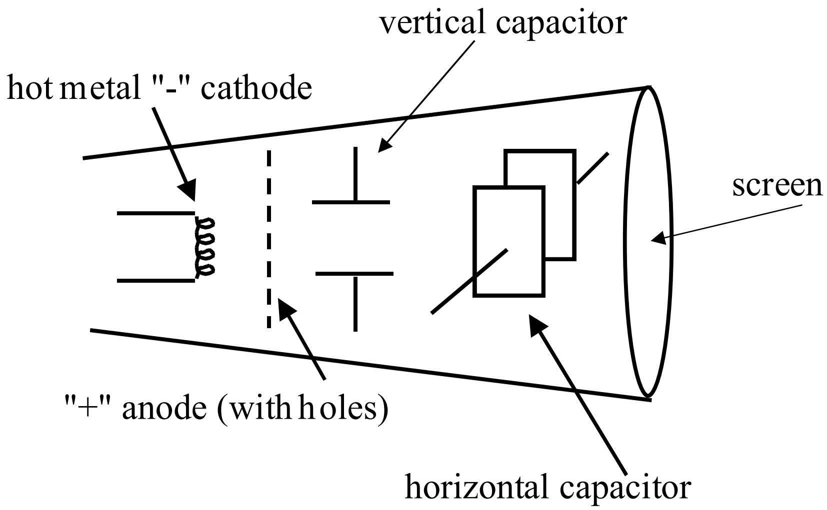
Recall the formula $C = \frac{1}{4\pi k}\frac{A}{d} = \varepsilon_0 \frac{A}{d}$.

That's if there's no dielectric. If there IS a dielectric, we just argued C is bigger. It turns out for most dielectric materials, C is bigger by some constant factor which depends only on the material, i.e.

$$C_{\text{with dielectric}} = K\frac{1}{4\pi k}\frac{A}{d} = K\,\varepsilon_0 \frac{A}{d}$$

K will be some constant for any given dielectric material (Giancoli has a table) Bigger K means you get a bigger C.
(Paper, e.g. has K=3, roughly.)

Capacitors are everywhere: in circuits, radios, computers, TV's...
It's handy to have a "charge storage" device!
The final (brief) topic of this chapter is a real world application of capacitors: *CRT*'s, or "cathode ray tubes".



A "cathode ray" is an old-fashioned name for electrons. The "cathode" is a heated piece of metal, set at a very low voltage. The "anode" is set at a high potential (so the "cathode" and "anode" basically form a capacitor) Electrons boil off the hot cathode, and then they are accelerated towards the high voltage ("+" charged) anode. The anode is a grid with lots of holes, so many electrons can fly right on by and cruise towards the screen.

They pass through a pair of capacitors (vertically and horizontally oriented) which have a voltage that "sweeps". As electrons pass through these capacitors, they feel the force from the E field, which bends the path of the electron. Since the voltage is swept, the electrons are also swept. They fly on by, and hit the screen, which glows where the electrons hit. So you see them sweeping by, and this makes the whole screen glow. By turning the anode on and off, you can make the electrons go through or not, thus making bright or dark spots, which allows you to make an image.

This device is used in old computer monitors and TV, oscilloscopes, EKG traces, you still see them all over. We'll play with an oscilloscope in lab, where you can control the voltages by hand and mess around with manipulating electron beams...